MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.302 Feedback Systems

Spring Term 2008 — Issued : February 12, 2008
Problem Set 2 — Due : *Wednesday*, February 20, 2008

Problem 1: For the following forward-path transfer function $G(s)$ and feedback-path transfer function $H(s)$

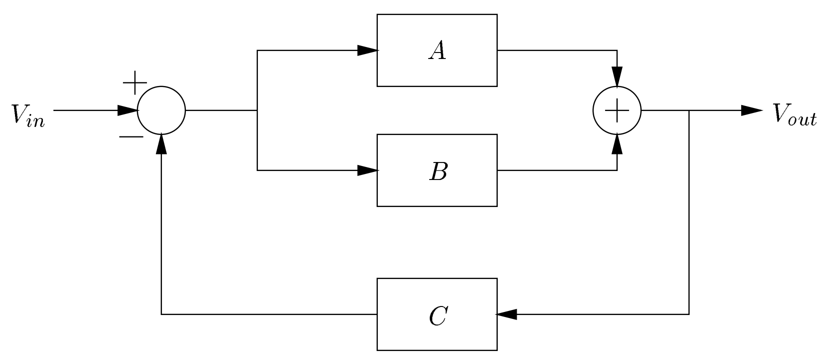
$$G(s) = \frac{K}{s^2} \qquad H(s) = \frac{s+1}{0.1s+1}$$

find the standard transfer functions $T(s)$, $L(s)$, $L_0(s)$, and $C/R(s)$, and simplify them into ratios of polynomials. Also, find the characteristic equation $P(s)$ and simplify it into a simple polynomial.

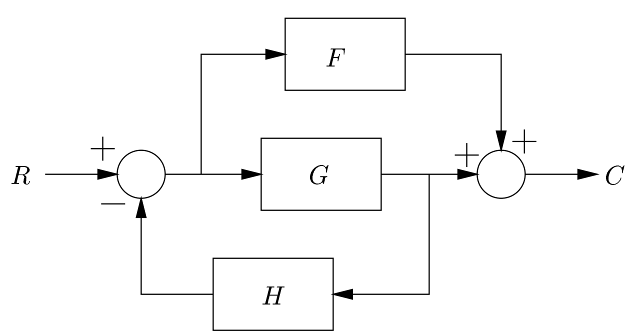
Definitions:

$T(s) = -GH$	Loop Transmission
$L(s) = GH$	Loop Transfer Function
$L_0(s) = L(s)/K$	Normalized Loop Transfer Function
$C/R(s) = \frac{G}{1+GH}$	Closed Loop Transfer Function

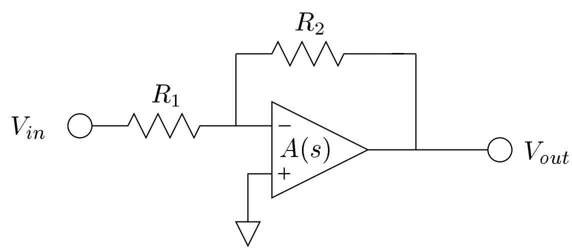
Problem 2: For the system shown below, determine how the fractional change in closed-loop gain depends on fractional changes in the parameters A, B, and C.

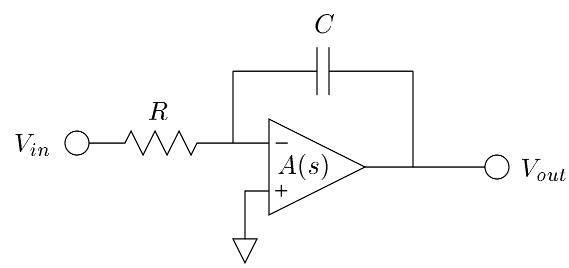


Problem 3: Simplify the following block diagram into a single block:

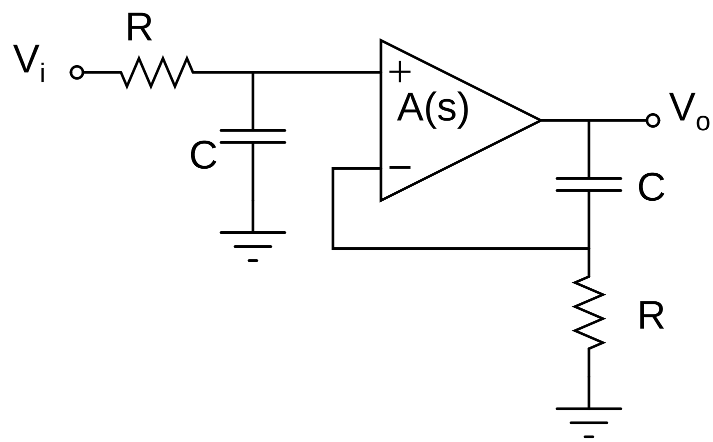


Problem 4: Derive the block diagrams for the op amp circuits below. Assuming that the op-amp gain $A(s)$ is constant and goes to infinity, do the answers check with expectations?





Problem 5: For the following op amp circuit, draw the block diagram and manipulate it into the unity feedback form. Also, what is the ideal transfer function?



