Information-Theoretic Considerations for Symmetric, Cellular, Multiple-Access Fading Channels—Part I

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***Abstract*—A simple idealized linear (and planar) uplink cellular multiple-access communication model, where only adjacent cell interference is present and all signals may experience fading, is considered. Shannon theoretic arguments are invoked to gain insight into the implications on performance of the main system parameters and multiple-access techniques.**

In Part I, we specialize to the linear model and address the case of practical importance where the cell-site receiver processes only the signals received at this cell site, and where the actively interfering users assigned to other cells (but not those within the cell) are interpreted as Gaussian noise (worst case assumption). We assume that the cell-site receiver is aware of the instantaneous signal-to-noise ratios for all users assigned to that cell while the users' transmitters do not have access to this side information. We first investigate several different scenarios and focus on the effect of fading and intercell interference and then provide a general formulation for an achievable rate region (inner-bound) of which all the different scenarios are special cases. The features of TDMA and wideband (WB) intracell multiple-access techniques are examined as well as the role of (optimized) fractional intercell time-sharing (ICTS) protocol. With no fading, orthogonality within the cell is optimal (not unique, however) with or without intercell interference. When fading is present and intercell interference is not dominant, WB intracell access is found advantageous due to an inherent fading averaging effect. This conclusion may reverse when intercell interference takes place and exceeds a given threshold; and then intracell orthogonal signaling (TDMA) is preferable. With intercell interference present, it is demonstrated that fractional ICTS may prove beneficial as compared to full exploitation of time (no sharing) at each cell, where the latter is known to be optimal under no fading conditions and an ultimate receiver which optimally processes all the information received at all cell sites.

In Part II, the model is extended to account for cell-site receivers that may process also the received signal at an adjacent cell site, compromising thus between the advantage of incorporating additional information from other cell sites on one hand and the associated excess processing complexity on the other. Various settings which include fading, TDMA, WB, and (optimized) fractional ICTS protocols are investigated and compared. In this case and for the WB approach and large number of users per cell it is found, surprisingly, that fading may enhance performance in terms of Shannon theoretic achievable rates. The linear model is extended to account for general linear and planar configurations. The effect of random number of users per cell is investigated and it is demonstrated that randomization is beneficial. Certain aspects of diversity as well as some features of TDMA and orthogonal code-division multiple-access (CDMA) techniques in the presence of fading are studied in an isolated cell scenario.

***Index Terms*— Achievable throughput, CDMA, cellular IT models, fading, Shannon theory, TDMA, wideband.**

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I. INTRODUCTION

With the increased interest in cellular wireless communication systems as is reflected by the extensive literature in the area and the emerging standards for practical systems ([1]–[10] and references therein), intensive efforts are invested to assess the theoretical and ultimate limitations of these systems. Towards this end, information-theoretic arguments have been recently employed to gain insight into the implications on performance of the main system parameters and protocols, resorting to relatively simple models which lend themselves to analytical investigation. Most of the models analyzed are basically single-user models, that is, the other interfering users in nonorthogonal accessing protocols are interpreted as an additive noise (usually Gaussian) component ([11]–[18] and references therein). The capacity of a single-user fading channel has also been extensively studied [4, and references therein], [19]–[26]. Recently, more elaborate models which address the basic multiuser features of the system were introduced and analyzed [27]–[50]. In some of the above mentioned multiple-access analyses [27], [29]–[31], the channel was treated as known and no time-varying fading phenomena was addressed; while in others [18], [32]–[48], the time-varying nature (fading) of the channel was accounted for. Shannon theoretic analyses which are mainly relevant to code-division multiple-access (CDMA) systems using single and/or multiaccess communication models with or without fading present are also well documented [43], [50]–[70]. Most of the multiple-access information-theoretic models [28], [36]–[43] are basically adapted for an isolated cell multiaccess system in which the cell site is interested in the reliable decoding of all users (signals) present which are accompanied by a Gaussian noise component. The effect of intercell interference, whenever present, has not been explicitly considered in these models. For uncoded communication the multiuser detection approach in a variety of fading and nonfading channels and a wide spectrum of modulation/detection techniques has been intensively investigated. See [71]–[73] and references therein, for a tutorial review. See [74]–[76] for information-theoretic considerations combined with multiuser detectors and [77] for single-user-based variants, and other linear-transformation-based processing techniques.

The efficiency of channel-accessing protocols using elementary Shannon theoretic arguments and treating active interfering intra- and intercell signals as noise is discussed in [13], [17], [61], and [78]–[80]. Cases where the channel is available to the receiver and transmitter and aggregate rates are of interest are discussed in [32], [36], [38]–[42], [44], [81], and [82]. The capacity in terms of number of active users per cell (or in properly normalized units of erlangs) or information rates (with a certain level of reliability) per second per cell per hertz and the relative efficiency of various accessing protocols is the subject of numerous papers. See, for example, [83]–[94].

