MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science
6.262 – Discrete Stochastic Processes

Problem Set #6

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Due: March 18, 2009

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(1) Exercise 3.17. In parts b) and c) the renewal-reward function R(t) is the "expected reward rate" or, more precisely, the "expected cost rate." It is a function with the property that the expected cost over any interval $[t_1, t_2]$ is $\int_{t_1}^{t_2} R(t)dt$.

(2) Exercise 3.23. Briefly compare your answers to parts a) and b) with the behavior of the system in Problem 2.23 from Problem Set # 3.

(3) Exercise 3.24.

(4) Exercise 3.25

(5) **<u>Problem F</u>:** A store sells TV's for $500, but if an hour passes with no sales they cut the price of the next television to $250. Once that TV is sold, the price for the next TV goes up to $500 for the next hour. Every customer who arrives buys immediately, and the store never closes. For parts a) and b), assume the customer arrivals are a Poisson process with rate 1 per hour.

a) Bill comes to work at 8:00 am and gets to go home for lunch at noon if six or more TV's have been sold by noon. How likely is that? Two slightly different approaches lead to two different (but equal) expressions. Give both, but you need not evaluate either of them.

b) If exactly 3 customers arrive in the first three hours of Bill's shift, what is the probability that the second of those customers pays $250 for his TV? Briefly explain your reasoning.

For parts c) and d) assume the customer arrivals are a renewal process with a general interarrival distribution $F_X(x)$ where X is the interarrival time measured in hours. (Assume X has a finite mean and a density $f_X(x)$, but give your answers in terms of $F_X(x)$.)

c) What is the expected price of the 30th TV sold? Explain your reasoning.

d) Someone phones the store at 3 pm next Friday to ask the current price of the television on sale. What is the expected value of the price at that time? (This question can only be answered if certain additional assumptions are made. These are not assumptions on $F_X(x)$. Specify the needed assumptions and explain briefly how they help in solving the problem.)

e) Are the answers to c) and d) identical? If they are, prove it. If not, give a counterexample. Are the answers to c) and d) identical in the special cases $F_X(x) = 1 - e^{-\lambda x}$ for $\lambda > 0$? Explain why or why not. In both parts, avoid using the specific formulas you derived in c) and d) and explain your insights.

The next two problems are intended to be relatively brief and easy.

(6) **<u>Problem G:</u>** Various "random incidence" facts **(**e.g., the large-time expected duration and long-term average duration for a renewal process equals $\left(\overline{X^2}/\overline{X}\right)$**)**, may seem less peculiar when they arise in simpler settings than renewal processes. Here is one simple example.

Divide the interval $[0, T]$ into n adjacent subintervals with deterministic lengths $x_1, x_2, \cdots, x_n$. The only constraints are that $x_k \geq 0$ and $\sum_{k=1}^{n} x_k = T$. The only stochastic element is to pick a time t from the uniform distribution on $[0, T]$. Find the expected value of the length of the interval in which t lands. Put the result into a form comparable to eq. (3.26) for renewal processes. Explain the relation between this model and calculation and the one for duration in renewal processes. How are they similar and how are they different?

What choice(s) of subinterval lengths minimizes the expected duration?

(7) **<u>Problem H:</u>** Consider two deterministic queues, both having customer interarrival times of exactly 1 minute. In the first queue, the service time is exactly 1 minute, the arrival of the next customer is always exactly synchronized with the departure of the last customer in service, and there are always exactly three customers in the system (i.e., two in queue and one in service) in the (marginally stable) steady state. In the second system the arrival process is the same, but the service time is always exactly 0.5 minute.

In which (if either) of these cases does Little's Theorem give the correct answer? In which (if either) of these cases are the statement and proof of Little's Theorem in Section 3.6.1 of the book correct?