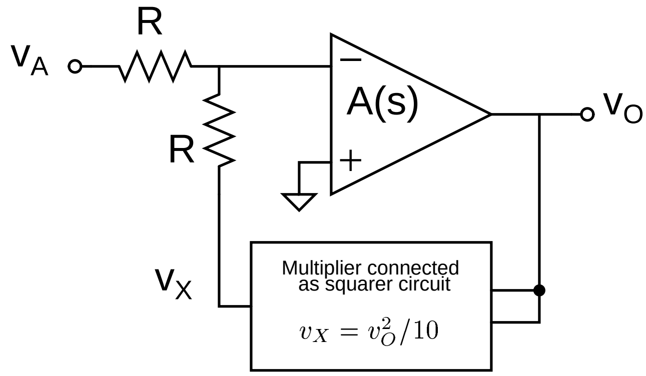
Problem 6: Sal Lektra wants to build an electric car using a DC motor that he found on Junkyard Wars. He mounts the motor in a car, with the shaft directly driving the wheels. The wheels have a diameter of 22 inches. When he tows the car behind his pickup at 50 mph, he measures 80 volts across the motor terminals. When 80 volts is connected to the motor terminals, it takes 40 seconds to reach 49 mph from a standing start. Ignore friction and inductance, and remember that there are 5280 feet in a mile.

Based on the above measurements, find the motor transfer function $\frac{\dot{\Theta}}{V_m}(s)$.

Problem 7: A square-rooting circuit, using a technique similar to that of the divider shown in lecture, is shown below. What is the ideal input-output relationship for this circuit (assuming an ideal op amp)? What is the input-output transfer function if the op amp transfer function is

$$A(s) = \frac{a_o}{\tau s + 1}?$$

Determine the range of input voltages for which the square rooter is stable for this $A(s)$. How does input level affect the speed of response?



Problem 8: A DC motor is to be used in a position servomechanism, but there is a choice as to how the machine can be connected. The load is an inertia connected to the motor through a flexible (springy) shaft. The motor can be connected as an armature-controlled machine supplied with constant field current from a current source, or as a field controlled machine supplied with constant armature current from a current source, as shown in Figures 1 and 2. These figures show the armature-controlled and field-controlled connections, respectively.

When operated in the armature-controlled mode (Figure 1), a variable voltage is applied to the armature as an input to the motor to change the shaft position. Alternatively, the variable control voltage is applied to the field terminals to vary the shaft position when the motor is operated in the field-controlled mode (Figure 2).

Derive the transfer functions that relate the angular displacement of the load inertia to the input voltage for the two different configurations illustrated in Figures 1 and 2. Make up variable names (preferably consistent with the lab handouts) for quantities you need in the block diagrams. Carefully specify your choices.

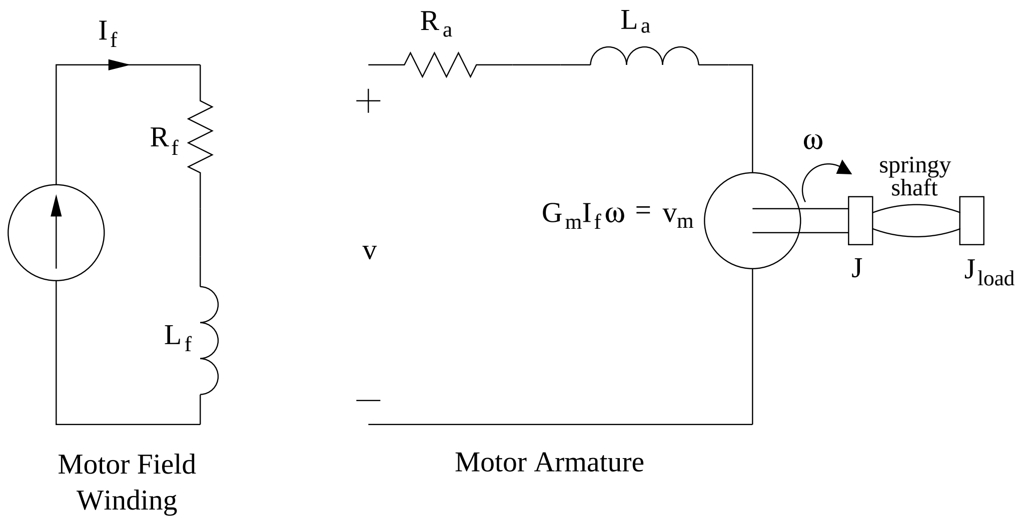


Figure 1: Armature control

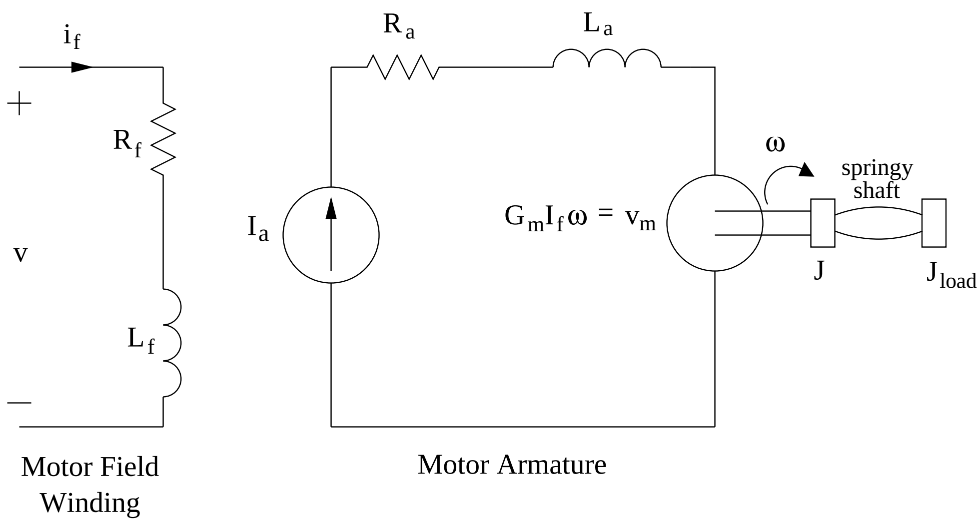


Figure 2: Field control