A simple model for cellular communication which accounts for the intercell interference was introduced in [27] (see also [29]). The model in [27] assumes either a planar (two-dimensional) or linear (single-dimensional) cellular array configuration where intercell interferences emerges from adjacent cells only and characterized by a single parameter $0 \le \alpha \le 1$. The problem is treated within the multiple-access channels information-theoretic framework [95] where a super-receiver is assumed which has a delayless access to the information received at all the cell sites. It is concluded [27] that under this model, the achievable rate per user (assuming all users operate at the same rate) increases or decrease with α (assuming $\alpha \ll 1$) depending on the value of the signal-to-noise ratio. Further, it is determined that under average power constraints, time-division multiple access (TDMA) within the cell (intracell) achieves optimal performance while any intercell time division (or time sharing) degrades performance. In [28], in contrast, it was determined that intracell TDMA is suboptimal when multipath fading is introduced under the assumption of no intercell interference $\alpha = 0$, i.e., an isolated cell scenario. Similar conclusions hold for frequency-division multiple access (FDMA) [43].

Here, we also focus on the uplink and introduce a simple, discrete-time, synchronous, idealized symmetric linear (or planar) model for the cellular system which is similar to that in [27]. We first specialize to the case of practical importance where the receiver at the cell site processes only the signal received at this cell site (no intercell site cooperation) and it is ignorant of the codebooks employed by users of other cells. The general problem falls within the class of multiple-access (the users within the cell) and interference (the users in other cells which transmit information to their own cell sites) Gaussian channel, the ultimate capacity region of which is a long-standing open problem [95], [96].

The actively interfering users assigned to other cells (but not those within the cell) are interpreted as Gaussian noise which can be viewed as a worst case assumption, and is in particular realistic when the codebooks of the interfering (adjacent-cell) users are not available at the cell-site receiver. The Gaussian approximation is also motivated by recent results on Euclidean-distance mismatched decoding in the multiple-access Gaussian channel under the random-coding (with Gaussian input distribution) regime [97], [98].

It is further assumed that the instantaneous signal-to-noise ratio (SNR) value for each relevant user is available to the receiver but not the transmitter, in contrast to cases where the transmitter is also aware of the fading state via a feedback channel [38]–[42], [44].

In this two-part paper, we investigate several different scenarios and focus on the effect of fading and intercell interference. In Part I, the features of TDMA and wide-band (WB), code-division multiple access (CDMA)-like, techniques are examined as well as intercell time-division protocols as partial time-sharing (or frequency-band) reuse. With no fading, intracell orthogonality is optimal (not unique, however) with or without intercell interference. When fading is present and the intercell interference is negligible, WB intracell approaches are found advantageous, in agreement with the results reported recently in [28] and [43]. This conclusion may reverse when intercell interference takes place and exceeds a given threshold and then intracell orthogonal TDMA is preferable. This is also the case when fading is assumed to affect only the transmitters in other cells, which models an idealized situation with perfect genie-aided unpenalized instantaneous power control within the cell. With intercell interference present, it is demonstrated that intercell partial (or full) time-division (and/or band-division) protocols may prove beneficial as compared to full exploitation of time/bandwidth resources at each cell. The latter is optimal for the setting considered in [27] (no fading and an ultimate receiver which optimally processes the signals required at all cell sites). A general formulation of an achievable rate region (inner bound) is given for which the above mentioned specific scenarios are special cases. The inherent information-theoretic significance of intercell protocols in the current model emerges in a natural way from the general formulation. In Part II, the model is extended to account for two cell-site processing, that is, the cell-site receiver processes the received signal from an adjacent cell in addition to its own received signal, and in this context various settings which include fading, intracell TDMA, as well as WB signaling and intercell time-sharing protocols are examined. This model compromises between the information-theoretic advantages of incorporating additional information from other cell sites on one hand and the associated excess processing complexity on the other. Certain aspects of diversity as well as different features of intracell orthogonal access techniques (TDMA versus CDMA) in a fading environment are discussed. Various aspects of random number of users and limiting behavior of the information rates are also addressed. For simplicity and brevity reasons, the results are presented for a flat-fading model but most of the results extend to multiray selective fading [4], [28], in a straightforward manner.

In the next section, the idealized, simple, linear, cellular, faded communication model and the underlying assumptions are introduced and the corresponding results are presented in Section III. The extensions to two-cell-site processing receivers and the two-antenna diversity systems are treated in Part II, Section I. Part II, Section II, addresses the planar cell configuration (and generalizations) and the discussion and summary in Part II, Section III, concludes this two-part paper. Technical details and proofs are placed in appendices.

