Problem 2.2  Consider the M/G/∞ queue in which each customer always finds a free server. Let $P_k(t) = P[N(t) = k]$ and assume $P_0(0) = 1$. Show that

$$P_k(t) = \sum_{n=k}^{\infty} e^{-\lambda t} \frac{(\lambda t)^n}{n!} \binom{n}{k} \left[ \frac{1}{t} \int_0^t [1 - F_X(x)]dx \right]^k \left[ \frac{1}{t} \int_0^t F_X(x)dx \right]^{n-k},$$

where $F_X$ is the cumulative distribution function of the service time $X$.

Problem 2.3  Consider an M/G/1 queue in which bulk arrivals occur at rate $\lambda$ and with a probability $g_r$ that $r$ customers arrive together at an arrival instant.

(a) Show that the $z$-transform of the number of customers arriving in an interval of length $t$ is $e^{-\lambda t [1 - G(z)]}$ where $G(z) = \sum g_r z^r$.

(b) Show that the $z$-transform of the random variables $v_n$, the number of arrivals during the service of a customer, is $X^*[\lambda - \lambda G(z)]$.

(c) Show that the generating function for queue size is

$$Q(z) = \frac{(1 - \rho)(1 - z)X^*[\lambda - \lambda G(z)]}{X^*[\lambda - \lambda G(z)] - z}.$$  

Using Little’s result, find the ratio $W/\bar{x}$ of the expected wait on queue to the average service time.