MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science
6.262 – Discrete Stochastic Processes

Problem Set # 11 | Issued: April 30, 2009
Due: May 6, 2009

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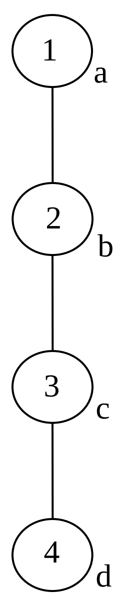
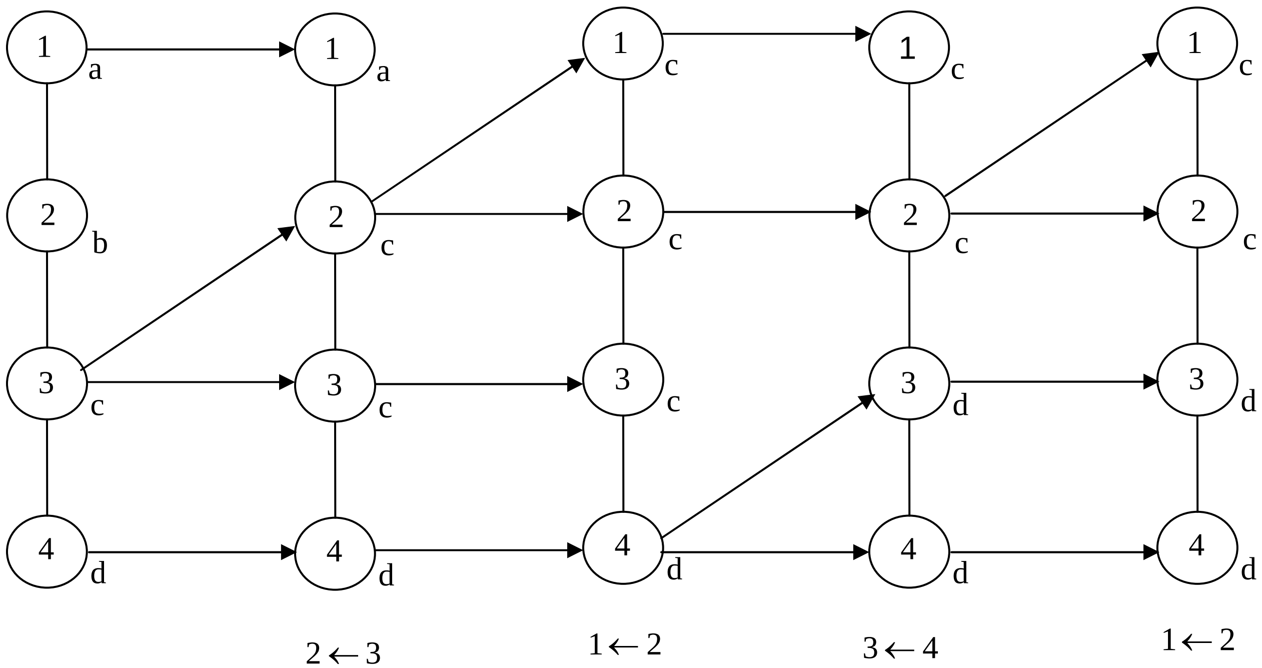
1) The Voter Problem (Part II).

Consider once again the voter model from Problem M in Problem Set 9, defined on a connected graph **G** with n vertices. Given an initial condition in which each vertex has a specified opinion (0 or 1), find the probability that the Markov process eventually settles into a state in which every vertex's opinion is 1. Though this Markov process has 2^n states, give an expression for this probability that can be evaluated for any such graph in at most $O(n^2)$ steps.

