Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.245: MULTIVARIABLE CONTROL SYSTEMS

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Problem Set 1 [1]

This assignment is a warm-up excercise (there was little taught in the class, so far), and contains only "theory" tasks (i.e. problems with "x.xT" identifiers), intended to check your understanding of the material from Chapter 2 of the lecture notes.

Task 1.1T

Answer the following Yes/No questions, and provide brief reasoning.

(a) System $S \in \mathcal{S}^{1,1}_{CT}(\mathbb{R})$ (i.e., a CT single input single output (SISO) system with boundary conditions set $\mathbb{R}$) is defined by

$$S(w, x_0) = \{y : \quad y(t) = x_0 + w(t/2) \ \forall \ t \geq 0\}.$$

Is this system

(i) linear?

(ii) time invariant?

(iii) causal?

(iv) L2 gain stable?

[1] Version of September 10, 2011, due on September 19, 2011.

(b) System $S \in \mathcal{S}_{DT}^{1,1}(\mathbb{R})$ (i.e., a DT single input single output (SISO) system with boundary conditions set $\mathbb{R}$) is defined by

$$S(w, x_0) = \{y : \quad y(t) = e^{-t}x_0 + |w(t+1)| \ \forall \ t \geq 0\}.$$

Is this system

(i) linear?

(ii) time invariant?

(iii) causal?

(iv) L2 gain stable?

Task 1.2T

In each of the (unrelated) statements (a)-(d) below, find the maximal value of real parameter r for which the statement is true, and provide a brief explanation.

(a) System $S \in \mathcal{S}_{CT}^{1,1}(\{0\})$, defined by

$$S(w, x_0) = \{y : \quad y(t) = 0 \text{ when } t + r < 0, \quad y(t) = w(t + r) \text{ when } t + r \geq 0\},$$

is L2 gain stable.

(b) System $S \in \mathcal{S}_{CT}^{1,1}(\{0\})$, defined by

$$S(w, x_0) = \{y : \quad y(t) = w(|r|t)\},$$

is causal.

(c) System $S \in \mathcal{S}_{DT}^{1,1}(\mathbb{R})$, defined by

$$S(w, x_0) = \{y : \quad y(t) = r + rt + e^{-t}x_0 + w(t + 1)\},$$

is time invariant.

(d) System $S \in \mathcal{S}_{DT}^{1,1}(\mathbb{R})$, defined by

$$S(w, x_0) = \{y : \quad y(t) = (r - 1)\sin(x_0) + rw(t^2)\},$$

is linear.

Task 1.3T

For each of the systems described in (a)-(d), calculate its L2 gain, and provide a brief explanation.

(a) System $S \in \mathcal{S}_{CT}^{1,1}(\{0\})$, defined by

$$S(w, x_0) = \{y : \quad y(t) = \sin(t)w(t)\}.$$

(b) System $S \in \mathcal{S}_{CT}^{1,1}(\{0\})$, defined by

$$S(w, x_0) = \{y : \quad y(t) = e^{-t}w(t)\}.$$

(c) System $S \in \mathcal{S}_{DT}^{1,1}(\{0\})$, defined by

$$S(w, x_0) = \{y : \quad y(t) = \sin(w(t+1))\}.$$

(d) System $S \in \mathcal{S}_{DT}^{1,1}(\{0\})$, defined by

$$S(w, x_0) = \left\{y : \quad y(t) = \frac{w(t)}{1 + |w(t)|}\right\}.$$