MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.302 Feedback Systems

Fall Term 2004 Issued : 9:00 am
Final Exam Due : 12:00 pm

Final Exam
December 15, 2004
180 minutes
Room 32-155

1. This examination consists of five problems. Work all problems.
2. This examination is closed book. No calculators are allowed. Helpful equations and the root-locus rules appear at the end of this packet.
3. You MUST summarize your solutions in the answer sheets included in this packet. Draw all sketches neatly and clearly where requested. Remember to label ALL important features of any sketches.
4. Make sure that your name is on each answer sheet and on each examination booklet.

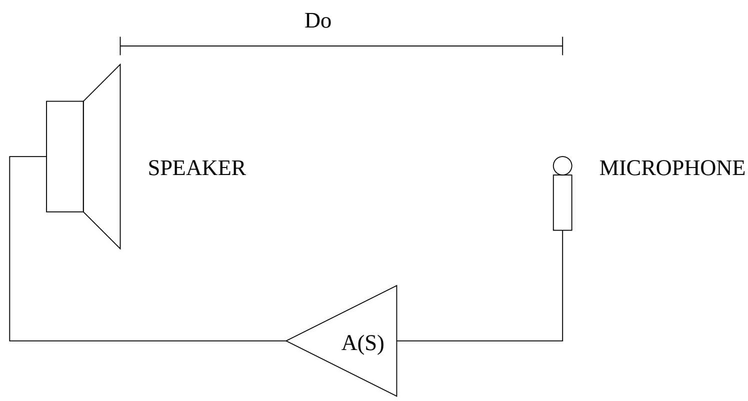
We encourage you to do the work for all of the problems in the answer sheets as well. If you find that the answer sheets do not contain enough space for your scratch work, you may do additional work in the accompanying examination booklet. Make sure that you clearly denote which problem is on each page of the examination booklet. Your examination booklet will also be read by the graders, but only if your answers appear on the answer sheets.

Good luck.

Problem 1 (20%)

Acoustic Feedback

In a somewhat unusual use of acoustic feedback, it is decided to use a speaker, a microphone, and an amplifier with well-chosen dynamics to create an oscillator. That is, to build a system with **one** pair of poles on the $j\omega$ axis.



Assume:

Speaker and microphone transfer functions are unity for all frequencies.

Speed of sound is $v_o = 340$ m/s.

The amplitude of sound from the speaker falls off as $\frac{K_o}{r^2}$, where r is the distance from the speaker.

(a) Write down or derive the Laplace transform of a pure time delay. In other words, express $\mathscr{L}\{h(t - \tau_o)\}$ in terms of $H(s)$.

(b) Draw a block diagram to represent the above system. Assume an inversion in the loop at low frequencies.

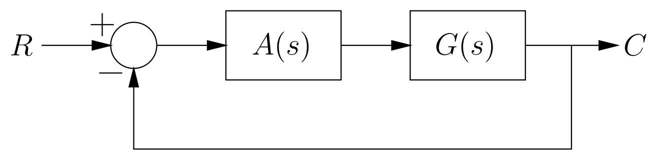
(c) Suppose that the amplifier is ideal: $A(s) = A_o$ for all frequencies. Show that this **prevents** us from realizing only a **single pair** of closed-loop poles on the $j\omega$ axis.

(d) Now suppose that $A(s) = \frac{A_o}{s}$. Let $K_o = 0.1\ m^2$. Choose A_o and D_o to make the system oscillate at 1000 rps.

Problem 2 (20%)

Control and Nyquist

Consider the following unity feedback system:



where

$$G(s) = \frac{1}{s(\frac{s^2}{(10^3)^2} + \frac{2*0.005s}{10^3} + 1)}$$

(a) If a pure proportional controller, $A(s) = K$, is used, what is the maximum value of K that results in a stable system with $\Phi_M > 45^o$? Show this result using a Nyquist plot.

