MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Quiz Solutions 07/17/97

Mishaps occur at one of 300 work sites. Each mishap is equally likely to happen in each of the 300 locations. In one year, there are in total 8 mishaps.

(1) $P\,(\text{all 8 mishaps occur at site \#274}) =$

$P\,(\text{first mishap occurs at \#274}) \times P\,(\text{second mishap occurs at \#274}) \times \cdots \times P\,(\text{eighth ...}) =$

$\frac{1}{300} \times \frac{1}{300} \times \cdots \times \frac{1}{300} = \frac{1}{300}^{8}$

(2) This is analogous to the birthday problem we solved in class.

$A1$: first mishap occurs in a place.

$A2$: second mishap occurs in a place with no prior mishaps.

$A3$: third mishap occurs in a place with no prior mishaps.

$A4$: fourth mishap occurs in a place with no prior mishaps.

$A5$: fifth mishap occurs in a place with no prior mishaps.

$A6$: sixth mishap occurs in a place with no prior mishaps.

$A7$: seventh mishap occurs in a place with no prior mishaps.

$A8$: eighth mishap occurs in a place with no prior mishaps.

$P\,(\text{no two mishaps occur at the same site}) =$

$P\,(A1 \text{ and } A2 \text{ and } A3 \text{ and } A4 \text{ and } A5 \text{ and } A6 \text{ and } A7 \text{ and } A8) =$

$P\,(A1) \times P\,(A2 \mid A1) \times P\,(A3 \mid A2 \text{ and } A1) \times \cdots \times$

$P\,(A7 \mid A6 \text{ and } A5 \text{ and } A4 \text{ and } A3 \text{ and } A2 \text{ and } A1) \times$

$P\,(A8 \mid A7 \text{ and } A6 \text{ and } A5 \text{ and } A4 \text{ and } A3 \text{ and } A2 \text{ and } A1) =$

$\frac{300}{300} \times \frac{299}{300} \times \frac{298}{300} \times \frac{297}{300} \times \frac{296}{300} \times \frac{295}{300} \times \frac{294}{300} \times \frac{293}{300} \approx 0.91$

(3) The number of mishaps at site #66 is $\text{Bin}\left(8, \frac{1}{300}\right)$.

$P\,(\text{exactly two mishaps at \#66}) = \binom{8}{2} \frac{1}{300}^{2} \frac{299}{300}^{6} = \frac{56 \times 299^6}{2 \times 300^8} \approx 0.0003$

(4) $P\,(\text{no mishaps at \#129} \mid \text{exactly two happened at \#66}) =$

$\frac{P(\text{"no mishaps at \#129" and "exactly two happened at \#66"})}{P(\text{exactly two happened at \#66})} = \frac{\binom{8}{2} \frac{1}{300}^{2} \frac{298}{300}^{6}}{\binom{8}{2} \frac{1}{300}^{2} \frac{299}{300}^{6}} = \frac{298}{299}^{6} \approx 0.98$

This could have been immediately obvious, because if two mishaps happened at #66, there are 6 mishaps that did not. The mishaps that did not happen at #66 have equal probability of occurring anywhere else. The probability that all six occur somewhere else than at #129 is just $\frac{298}{299}^{6}$.

(5) If each of the sites from #1 to #30 have the same number of mishaps, they must all have exactly zero mishaps (there are only eight mishaps in all). The probability that a single mishaps occurs at one of the sites between 1 and 30 is $\frac{30}{300}$ and the probability that it does not occur there is correspondingly $\frac{270}{300}$. For eight independent mishaps we have $\frac{270}{300}^8 \approx 0.43$

Note! For this last part, we can not first figure out the probability there are no mishaps at #1, and raise that answer to the 30 power. For that we would need independence, but if there were no mishaps at #1, that increases the probability of mishaps at #2, etc.

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