II. System Model

Consider a simple, idealized, discrete-time, synchronous, infinite linear cellular, uplink communication model. The model is basically similar to that considered in [27] where it is assumed that the received signal at each cell site is interfered by the active users of the adjacent cells only. Let y_j^m stand for the received sample at cell m and discrete time j which is given by

$$y_j^m = \sum_{\ell=1}^{K} a_{\ell,j}^m x_{\ell,j}^{m-1} + \alpha \sum_{\ell=1}^{K} b_{\ell,j}^m x_{\ell,j}^m + \alpha \sum_{\ell=1}^{K} c_{\ell,j}^m x_{\ell,j}^{m+1} + n_j^m. \tag{2.1}$$

Here $x_{\ell,j}^m$ represents the jth-channel symbol of the ℓth user in the mth cell. The nonnegative coefficients $a_{\ell,j}^m$, $b_{\ell,j}^m$, and $c_{\ell,j}^m$ denote a possible fading processes experienced by the ℓth user at time j and cell m, where $a_{\ell,j}^m$ stand for the fading for the users of cell m while $b_{\ell,j}^m$, $c_{\ell,j}^m$ the fading of users in the adjacent cell $(m-1)$ and $(m+1)$, respectively, which affect cell m. It is assumed, unless otherwise stated, that $\{a_{\ell,j}^m\}$, $\{b_{\ell,j}^m\}$, and $\{c_{\ell,j}^m\}$, when viewed as individual random processes in the time index j, are all statistically independent of each other, identically distributed (having the same probability law), and strictly stationary and ergodic. Unless otherwise stated, all the fading random variables are normalized to unit power

$$E(a_{\ell,j}^m)^2 = E(b_{\ell,j}^m)^2 = E(c_{\ell,j}^m)^2 = 1$$

where E designates the expectation operator. The nonfading case is treated as a special case where all the variances of the corresponding fading random variables are set to zero. The inputs (codewords) of each user are assumed to be subjected to an average-power constraint P [99, ch. 7], that is,

$$\frac{1}{N} \sum_{j=1}^{N} E(x_{\ell,j}^m)^2 \le P, \qquad \underset{m\text{- integer}}{\ell = 1, 2, \cdots, K} \tag{2.2}$$

where N stands here for the codeword length and the expectation is taken over the respective codebooks selected by all the different users. Equation (2.1) implies perfect power control within the cell when no fading is present ($a_{\ell,j}^m = b_{\ell,j}^m = c_{\ell,j}^m = 1$, i.e., all intracell signals are received at the same power). The fading processes model then the time-varying changes which are not provided to the transmitter due to the lack of an adequately fast reliable feedback link. Our discrete-time models follow directly from the ideally bandlimited flat fading model [28], under the assumption of relatively negligible Doppler spread [36], which usually is valid in cases of relatively slowly moving users and reasonable system bandwidth [6]–[10]. Though the model considered in (2.1) is basically a single-ray model, the associated results can be extended to account for multipath fading as shortly discussed in Part II of the paper.

It is assumed, as is also done in [27] and [28], that there are exactly[1] K active users in each cell, signaling at the same information rate R nats per channel use (unless otherwise stated). The time-discrete model in (2.1) pre-assumes a symbol- and frame-synchronous system [100], [101] though, as will become clear in the following, the frame-synchronous assumption can be relaxed in certain cases when time sharing is not employed. Both these assumptions are obviously idealized and are invoked for the sake of simplicity and tractability. The received signal (y_j^m) at cell site m is accompanied by a white Gaussian noise which is represented by the zero-mean, independent and identically distributed (i.i.d.) Gaussian sequence $\{n_j^m\}$ with variance $E((n_j^m)^2) = \sigma^2$.

The intercell interference is modeled by a single parameter $\alpha \ge 0$ which represents the attenuation of the adjacent cell signals $(m-1, m+1)$ when received at cell site m. For the sake of simplicity, we assume that $\{x_{\ell,j}^m\}$ and $\{a_{\ell,j}^m\}$, $\{b_{\ell,j}^m\}$ and $\{c_{\ell,j}^m\}$ are real-valued signals. The extension to complex proper processes [102] is straightforward and involves only power and rate rescaling by a factor of 2.

Our primary assumptions in Sections III is that the cell site bases the detection of all the K users that belong to this cell only on the information received at that cell site. This is markedly different from the assumption made in [27], where an ultimate receiver which observes the information received at all cell sites was used. Further, we assume that the cell-site receiver is ignorant about the codebooks used by the users assigned to other cells. In Part II, these assumptions are somewhat relaxed and the cell-site receiver is allowed to process the received signals of two adjacent cells, where for this case it is assumed that the receiver is equipped with the codebooks of all the users ($2K$ of them) who are assigned to these two relevant cells.

The cell-site receiver (say at cell m) is assumed to be aware of the instantaneous signal-to-noise ratio of each user, that is,

$$\mathrm{SNR}_{\ell,j}^m \triangleq \mathrm{SNR}\, (a_{\ell,j}^m)^2 \Big/ \left\{ 1 + \alpha^2\, \mathrm{SNR} \sum_{\ell=1}^{K} [(b_{\ell,j}^m)^2 + (c_{\ell,j}^m)^2] \right\} \tag{2.3}$$

where $\mathrm{SNR} \triangleq P/\sigma^2$, but the transmitter is aware only of the statistical features of the fading parameters as no reliable feedback (with a small enough delay) from the receiver to the transmitter is assumed to be available.

