A few of you have expressed concern over the C.012 material for this class. There is no need to worry. You will not need much of C.012, but the transistors and the parts you do need will become obvious because of how often they come up. The purpose of this recitation is to help you to come to grips with the transistor physics that you will need.

We start nonlinear modeling and by facing the ugly reality of a highly nonlinear device whose inner workings make our heads spin:

\[
I_C = I_S \left( \frac{e^{\frac{V_{CE}}{K_T}} - 1}{\frac{V_{BS}}{K_T}} \right).
\]

Right away, we have a problem. Linear systems we can handle. Nonlinear systems we cannot.

So what do we do? We close our eyes, pretend that it is a linear device, and then arrange our design so that we're not punished for our crimes.

The math in these situations is clumsy at best.
You know this as "linearizing," or "deviating a small signal model." We take an expression like

\[ I_c = I_s e^{q V_{BE}/kT} = F(V_{BE}) \]

and write instead

\[ \Delta I_c = \frac{dI_c}{dV_{BE}} \Delta V_{BE} \]

Hmm. We need to find an appropriate \( k \) somehow, even though our approximate expression looks nothing like the original.

Based on what we've written, we say

\[ k = \frac{\Delta I_c}{\Delta V_{BE}} = \frac{F(V_{BE} + \Delta V_{BE}) - F(V_{BE})}{\Delta V_{BE}} \]

Clearly we're taking a risk here. Suppose we choose

We're throwing away all of the non-linear behavior in favor of a simple proportionality factor \( k \). What if we choose an interval \( \Delta V_{BE} \) and operation point \( V_{BE} \)?

Consider:

\[ F(V_{BE}) \]

\[ \Delta F = 0 \]

\[ V_{BE} \quad V_{BE} + \Delta V_{BE} \]
Our Levene model would say that $f$ does not depend on $V_{BE}$!

We can minimize our error by respecting the size of $\Delta N_{BE}$.

Over a small range, not such a bad approximation! And we do better by making $\Delta N_{BE}$ even smaller:

$$k = \lim_{\Delta N_{BE} \to 0} \frac{f(V_{BE0} + \Delta N_{BE}) - f(V_{BE0})}{\Delta N_{BE}}$$

Do you recognize this? Think high school calculus:

$$k = \frac{d\, f}{d\, N_{BE}} \bigg|_{N_{BE} = V_{BE0}}$$

So we approximate the nonlinear behavior as a Levene function, with the caveat that we stay "close" to $V_{BE0}$. In the case of our exponential function:

$$I_C = I_S e^{\frac{V_{BE}}{kT}}$$
\[ I_c = \left( \frac{d}{dN_{BE}} \left( I_s e^{q (V_{BE} - V_T)/kT} \right) \right) N_{BE} \]

\[ = \left( \frac{q}{kT} I_s e^{q N_{BE}/kT} \right) N_{BE} \]

\[ = \frac{q}{kT} I_c \cdot N_{BE} \rightarrow \text{For } g_m = \frac{q}{kT} I_c \]

Don't lose track of what happened here. We've used our transistor just underwent a transformation.

**Class Exercise**: Now your turn.

Support that we discovered a strange new device.

\[ I_0 \text{ depends on } N_L \text{ according to} \]

\[ I_0 = \sin (k_0 N_L) \]

1) Determine the transconductance of this device.
2) Bias the circuit so that the incremental voltage gain is 20.
We use this technique all over the place in deeply analog electronics. We take devices that are OFF nonlinear, differentiate about our operating point, and from then on it's a nonlinear world.

Where else do we use this in our model? Recall the full hybrid IT:

None of the elements \( R_I, C_{II}, C_M \), or \( K \) are linear. That is to say, for \( C_{II} \) it is not true that the change we supply to \( C_{II} \)
Varies linearly with the voltage we 

\[ Q_{n} = C \, V_{n}, \]

we have

\[ Q_{n} = f(V_{n}). \]

It doesn't matter. At the end of the day, we use 
device physics to get an exact form of \( f(V_{n}) \).

Then we differentiate, and say

\[ C_{n} = \frac{dQ_{n}}{dV_{n}} = \frac{d}{dV_{n}} f(V_{n}). \]

Read the text carefully, and you will see how 
this technique is used so often. Once you understand, 
you need only remember results:

\[ I_{c} = I_{s} e^{qV_{n}/kT} \]

\[ I_{b} = I_{c} / \beta_{i} \]

\[ g_{m} = \frac{qI_{c}}{I_{c}} \]

\[ c_{h} = e g_{m} \, c_{f} \]

And so on.