MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Success of Bag Searches 06/17/97

Random variable Z is the time (in minutes) it takes to find a lost bag.

$$Z = \begin{cases} 5 & \text{w.p. } 0.5 \\ 10 & \text{w.p. } 0.4 \\ 20 & \text{w.p. } 0.1 \end{cases} \quad (1)$$

(note: w.p. stands for with probability. The above is exactly the same as saying, $P(Z = 5) = 0.5$, $P(Z = 10) = 0.4$, $P(Z = 20) = 0.1$.)

On the BOS-DCA route we have searched for 12 minutes and no bags were found (the search was interrupted).

$$Q = \text{"initial probability that the bag in question is on board"} = \frac{2}{3}. \quad (2)$$

We want P (bag on board j not found in 12 minutes).

Let us start by defining the events:

C : Bag is on board.
D : Bag is not found in first 12 minutes.

$$P(C|D) = \frac{P(C\&D)}{P(D)} = \frac{P(D|C)P(C)}{P(D\&C) + P(D\&\overline{C})} = \frac{P(D|C)P(C)}{P(D|C)P(C) + P(D|\overline{C})P(\overline{C})} \quad (3)$$

Here, we are given $P(C) = Q = \frac{2}{3}$ and from the distribution of Z we get,

$$P(D|C) = P(\text{"search would take more than tvelve minutes (when there is a bag)"}) \quad (4)$$
$$= P(Z > 12) = P(Z = 20) = 0.1. \quad (5)$$

We also have that $P(D|\overline{C}) = 1$ because if the bag is not on board, there is no chance we will find it (on board) within 12 minutes.

Plugging in to our formula above we get,

$$P(C|D) = \frac{P(D|C)P(C)}{P(D|C)P(C) + P(D|\overline{C})P(\overline{C})} = \frac{0.1 \cdot \frac{2}{3}}{0.1 \cdot \frac{2}{3} + 1 \cdot \frac{1}{3}} = \frac{1}{6} \quad (6)$$

So the probability has shrunk from $\frac{2}{3}$ to $\frac{1}{6}$.

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