Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.245: MULTIVARIABLE CONTROL SYSTEMS

by A. Megretski

Problem Set 2 Make-Up Tasks [1]

Task 2.1TM

For each of the statements (a)-(f) below, they whether it is true or false. For a false statement, give a counterexample. For a true statement, give a *short* explanation or a textbook reference.

(a) If a real n-by-n symmetric matrix Q and a vector $x \in \mathbb{R}^n$ are such that $x'Qx = 0$ then $Qx = 0$.

(b) If a real n-by-n symmetric positive semidefinite matrix Q and a vector $x \in \mathbb{R}^n$ are such that $x'Qx = 0$ then $Qx = 0$.

(c) If a real n-by-n matrix Q and a vector $x \in \mathbb{R}^n$ are such that $u'Qu \geq 0$ for all $u \in \mathbb{R}$, and $x'Qx = 0$, then $Qx = 0$.

(d) If real matrices $A = A'$, B are such that

$$R = \begin{bmatrix} A & B \\ B' & 0 \end{bmatrix}$$

is positive semidefinite then $B = 0$.

[1] Version of November 29, 2011.

(e) If L2 gain of a CT system $S \in \mathcal{S}_{CT}^{m,k}(\{0\})$ equals 1 then

$$\inf_{T \geq 0} \int_0^T \{|w(t)|^2 - |v(t)|^2\} dt > -\infty$$

for all $w \in \mathcal{L}^m$ and $v \in S(w, 0)$.

(e) If system $S \in \mathcal{S}_{DT}^{m,k}(\{0\})$ is such that

$$\inf_{T \geq 0} \int_0^T \{|w(t)|^2 - |v(t)|^2\} dt > -\infty$$

for all $w \in \mathcal{L}^m$ and $v \in S(w, 0)$ then L2 gain of S is strictly smaller than 1.

Task 2.2TM

For each of the situations described below, find the set of possible values for the L2 gain of system S. For the L2 gain values which are possible, give specific examples. When you state that some L2 gain values are not possible, give a proof of that. For the proofs, please try using the technique of quadratic dissipation inequalities (IQCs). While the statements are simple enough to have many alternative derivations, the intention of this task is to make you familiar with the IQC technique, before moving on to more complicated examples.

(a) S is a DT system which is strictly passive, in the sense that

$$\inf_{T \in \mathbb{Z}_+} \sum_{t=0}^{T} \left\{ w(t)' y(t) - |y(t)|^2 \right\} > -\infty$$

for every input/output pair (w, y).

(b) $S = S_2 \circ S_1$ is series interconnection of $S_1 \in \mathcal{S}_{CT}^{1,1}(\{0\})$ and $S_2 \in \mathcal{S}_{CT}^{1,1}(\{0\})$, where L2 gain of both S_1 and S_2 equals 1.

(c) $S = S_1 - S_2$ is the difference interconnection of $S_1 \in \mathcal{S}_{DT}^{1,1}(X_1)$ and $S_2 \in \mathcal{S}_{DT}^{1,1}(X_2)$, where X_1, X_2 are some sets, and L2 gain of both S_1 and S_2 equals 1. .

(d) $S = (-\Delta)^\circ$ where $\Delta \in \mathcal{S}_{CT}^{m,m}(X)$ has L2 gain of 1 and is passive in the sense that

$$\inf_{T \geq 0} \int_0^T w(t)' y(t) dt > -\infty \quad \text{whenever} \quad y \in S(w, x_0), \quad w \in \mathcal{L}^m, \; x_0 \in X,$$

X is some set, and the unity feedback interconnection defined by $-\Delta$ is well posed.