(b) Sketch the step response of the system when a value of K slightly less than that determined in part (a) is used. Show important times and amplitudes (initial, final value, initial slope, time between subsequent peaks). Why is the degree of stability of the system not reflected by its phase margin? (1-2 sentence explanation with relevant number(s) if needed.)

(c) Now use a controller $A(s) = Ke^{-sT}$. Find the value of T that results in the best system stability. Plot the Nyquist diagram for this system and determine K for $\Phi_M = 45^o$. What is the gain margin for the system with this compensation?

Problem 3 (20%)

Compensator Variety Pack

A certain negative feedback system has a forward-path transfer function $A(s)G(s)$ and unity feedback.

The plant $G(s)$ can be:

1.
$$\frac{1 - 0.1s}{1 + 0.1s}$$

2.
$$\frac{1}{s(0.01s + 1)^2}$$

3.
$$\frac{1}{s^2}$$

4.
$$\frac{1}{(s+1)(s-1)(0.05s+1)}$$

The compensator $A(s)$ can be:

(a)
$$\frac{K(10^{-3}s + 1)}{10^{-4}s + 1}$$

(b)
$$\frac{Ks}{10^{-3}s + 1}$$

(c)
$$\frac{K}{s}$$

(d)
$$\frac{K(s+1)}{0.05s + 1}$$

(e)
$$\frac{K(0.2s + 1)}{s}$$

For each plant, choose a compensator that results in a stable system with $\Phi_M = 45^o$ to 60^o. If the specified phase margin can be achieved with more than one compensator, choose the one that results in the highest crossover frequency.

Problem 4 (20%)

Describing Functions

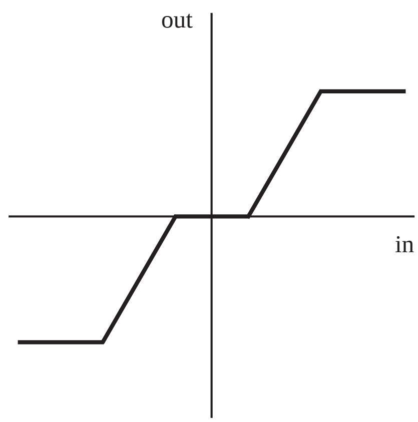
A negative feedback system has a loop transmission $KG(s)$.

It is found that the system is:

- stable for $0 < K < 10^{-3}$,
- unstable for $10^{-3} < K < 0.1$,
- stable for $0.1 < K < 1$,
- unstable for all $K > 1$.

(a) Sketch a $G(s)$ on the gain-phase plane in the answer sheet that will exhibit this behavior.

The linear gain K in the loop is replaced with a nonlinearity that has the general form shown below.



The parameters of the nonlinearity are such that

- $G_D(E) = 0,\ E < 0.01$
- $G_D(E)$ increases monotonically with E for $0.01 < E < 0.1$, reaching 0.8 at $E = 0.1$
- $G_D(E)$ decreases monotonically with E for $E > 0.1$, approaching $\frac{1}{10E}$ for $E \gg 0.1$.

(b) It is found that the system can sustain constant-amplitude oscillations at either frequency f_1 or f_2, where $f_2 > f_1$. Estimate the values of E that exist for each of the frequencies of oscillation.

(c) An additional linear gain of 1.3 is added to the loop, and it is found that the system can now oscillate at a new frequency f_3. What is the approximate value of E when the system is oscillating at f_3?

Problem 5 (20%)

Capturing those 'Kodak' moments

You are working on your final design project in 6.302. To document your ideas and hard work for yourself and posterity, you decided to take photos of all your work.

To do that, you have acquired a nice 35-mm camera for an unbeatable price of $0.99. The seller warned you however, that the camera has undergone some major stress while he was snowboarding one of the half-pipes at Mount Sunapee. Since then, a strange noise comes out of the camera while it is focusing.

