Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.302

Feedback Systems

Final Examination

18 December 2000

4-270

9:00 a.m. - 12:00 p.m.

This examination consists of 4 problems. The relative weight for each problem is indicated in the question booklet, for a total of 100 points. Please work ALL of the problems.

You **must** summarize your final solutions in the answer sheets included with this examination.

We would encourage you do do the work for all of the problems on these answer sheets as well. If you find that these answer sheets do not contain enough space, however, you may do your additional work in the accompanying examination booklet. This booklet will also be read by the graders.

Make sure that your name is on each answer sheet and on each booklet. Additionally, make sure that you clearly denote which problem you are working on each page of the booklet.

This examination is closed book. A formula sheet is available on the last page of the examination.

Remember to

- summarize your final solutions on the appropriate answer sheets.
- draw all sketches neatly and clearly as requested.
- label ALL important features on your sketches.

Good luck!

Problem 1: 25%

Name:

A system with minor loop compensation is shown in Figure 1.

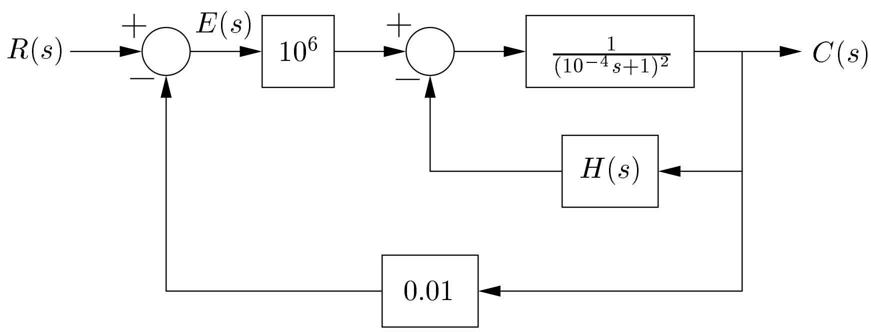

Figure 1: Minor Loop System

a. Do we want $H(s) = Ks$ or $H(s) = \frac{K}{s}$?

b. Find K such that $\omega_c = 10^6$ rps and $\phi_M = 45°$.

c. What is the dynamic tracking error $\left|\frac{E}{R}\right|$ at 10^3 rps?

d. Your customer wants $\omega_c = 10^6$ rps, $\phi_M = 45°$ and $\left|\frac{E}{R}\right| < 5 \times 10^{-6}$ at 10^3 rps; choose the $H(s)$ which satifies all of these criteria.

$$H_1(s) = \frac{10^{-6}s^2}{10^{-4}s + 1} \qquad H_2(s) = \frac{10^{-4}s + 1}{10^{-6}s^2}$$

$$H_3(s) = \frac{10^{-5}s^2}{10^{-3}s + 1} \qquad H_4(s) = \frac{10^{-3}s + 1}{10^{-5}s^2}$$

$$H_5(s) = \frac{10^{-4}s^2}{10^{-2}s + 1} \qquad H_6(s) = \frac{10^{-2}s + 1}{10^{-4}s^2}$$

e. What is $\frac{E}{R}$ evaluated at 10^3 rps now?

Problem 2: 30%

Name:

The block diagram for a series compensated feedback system is shown below in Figure 2.

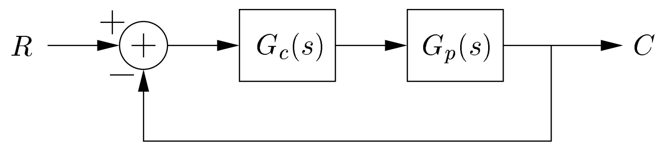

Figure 2: System to be Compensated

There are a number of plant transfer functions $G_p(s)$ that will be used in this system, including

a.

$$G_p(s) = \frac{1}{s^2}$$

b.

$$G_p(s) = e^{-s}$$

c.

$$G_p(s) = \frac{1}{s\left[\frac{s^2}{(100)^2} + \frac{2(0.01)s}{100} + 1\right]}$$

d.

$$G_p(s) = \frac{1 - 0.1s}{1 + 0.1s}$$

e.

$$G_p(s) = \frac{1}{(0.1s + 1)(0.1s - 1)}$$

f.

$$G_p(s) = \frac{s^2}{(s+1)^3(\frac{s}{300} + 1)}$$

The possible choices for compensator transfer functions are as follows.

