Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.245: MULTIVARIABLE CONTROL SYSTEMS

by A. Megretski

Problem Set 8 [1]

The problem set deals with H-Infinity optimization and Hankel Optimal model reduction.

Problem 8.1T

For all values of parameter $a \in \mathbb{R}$ for which the CT H-Infinity optimization setup

$$\dot{x}(t) = u(t) + w_1(t), \quad e(t) = ax(t) + u(t), \quad y(t) = x(t) + w_2(t)$$

is well-posed, find the optimal controller and the minimal cost.

Problem 8.2T

For all values of parameter $a \in \mathbb{R}$ find a Hankel optimal reduced model of order 1 for

$$G(z) = \frac{1-a^2}{z} + \frac{a}{z^2}.$$

Problem 8.3P

Consider the task of finding the minimal control effort required to stabilize the CT system with transfer function

$$P(s) = \frac{e^{-s}}{s-1}$$

[1] Version of November 28, 2011, due by the end of December 11, 2011.

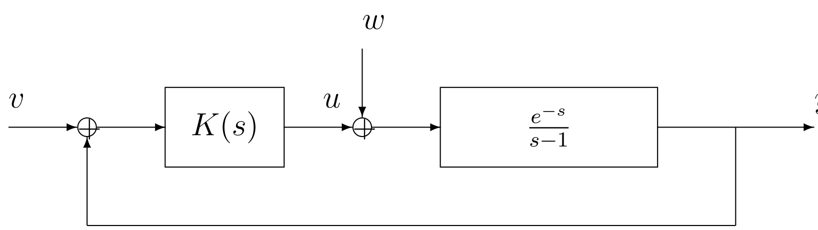

Figure 1: Problem 8.3 setup

using a fixed order controller $K = K(s)$, as shown on Figure 1. More specifically, you are asked to use Hankel optimal model reduction and H-Infinity optimization to develop an algorith for finding controllers $K = K(s)$ of a given order which stabilize the system from Figure 1 (so that the closed loop L2 gain from $[v; w]$ to $[u, y]$ finite), and make the closed loop L2 gain from w to u as small as possible.

You are not expected to look for the true optimal solution (there are no efficient methods for doing this), but rather to explore ways in which a good low order approximation of an H_∞ class transfer function containing e^{-s}, such as

$$G(s) = \frac{e^{-s}}{s+1}, \quad \text{or} \quad G(s) = \frac{e^{-s} - e^{-1}}{s-1},$$

can be used to set up some H-Infinity optimization to aid in designing a good controller.

Produce a table of values of best $w \to u$ L2 gains achieved by your algorithm with controllers of order m for $m \in \{1, 2, 4, 8, 16\}$ (it's OK if some of the numbers in the table equal ∞).