MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science
6.262 – Discrete Stochastic Processes

Problem Set #9 Issued: April 17, 2009
Due: April 22, 2009

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1) Please go over your midterm quiz and re-work every part on which you did not get a perfect score. Redo only the parts that were marked down. For example, if on Problem 3, part b) you got perfect scores on b1) and b3) but not b2), you only need to redo the part of b2) for which you lost credit.

In addition, on some questions there is a very simple way to do them, though some of you did them in a very difficult way instead. These are mentioned below. If you did these in a difficult way and nonetheless earned a perfect score on that part, please redo them the way we suggest below, which should require more insight but less computation. The insights will be worthwhile for you to gain.

Please try to do part b) of Problem 1 using only renewals, rather than solving 8 equations in 8 unknowns for 1^{st} passage times in a Markov chain.

On Problem 2, part f) is almost immediate once you have correctly done parts d) and e). Furthermore, five out of six parts in Problem 2 have a solution that does not require you to solve a system of equations.

On Problem 3, it is very helpful in doing calculations to note that, for two random variables A and B, max(A,B) $\leq$ x $\Leftrightarrow A \leq x$ & $B \leq x$. Therefore, if A and B are independent, $F_{\max(A,B)}(x) = F_A(x)F_B(x)$.
A second observation is that it is easy for you to show, for nonnegative numbers x_1, x_2, y_1, y_2, that

$$\max\{x_1 + x_2, y_1 + y_2\} \leq \max\{x_1, y_1\} + \max\{x_2, y_2\},$$

and this fact can be very helpful in Problem 3. A third useful fact for justifying certain steps is that max{x,y} and min{x,y}are both continuous functions of x and y. This can be helpful when considering certain limits.

Note on Regrading of Quizzes: If you feel we have misgraded any part of your quiz, you may give Prof. Wyatt the quiz (or a copy) with a separate written sheet next Wednesday saying what parts you do not feel were fairly or accurately graded, with a carefully written explanation for your reasoning. We will regrade all such problems, and this may raise *or lower* the score you have received. Do not, ever, under any circumstances, write on the quiz itself (or erase anything that is there.) We will not discuss the grading on your quiz with you in person, but we will carefully consider your written arguments.

We intend to be fair, but we also insist on clearly written explanations, given the generous amount of time you had to complete the quiz.

If you did not get a perfect score on a problem, but feel that you deserved a perfect score and have submitted your problem for regrading, you do not need to redo that part for this homework. But if you do not get a perfect score after regrading, please redo that part and hand it in with Problem Set #10.

2) Problem 6.3

3) Problem 6.6. The term (??) refers to eq. (6-34).

4) Problem 6.8

5) Problem 6.11. Before doing part a), please draw a Markov process diagram showing the states and the transition rates between them.

6) Problem M **The Voter Model (Part I)**

The "Voter Model" is a class of Markov Processes that have been developed to model the spread of opinions among a group of people. Using the notation of Problem M, the voter model is based on a connected graph G with n nodes (vertices), l links (edges) and no self loops. Each vertex represents a person, who can hold opinions 0 or 1 on the topic in question (e.g., if the topic is recent national politics, possibly a 0 for Republicans and a 1 for Democrats). An edge connects two people (i.e., vertices) who discuss this topic, while two people (vertices) who not discuss the topic or do not know one another at all have no edge between them. Each vertex has an independent Poisson process with rate 1 that indicates when that individual may change its opinion. In the simplest model, when the Poisson process $N_k(t)$ for vertex k has an arrival, vertex k reconsiders its opinion and adopts the opinion held by one of his neighbors, i.e., by another vertex to whom he is connected by a link. All his neighbors are chosen with equal probability, independent of their opinions at any time and independent of the choice of neighbor made by any vertex at any other time.

a) Draw the Markov process diagram for a 3-vertex voter model in which each vertex is connected by an edge to the other two.

a) How many states does a general Markov process voter model (with n > 0 nodes and l > 0 links) have? How many of those can be reached on a single transition from a given state? Let d_k denote the degree of the k-th vertex, $1 \le k \le n.$ What are the possible values for the transition rate $q_{i,j,}$ $i \neq j$?

b) Find all the classes of recurrent states for this Markov process, (i.e., for its embedded Markov chain.)