Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.242, Fall 2004: MODEL REDUCTION [*]

Test 2 preparation problems[1]

Problem 7.1

Construct a strictly proper rational transfer function $G = G(s)$ with no poles in the closed right half plane for which there exists no proper first order transfer function $\hat{G} = \hat{G}(s)$ such that

$$G(0) = \hat{G}(0), \quad G'(0) = \hat{G}'(0), \quad G''(0) = \hat{G}''(0).$$

Problem 7.2

Strictly proper transfer function $\Delta = \Delta(s)$ with no poles in the closed right half plane is such that $\Delta(1) = 1$, $\Delta(2) = a$, and $\Delta(0) = 100$, where $a \in \mathbf{R}$ is a parameter. Find the maximal lower bound for the H2 norm $\|\Delta\|_{H2}$, defined by

$$\|\Delta\|_{H2}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\Delta(j\omega)|^2 d\omega.$$

Problem 7.3

Is it true that every proper transfer function $G = G(s)$ with no poles in the closed right half plane has a state space model in which the controllability Gramian is an identity matrix? Give a proof or a counterexample.

[1] Version of November 1, 2004.

Problem 7.4

For which $a \in \mathbf{R}$ is it possible to find a proper rational transfer function $G = G(s)$ with no poles in the closed right half plane such that $\|G(s) - 1/(s^2 - 3s + 2)\| \le a$?

Problem 7.5

$G = G(s)$ is a proper 10-th order rational transfer function with no poles in the closed right half plane, and has Hankel singular numbers $\sigma_{2k-1}(G) = \sigma_{2k}(G) = 1/k$ for $k = 1, 2, 3, 4, 5$. $\hat{G}_6$ is its (uniquely defined) 6-th order strictly proper Hankel optimal reduced model. Find lower and upper bounds for Hankel singular numbers $\sigma_k(G - \hat{G}_6)$ of the error system.

Problem 7.6

Is it true that maximum $f(x) = \max\{f_1(x), f_2(x)\}$ of two quasi-convex functions $f_i : \mathbf{R} \mapsto \mathbf{R}$ is always quasi-convex? Give a proof or a counterexample.

Problem 7.7

Matrices A, B, C, where A is a Hurwitz matrix, are such that

$$C(sI - A)^{-1}B = \left(\frac{s-2}{s+2}\right)^{1000} - 1.$$

What will be the result of minimizing a in the semidefinite program

$$\begin{bmatrix} HA + A'H & HB & C' \\ B'H & 2a & y \\ C & y & 1 \end{bmatrix} > 0,$$

where $a, y \in \mathbf{R}$ and symmetric matrix $H = H'$ are decision parameters?