Problem Set 1

This assignment is a warm-up exercise (there was little taught in the class, so far), and contains only "theory" tasks (i.e. problems with "x.xT" identifiers), intended to check your understanding of the material from Chapter 2 of the lecture notes.

Task 1.1T

Answer the following Yes/No questions, and provide brief reasoning.

(a) System $S \in S_{CT}^{1,1}(\mathbb{R})$ (i.e., a CT single input single output (SISO) system with boundary conditions set $\mathbb{R}$) is defined by

$$S(w, x_0) = \{ y : y(t) = x_0 + w(t/2) \forall t \geq 0 \}.$$

Is this system

(i) linear?

(ii) time invariant?

(iii) causal?

(iv) L2 gain stable?

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1Version of September 10, 2011, due on September 19, 2011.
(b) System $S \in S_{DT}^{1,1}(\mathbb{R})$ (i.e., a DT single input single output (SISO) system with boundary conditions set $\mathbb{R}$) is defined by

$$S(w, x_0) = \{ y : \quad y(t) = e^{-t}x_0 + |w(t+1)| \quad \forall \ t \geq 0 \}.$$ 

Is this system

(i) linear?
(ii) time invariant?
(iii) causal?
(iv) L2 gain stable?

**Task 1.2T**

In each of the (unrelated) statements (a)-(d) below, find the maximal value of real parameter $r$ for which the statement is true, and provide a brief explanation.

(a) System $S \in S_{CT}^{1,1}(\{0\})$, defined by

$$S(w, x_0) = \{ y : \quad y(t) = 0 \text{ when } t + r < 0, \quad y(t) = w(t + r) \text{ when } t + r \geq 0 \},$$

is L2 gain stable.

(b) System $S \in S_{CT}^{1,1}(\{0\})$, defined by

$$S(w, x_0) = \{ y : \quad y(t) = w(|r|t) \},$$

is causal.

(c) System $S \in S_{DT}^{1,1}(\mathbb{R})$, defined by

$$S(w, x_0) = \{ y : \quad y(t) = r + rt + e^{-t}x_0 + w(t + 1) \},$$

is time invariant.

(d) System $S \in S_{DT}^{1,1}(\mathbb{R})$, defined by

$$S(w, x_0) = \{ y : \quad y(t) = (r - 1) \sin(x_0) + rw(t^2) \},$$

is linear.
**Task 1.3T**

For each of the systems described in (a)-(d), calculate its L2 gain, and provide a brief explanation.

(a) System $S \in S_{CT}^{1,1} \{0\}$, defined by

$$S(w, x_0) = \{y : y(t) = \sin(t)w(t)\}.$$

(b) System $S \in S_{CT}^{1,1} \{0\}$, defined by

$$S(w, x_0) = \{y : y(t) = e^{-t}w(t)\}.$$

(c) System $S \in S_{DT}^{1,1} \{0\}$, defined by

$$S(w, x_0) = \{y : y(t) = \sin(w(t + 1))\}.$$

(d) System $S \in S_{DT}^{1,1} \{0\}$, defined by

$$S(w, x_0) = \left\{ y : y(t) = \frac{w(t)}{1 + |w(t)|} \right\}.$$