We focus mainly on the case where all the users in the cellular system operate exactly at the same average rate of R nats per channel use.

In the following, unless otherwise stated, the signals of adjacent-cell interfering users $\{x_{\ell,j}^{m+1}\}$, $\{x_{\ell,j}^{m-1}\}$ are chosen from codebooks which are unknown at cell site m, and are treated as i.i.d. Gaussian noise samples. This is justified by assuming that the codebooks of all users are chosen randomly, governed by an underlying i.i.d. Gaussian distribution per symbol [95], and *independently* for each message transmission. In fact, the Gaussian assumption is a worst case and constitutes a saddle point as follows by a direct extension of the single-user case [103] (see Appendix I for short details).

The observation made in [97] and [98] indicates that for a nonfaded channel, a receiver which uses a Gaussian based optimal metric (Euclidean distance) cannot surpass the Gaussian

[1] In Part II, Section IV, the effect of a random Poisson distributed number of users in adjacent cells on the interference characteristics is examined.

capacity region in the case of an ergodic additive multiple-access non-Gaussian channel when Gaussian distributed codewords are selected. Recently [98], it has been also shown that similar results hold in a fading environment when the instantaneous signal-to-noise ratio is available at the receiver which in turn employs the optimally joint (for all users assigned to the specific cell) weighted (according to the instantaneous SNR values) Euclidean distance decoding. It is emphasized that the results of [97] and [98] apply to the mismatched Euclidean metric-based decoder while the saddle-point result assumes optimal maximum-likelihood decoding of the non-Gaussian channel. For small intercell interference $(\alpha \rightarrow 0)$, the results in [104] support this assumption (even if the codebooks of the adjacent-cell users were available to the cell-site receiver) and to some extent so do the important observations on the output statistics induced by a single-user capacity achieving codes [105]. Under these assumptions, no loss of generality is implied by considering $m = 0$ and in short-hand notations where the irrelevant index m is suppressed (and ± 1 is replaced by the respective $\pm$ signs) in (2.1), the relevant received signal is given by

$$y_j = \sum_{\ell=1}^{K} a_{\ell,j} x_{\ell,j}^0 + \alpha \sum_{\ell=1}^{K} b_{\ell,j} x_{\ell,j}^- + \alpha \sum_{\ell=1}^{K} c_{\ell,j} x_{\ell,j}^+ + n_j. \tag{2.4}$$

In the next section, the information-theoretic implications, under the current assumptions, of intracell multiaccess techniques (as WB and TDMA) and intercell fractional time-division protocols, with and without fading are presented, and formulated in a unified framework.

III. Achievable Rates: Linear-Cellular Model

In this section, we focus on the information-theoretic implications as predicted by the achievable rates of reliable communication of various intracell accessing protocols as WB and TDMA as well as fractional intercell time sharing (division). The implications of fading and of the intercell interference factor α are also considered. We start by examining special cases and then provide a unified formulation.

A. Wideband and TDMA Multiaccess

By wideband (WB) it is meant that each user in each cell uses all the bandwidth/time/power resources available to him/her and transmits information continuously at the same rate. The achievable rate R_{WB} of each user within a cell in nats per channel use equals

$$R_{\text{WB}} = \frac{1}{2K} E_{\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}} \log \left(1 + \frac{\text{SNR} \displaystyle\sum_{\ell=1}^{K} a_\ell^2}{1 + \alpha^2 \text{ SNR} \displaystyle\sum_{\ell=1}^{K} (b_\ell^2 + c_\ell^2)} \right) \tag{3.1}$$

where $E_{\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}}$ stands for the expectation with respect to the i.i.d. random vectors $\boldsymbol{a} = \{a_\ell\}_{\ell=1}^K$, $\boldsymbol{b} = \{b_\ell\}_{\ell=1}^K$, and $\boldsymbol{c} = \{c_\ell\}_{\ell=1}^K$ and natural logarithms are used throughout. This result is a special case of a general formulation to be presented in the end of this section which is proved in Appendix I. See also [35]–[47] for a capacity-region derivation under fading conditions.

It should be emphasized that the rate (3.1) is not necessarily the highest rate possible even under generally the nonoptimum (to be shown) WB approach if the cell site is provided with the information of the adjacent-cell users' codebooks. The general problem is closely related to the determination of the capacity region of a multiple-access and interference channel which in general has not yet been solved [95], [96]. This point will be further demonstrated later in this section.