1.

$$G_c(s) = \frac{K}{s}$$

2.

$$G_c(s) = \frac{K(s+1)}{s}$$

3.

$$G_c(s) = \frac{K(10s+1)}{(100s+1)}$$

4.

$$G_c(s) = \frac{K(10^{-2}s+1)}{(10^{-3}s+1)}$$

5.

$$G_c(s) = \frac{K}{s^2}$$

6.

$$G_c(s) = \frac{K}{(0.01s+1)^4}$$

For each plant, select a compensator and determine the appropriate value of K such that the system is stable with a phase margin between 40° and 70°, and the low frequency magnitude of the loop transmission is high.

Note that some of the compensators may be used with more than one plant, just as some might not work well with any plants.

Problem 3: 20%

Name:

Consider the following four loop transfer functions:

$$L_1(s) = \frac{10^6}{s} \qquad L_2(s) = \frac{10^6}{s+1} \qquad L_3(s) = \frac{10^{10}(10^{-4}s+1)}{s^2} \qquad L_4(s) = \frac{10^6(10^{-4}s+1)}{(10^{-2}s+1)^2}$$

a. For each loop transfer function, plot an asymptotic Bode plot.

b. For each loop transfer function, find the open loop DC gain, the crossover frequency ω_c, and the phase margin ϕ_M.

c. Assuming that the above loop transfer functions describe op amp circuits with unity feedback, find the error transfer function for each loop transfer function.

d. Find the steady state error to a 1 V step input for each loop transfer function.

e. Using synthetic division, find the first three error coefficients of the error series, e_0, e_1, and e_2, for each loop transfer function.

f. For a unit ramp input, the steady state error grows as

$$e_{ss} = e_0 t + e_1$$

Find the steady state error e_{ss} to an input ramp with a slope of 1 V/μs for each transfer function. Comment on the relative magnitude and measurability of these errors.

Problem 4: 25%

Name:

A certain negative feedback loop combines two systems; a linear element which possesses a transfer function $KG(s)$, and a nonlinearity which has both a dead zone and saturation. This feedback system is shown in Figure 3.

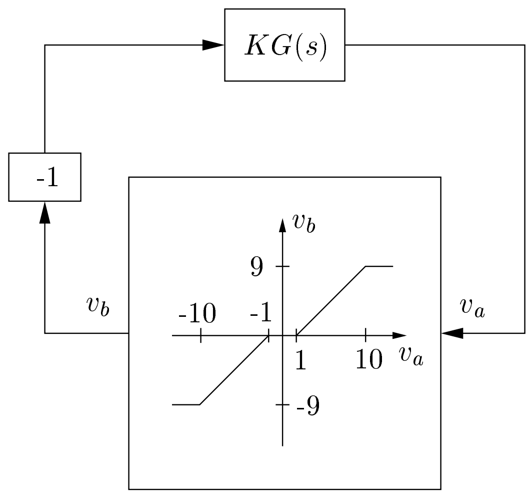

Figure 3: Feedback System

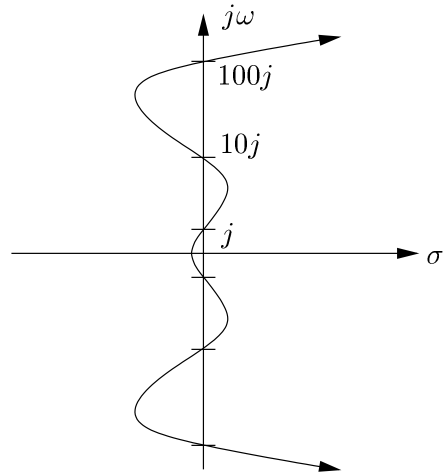

Figure 4: Root Locus Plot of $KG(s)$

The nonlinear element is characterized by its frequency independent describing function $G_D(E)$, where E is the amplitude of the sinusoidal excitation applied to the input of the element.

a. What is $G_D(0.5)$?

b. Estimate the value of $G_D(10)$. Since we don't want you to integrate, we will accept a reasonable error in your estimate.

c. For $E \gg 10$, $G_D(E)$ approaches a limiting relationship dependent upon E. What is this relationship?

The dominant branches of the root locus for $KG(s)$ are as shown in Figure 4. The axis crossing at $s = j$ occurs for $K = 1$, the axis crossing at $s = 10j$ occurs for $K = 1000$, and the axis crossing at $s = 100j$ occurs for $K = 10^4$. Furthermore, the system is tested with $K = 2 \times 10^4$, and it is found that it can exhibit stable amplitude limit cycles at 1, 10, or 100 rps, depending on the initial conditions.

Consider the following ranges of E:

- ⋄ $0 < E < 1$.
- ⋄ $1 < E < 10$.
- ⋄ $10 < E < 50$.
- ⋄ $E > 50$.

d. E is in which range when the system is oscillating at 1 rps?

e. E is in which range when the system is oscillating at 10 rps?

f. E is in which range when the system is oscillating at 100 rps?