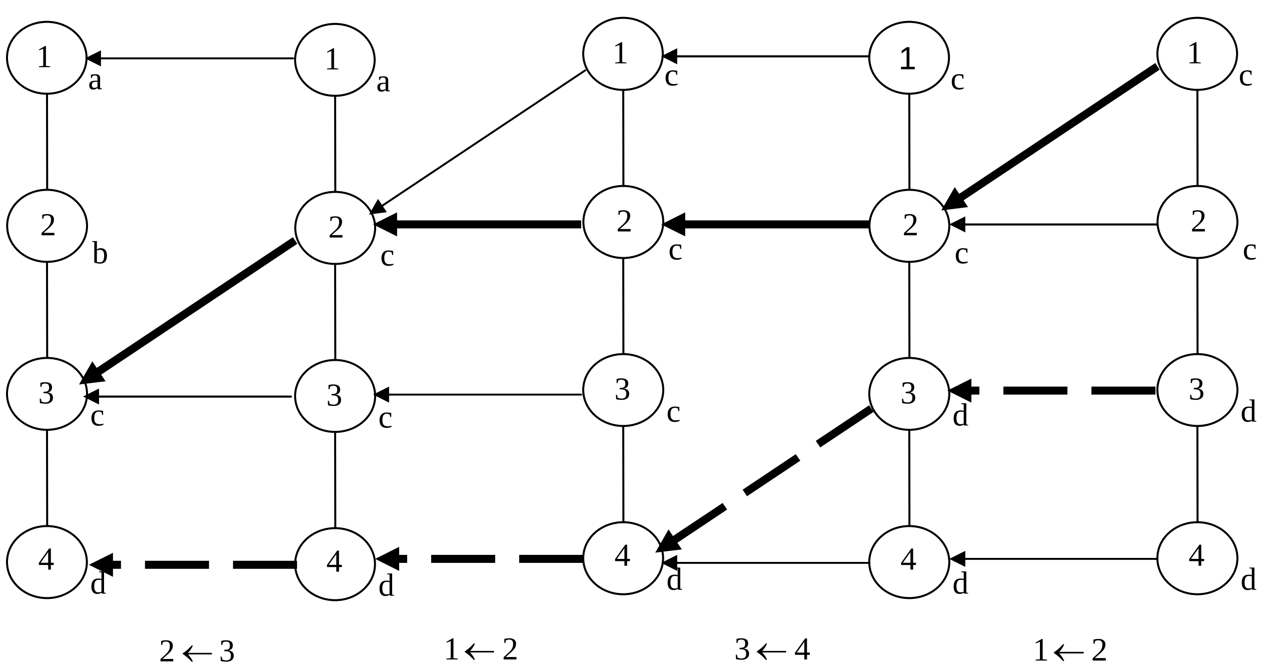
This continues the line of thought we had begun to develop in Problem M, and in Problem L in Problem Set 8. However, it seems essentially impossible to solve this without using the following insight, which we introduce via an example of a 4-vertex graph with initial opinions at the 4 vertices of a, b, c, and d.



We denote a transition in which the Poisson clock for vertex #i has an arrival and immediately vertex #i takes the opinion of one of its neighbors, vertex #j, by $i \leftarrow j$. Suppose for example the first 4 transitions are, in sequence: $2 \leftarrow 3,\ 1 \leftarrow 2,\ 3 \leftarrow 4,\ 1 \leftarrow 2$. The opinions held by all the vertices are shown immediately after each transition in the figure below:



Then it is possible to trace a path backward in time, beginning with any vertex k, to the neighbor $\mathcal{N}$(k) from which vertex k gained its final opinion, then to the neighbor $\mathcal{N}(\mathcal{N}($k)) from which $\mathcal{N}$(k) received the opinion it passed on to neighbor k, etc. The path backward from vertex 1 is shown below with bold solid arrows and the path backward from vertex 3 is shown with bold dashed arrows. Note that 1) such a path must exist beginning from each vertex at the final time under consideration, 2) the opinion along such a path never changes, though 3) the path itself is random. You may assume the set of possible reverse paths is aperiodic.



Note: These problem numbers refer to the versions of Chapter 7 that are now on the course website.

2) Problem 7.15) (The density should be $f_X(x) = \begin{cases} e^{-x} \big/ (e - e^{-1}), & -1 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$

3) Problem 7.20)

4) Problem 7.23)

5) Problem O

Consider a gambler with an initial wealth W (W is an integer >0). At each game, his wealth either increases by 1 with probability p, or else decreases by 1 with probability (1-p).

a) Give an expression for the probability that he doubles his wealth before going broke. Please simplify the expression so that its behavior as a function of p can easily be understood, and evaluate it at p = 0 and p = 1.

b) Give an expression for the expected number of games it will take until he either doubles his wealth or goes broke. Again, please simplify as much as possible and evaluate at p = 0 and p = 1.