Note that (3.1) implies, as mentioned also in Appendix I, that the receiver must be aware only of the instantaneous signal-to-noise ratios of all users within the cell, $\text{SNR}_{\ell,j}^o$ (2.3) $\forall \ell = 1, 2, \cdots, K$ and $\forall j$, but not of all the individual fading values of the desired and interfering signals $\{a_{\ell,j}\}$, $\{b_{\ell,j}\}$, and $\{c_{\ell,j}\}$. This assumption is quite practical in cases where the dynamics of the fading process (that is, the Doppler spread [106]) is much smaller than the bandwidth available which translates here to the observation that the independent processes $\{a_{\ell,j}\}$, $\{b_{\ell,j}\}$, and $\{c_{\ell,j}\}$ are individually highly correlated for each ℓ, when viewed as processes in the discrete-time index j.

We turn now to examine a TDMA multiaccess protocol within a cell (intracell) while no time division is exercised among different cells (all cells use the full time resources). Every user transmits with average power KP at its assigned slot (that occurs at rate $1/K$ of the channel signaling rate). It follows straightforwardly (again, a special case of the general formulation to be presented) that the achievable rate R_{TDMA} equals

$$R_{\text{TDMA}} = \frac{1}{2K} E_{a,b,c} \log \left(1 + \frac{a^2 \text{ SNR}_T}{1 + \alpha^2 \text{ SNR}_T (b^2 + c^2)} \right) \tag{3.2}$$

where $\text{SNR}_T \triangleq K \cdot \text{SNR} = KP/\sigma^2$ is the total signal-to-noise ratio of the system. The denominator in the argument of the logarithm reflects the fact that for each user there are only two active interferers each located in a neighboring cell. Again, $E_{a,b,c}$ stands for the expectation with respect to the individual i.i.d. random variables a, b, and c. We will consider several special cases and start with the simplest one.

B. No Interference $(\alpha = 0)$, No Fading $(a_{\ell,j} = b_{\ell,j} = c_{\ell,j} = 1)$

As is well known [95] in this case, the WB and TDMA strategies yield the same optimal result

$$R_{\text{WB}} = R_{\text{TDMA}} = \frac{1}{2K} \log(1 + \text{SNR}_T). \tag{3.3}$$

The no-interference case $(\alpha = 0)$ leads to isolated cell considerations and in this case the rate in (3.3) yields the highest achievable rate as intercell sharing protocols and the interfering adjacent cell users are irrelevant here.

C. No Interference ($\alpha = 0$), Fading Present

In this case, the superiority of the WB approach over TDMA is demonstrated in the following inequality:

$$R_{\rm WB} = \frac{1}{2K} E_{\boldsymbol{a}} \log \left(1 + {\rm SNR}_T \sum_{\ell=1}^{K} \frac{1}{K} a_\ell^2\right) \geq \frac{1}{2K} \sum_{\ell=1}^{K} \frac{1}{K} E_a \log(1 + {\rm SNR}_T\, a_\ell^2) = R_{\rm TDMA} \quad (3.4)$$

which follows by Jensens' inequality. This interesting conclusion has been pointed out in a continuous-time setting and multipath fading environment in [28], and in the context of FDMA in [43].

An interesting observation for the WB strategy is that for a large number of users $K \gg 1$, the penalty incurred due to the fading process is negligible and

$$R_{\rm WB} \underset{K \gg 1}{\approx} \frac{1}{2K} \log(1 + {\rm SNR}_T). \quad (3.5)$$

This follows from the proposition proved in Appendix II which stems straightforwardly from the law of large numbers.

Consider now a special case were the random variables $\{a_\ell\}$ are i.i.d. Rayleigh and thus the random variable

$$S = \sum_{\ell=1}^{K} a_\ell^2$$

is chi-square (Appendix III, (AIII.1)) distributed with $2K$ degrees of freedom (Appendix III, (AIII.2), with $\sigma_a^2 = 1/2$). In this case

$$R_{\rm TDMA} = \frac{1}{2K} \int_0^\infty e^{-s} \log(1 + {\rm SNR}_T \cdot s)\, ds = -\frac{1}{2K} e^{({\rm SNR}_T)^{-1}} E_i(-({\rm SNR}_T)^{-1}) \quad (3.6)$$

where

$$E_i(x) = \int_{-\infty}^{x} (e^\alpha/\alpha)\, d\alpha$$

is the exponential integral function [107, Sec. 8.21] (see also [11] and references therein); while

$$R_{\rm WB} = \frac{1}{2K} \int_0^\infty \frac{s^{K-1}}{\Gamma(K)} e^{-s} \log(1 + {\rm SNR} \cdot s)\, ds = \frac{(-1)^K}{2K!} \frac{\partial^{K-1}}{\partial \mu^{K-1}} \left[\frac{-1}{\mu} e^{\mu({\rm SNR})^{-1}} E_i(-\mu({\rm SNR})^{-1})\right]_{\mu=1} \quad (3.7)$$