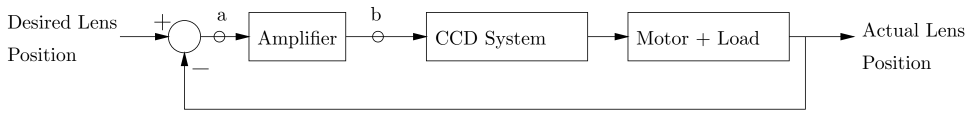
Figure 1: Minolta 70

As your design project moves along you snap lots of pictures. You hear that moving noise each time you take a photo, and it seems like the camera's lens is constantly moving in and out of focus.

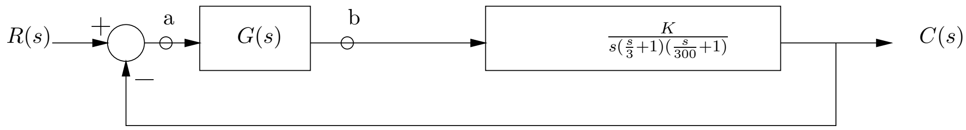
You develop the photos and find that the focus varies a lot, from very blurry to sharp. The autofocus feature seems to be busted.

Ambitious and brilliant, you read up some on autofocusing in your camera, determined to fix the problem. Here is what you find...

A charge-coupled device (CCD) is used as part of an automatic focusing system in your 35-mm camera. Automatic focusing is implemented by focusing the center of the image on a charge-coupled device array through two lenses. The separation of the two images on the CCD is related to the focus. The camera senses the separation, and a computer drives the lens and focuses the image (Popular Photography). The automatic focus is a position control system, where the desired position of the lens in an input selected by pointing the camera at the subject. The output is the actual position of the lens.



(a) CCD automatic focusing system.



(b) Functional block diagram.

Figure 2: CCD position control system

The Minolta camera you acquired (shown above) uses a CCD automatic focusing system and the block diagram in Figure 2 shows the automatic focusing feature represented as a position control system.

For parts (a) - (c) assume $G(s) = 1$

(a) Assuming the simplified model shown, with $K = 3$, draw the open-loop Bode magnitude plot, and, based on this plot, the approximate closed loop plot on the same axes (Use asymptotic approximations). Clearly label each plot's breakpoint frequencies, slopes, magnitudes, and crossover.

(b) How high do you think K must be for the performance you observe?

(c) Now, you accidentally drop your camera and it looks like your gain K has changed yet again, and you measure it to be $K = 3$. To your great joy, your camera seems to work much better now! Explain what happened with one relevant metric.

(d) Profs Dawson and Kaertner come by while you fiddle with stuff and suggest that you modify the loop altogether with a lead compensator. So you open up the camera, and place your lead network

$$G(s) = G_{lead} = \frac{1}{\alpha}\frac{\alpha\tau s + 1}{\tau s + 1}$$

where $\alpha = 10$ between nodes a and b. Assume that gain can be adjusted by tweaking K. You plan to get 55^o of phase margin. What is the time constant τ of the pole in the lead compensator? What is the crossover of the compensated system?

(e) Profs approve your paper design and ask how you plan to realize this circuit. Draw a simple circuit topology to realize the compensator you designed above using **PASSIVE** network with resistor(s) and/or capacitor(s). You DO NOT need to provide values. Show where the **input** and **output** nodes are.

Wow, things are great in the lab, your improved camera is fast and stable. You take it skiing and suddenly you find that the autofocus is misbehaving again. While skiing you run into your favorite TAs who suggest that the model of your camera varies depending on the outdoor temperature. They suggest that you sacrifice the speed of your camera and use a different type of compensation to make your loop stable regardless of temperature.

(f) Give the name for suggested compensator type?

Circle the transfer function which is likely to realize it.

$\frac{K_2}{s}$ or $K_2 s$

(g) **Optional** (Note: 1 pt of extra credit can be applied to total on this problem ONLY, not to exceed total points given to this problem. Not doing this problem will not affect your grade.)

Your improved camera gets so much attention that you have a photo taken which then gets published in *The Tech* (shown below). For **1 point** of extra credit, draw your face in the photo (see answer sheet).