is given in terms of the exponential integral function and its derivatives (which can be expressed explicitly [107]). The asymptotic behavior is determined by

$$R_{\rm WB} \approx \begin{cases} \dfrac{\rm SNR}{2}, & {\rm SNR} \to 0 \\ \dfrac{1}{2K} \log(1 + {\rm SNR}_T) + (\psi(K) - \ln K), & {\rm SNR} \to \infty \end{cases} \quad (3.8)$$

where $\psi(\nu)$ is Euler's psi function [107, Sec. 8.36] and for K integer

$$\psi(K+1) = -\mathbb{C} + \sum_{\ell=1}^{K} (1/\ell)$$

where $\mathbb{C} \approx 0.577215$ is the Euler constant [107, Sec. 9.73]. Note that [107, Sec. 8.36.7, eq. (2)] for $K \gg 1$, indeed as expected (3.5), the asymptotic terms that (3.8) yields $(1/2K)\log(1 + {\rm SNR}_T)$ where the effect of the fading is averaged out and eventually disappears. For ${\rm SNR}_T \gg 1$, it follows by asymptotic results of the exponential integral function that

$$R_{\rm TDMA} \underset{{\rm SNR}_T \gg 1}{\approx} \frac{1}{2K} (\log(1 + {\rm SNR}_T) - \mathbb{C}). \quad (3.9)$$

Equation (3.9) indicates a penalty of ~1.78 dB in terms of power efficiency which accounts in this case (${\rm SNR}_T \gg 1$) for the fading process.

We proceed to examine the more interesting case where intercell interference $\alpha > 0$ is present, and first investigate the case with no fading.

D. Intercell Interference ($\alpha > 0$), No Fading ($a_{\ell,j} = b_{\ell,j} = c_{\ell,j} = 1$)

By (3.1) and (3.2), it follows immediately that

$$R_{\rm WB} = R_{\rm TDMA} = \frac{1}{2K} \log\left(1 + \frac{{\rm SNR}_T}{1 + 2\alpha^2\, {\rm SNR}_T}\right) \quad (3.10)$$

which again demonstrates the equivalence of the WB and TDMA intracell accessing strategies in this case. Indeed, under these strategies our simple model is interference-limited and for $\alpha^2\, {\rm SNR}_T \gg 1$, that is, a large total normalized (to the Gaussian noise variance σ^2) interference power, the limiting performance is governed by

$$R_{\rm WB} = R_{\rm TDMA} \approx \frac{1}{2K} \log\left(1 + \frac{1}{2\alpha^2}\right), \quad 2\alpha^2\, {\rm SNR}_T \gg 1. \quad (3.11)$$

It is in order to emphasize at this point that the results derived under the assumption made here that other cell users act like Gaussian interference and that perfect power control is available, may change dramatically when the cell site has knowledge of the codebooks used in the adjacent cells, or if power control is relaxed [46]. The achievable rate under the Gaussian interference assumption (with either WB or TDMA) and perfect power control is given in (3.10), which demonstrates an interference-limited behavior in (3.11).

For comparison, if the intracell signals are also interpreted as Gaussian noise (i.e., single-user decoding (SUD) is done at the cell site, as is usually done in practice [61]), the achievable rate for TDMA is unchanged (3.10), but for WB the achievable rate is

$$R_{\rm WB(SUD)} = \frac{1}{2} \log\left(1 + \frac{\rm SNR}{(K-1)\, {\rm SNR} + 2\alpha^2\, {\rm SNR}_T + 1}\right). \quad (3.12)$$

Let K become large and then let $\mathrm{SNR}_T \to \infty$. Then (3.12) becomes

$$R_{\mathrm{WB(SUD)}} \approx \frac{1}{2K}\left(\frac{1}{1+2\alpha^2}\right). \tag{3.13}$$

Thus $R_{\mathrm{WB(SUD)}}$ too demonstrates an interference-limited behavior. But this interference-limited behavior no longer occurs when the cell-site receiver knows the codebooks of the transmitters in the adjacent cells, and decodes these users too. Using such an "adjacent-cell decoder" (ACD), we can achieve the rate

$$R_{\mathrm{WB(ACD)}} = R_{\mathrm{TDMA(ACD)}} = \min(R_1, R_2) \tag{3.14a}$$

where

$$R_1 = \frac{1}{6K}\log(1+\mathrm{SNR}_T + 2\alpha^2\ \mathrm{SNR}_T) \tag{3.14b}$$

$$R_2 = \frac{1}{4K}\log(1+2\alpha^2\ \mathrm{SNR}_T). \tag{3.14c}$$

This rate is obtained by considering the cell and its two adjacent cells as a single multiple-access channel. Equation (3.14b) represents the total rate of the $3K$ users (K within the cell and $2K$ in the adjacent cells); and (3.14c) represents the total rate for the two adjacent cells, when the signals from the main cell are known [95].

Now it is easily verified that

$$R_{\mathrm{WB}} > R_{\mathrm{WB(ACD)}}, \qquad \mathrm{SNR}_T < \mathrm{SNR}_{\mathrm{th}} \tag{3.15a}$$

$$R_{\mathrm{WB}} \le R_{\mathrm{WB(ACD)}}, \qquad \text{otherwise} \tag{3.15b}$$

where the threshold SNR is

$$\mathrm{SNR}_{\mathrm{th}} = \frac{1+4\alpha^2 - 8\alpha^4 + \sqrt{1+8\alpha^2}}{16\alpha^4}. \tag{3.15c}$$

(Also, when $\mathrm{SNR}_T = \mathrm{SNR}_{\mathrm{th}}$, then $R_1 = R_2$.) Thus we can conclude that when the cell-site decoder knows the codebooks of the users in the adjacent channels, the system is not interference-limited. This result extends also to more complicated linear and planar cellular models, as will be mentioned in Part II.

The fact that WB (or TDMA) predicted result (3.11), where each cell utilizes the full time/bandwidth resources is not optimal in general for $\alpha > 0$, even under the assumption that the cell-site decoder is unaware of the codebooks of the adjacent-cell users for an unfaded or faded scenario, is demonstrated by allowing for an intercell time-sharing (division) protocol.

1) Intercell Time Sharing: Consider a fractional intercell time-sharing (ICTS) protocol described in Fig. 1, where the transmission time is divided into frames of N_F channel symbols per frame and where the users in the even-numbered cells use the starting $1-\theta$ fraction of the frame (that is, modulate the first $(1-\theta)N_F$ symbols) while odd-numbered cell users employ the ending $1-\theta$ fraction of the frame (modulate the last $(1-\theta)N_F$ symbols) where $0 \le \theta \le 1/2$ is the intercell fractional time-sharing (division) parameter. The overlapped time fraction which is interfered is $1-2\theta$ of the frame and for $\theta = 0$ no intercell time division is exercised while for $\theta = 1/2$ full time division, which absolutely eliminates the interference, is employed. The transmission rate

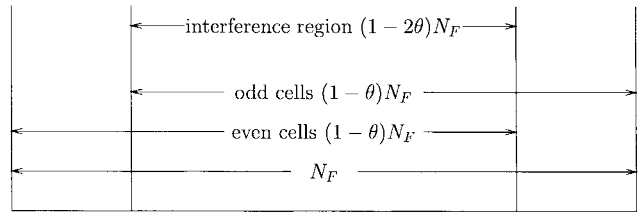

Fig. 1. Intercell time-sharing (ICTS) protocol, $0 \le \theta \le 1/2$, intercell fractional time-sharing parameter.

with either WB or TDMA intracell strategy where neither the transmitter nor the receiver are making use of the *a priori* agreed upon knowledge on the location of the interfered fraction of the frame is

$$R_{\mathrm{ICTS1}} = \frac{1-\theta}{2K}\log\left(1+\frac{\mathrm{SNR}_T/(1-\theta)}{1+2\alpha^2\,\dfrac{\mathrm{SNR}_T(1-2\theta)}{(1-\theta)^2}}\right). \tag{3.16}$$

The equation follows by noticing that whenever a transmitter is active (a fraction $(1-\theta)$ of the time) it uses average power of $P/(1-\theta)$ and the average interference power is $(KP/(1-\theta))\cdot((1-2\theta)/(1-\theta))$, where $(1-2\theta)/(1-\theta)$ is the fraction of time within the transmission in which active interference takes place. When the transmitters and receiver exploit the fact that the fraction of time the interference occurrence is *a priori* available, but the transmitters do not attempt to optimize their powers, the resultant rate is

$$R_{\mathrm{ICTS2}} = \frac{1-2\theta}{2K}\log\left(1+\frac{\mathrm{SNR}_T/(1-\theta)}{1+\dfrac{2\alpha^2\ \mathrm{SNR}_T}{1-\theta}}\right) + \frac{\theta}{2K}\log\left(1+\frac{\mathrm{SNR}_T}{(1-\theta)}\right). \tag{3.17}$$

The equation readily follows by noticing that a fraction $1-2\theta$ of the time there is interference of power $2\alpha^2 KP/(1-\theta)$ as each transmitter employs power $P/(1-\theta)$ while for a fraction of θ, no interference is present at all and for another fraction of θ no transmission is attempted. See [79] for similar arguments termed there as adaptive TDMA. By the convexity of $\log(1+a/x)$ as a function of $x \ge 0$, it follows as expected (as more information is incorporated in the receiver and transmitter) that $R_{\mathrm{ICTS2}} \ge R_{\mathrm{ICTS1}}$. The same rate R_{ICTS2} is also achievable from (3.1). This is since the different interference levels can be viewed as a fading processes which assumes only two values. Optimizing the transmitted power in the interfered and uninterfered slots yields even a better result given here by

$$R_{\mathrm{ICTS3}} = \frac{1-2\theta}{2K}\log\left(1+\frac{\mu_1}{1+2\alpha^2\mu_1}\right) + \frac{\theta}{2K}\log(1+\mu_2) \tag{3.18}$$

where μ_1 and μ_2 solve the equations as shown in (3.19a) at the bottom of this page, and

$$\mu_2 = \begin{cases} 2\ \mathrm{SNR}_T, & \text{for } \theta = 0 \\ \mathrm{SNR}_T/(1-\theta), & \text{for } \alpha = 0 \\ (\mathrm{SNR}_T - (1-2\theta)\mu_1)/\theta \end{cases}. \tag{3.19b}$$

The rates of interest R_{ICTS1}, R_{ICTS2}, or R_{ICTS3} should then be optimized over $0 \le \theta \le 1/2$ to determine the best intercell fractional time-sharing strategy. It is evident that θ increases with the increase of the normalized interference power $\alpha^2\ \mathrm{SNR}_T$ and there is a threshold beyond which the optimal intercell cooperation strategy dictates full time division (sharing), that is, $\theta = 1/2$, which in our model implies no intercell interference. For example, by considering the zeros of the derivative of R_{ICTS2} with respect to θ

$$\begin{aligned} 0 = \frac{\partial R_{\mathrm{ICTS2}}}{\partial \theta} &= -2\log\left(1 + \frac{\mathrm{SNR}_T}{(1-\theta) + 2\alpha^2\ \mathrm{SNR}_T}\right) \\ &+ \log\left(1 + \frac{\mathrm{SNR}_T}{1-\theta}\right) \\ &+ \frac{(1-2\theta)\ \mathrm{SNR}_T}{(1-\theta+2\alpha^2\ \mathrm{SNR}_T)(1-\theta+(2\alpha^2+1)\ \mathrm{SNR}_T)} \\ &+ \frac{\theta\ \mathrm{SNR}_T}{(1-\theta)(1-\theta+\mathrm{SNR}_T)} \end{aligned} \tag{3.20}$$

it is concluded that the optimal fractional ICTS strategy (θ_{opt}) is either $\theta_{\mathrm{opt}} = 0$ (no ICTS) or $\theta_{\mathrm{opt}} = 1/2$ (full ICTS) according to

$$\theta_{\mathrm{opt}} = \begin{cases} 0, & \mathrm{SNR}_T < (1-4\alpha^2)/(8\alpha^4) \\ 1/2, & \text{otherwise} \end{cases}. \tag{3.21}$$

The same conclusion follows for the ICTS1 strategy (3.16). When power is optimized, as is done in the ICTS3 strategy, fractional ICTS $0 < \theta < 1/2$ may turn optimal as is demonstrated in Fig. 2, where KR_{ICTS3} is shown versus $0 \le \theta \le 1/2$ for $\mathrm{SNR}_T = 10$ (10 dB), 70 (18.45 dB), and 100 (20 dB) and $\alpha = 0.2$. The regions of α^2, SNR_T for which no ICTS $(\theta = 0)$, fractional ICTS $(0 < \theta < 1/2)$, and full ICTS $(\theta = 1/2)$ are advantageous are readily found by the curves specified by $\partial R_{\mathrm{ICTS3}}/\partial\theta = 0$ for $\theta = 0$ and $\theta = 1/2$, and are shown in Fig. 3.

As is readily seen from the expressions of R_{ICTS1}, R_{ICTS2}, and R_{ICTS3} the fractional intercell time sharing eliminates the interference-limited behavior of the system with no intercell time division with either WB or TDMA intracell accessing. For example, full intercell cooperation, that is $\theta = 1/2$, results in no intercell interference and in this case for any α

$$R_{\mathrm{ICTS1}} = R_{\mathrm{ICTS2}} = R_{\mathrm{ICTS3}} = \frac{1}{4K}\log(1 + 2\mathrm{SNR}_T). \tag{3.22}$$

To see that the benefits of intercell time sharing *do not* stem from the suboptimal detection in the WB case with no intercell time sharing $(\theta = 0)$ due to the Gaussian assumption invoked to model the intercell interference, consider the case $\alpha = 1$. For this case, there is full symmetry (power-wise) between the current cell users and the interfering signals. Therefore, the achievable rate R_{IC} of the interference channel providing the cell-site receiver with the codebooks of all the users in the neighboring cells is

$$R_{\mathrm{IC}} = \frac{1}{6K}\log(1 + 3\mathrm{SNR}_T). \tag{3.23}$$

This is since for a fully symmetric case the capacity of the interference channel equals that of the multiple-access channel [95], [96], which for the case at hand has $3K$ users, (K from the current cell and K from each neighboring cell). The superiority of the rates in (3.22) as compared to those in (3.23) is readily verified.

Note that though the rate (3.23) corresponds to the interference channel, it is not the optimum that can be achieved (as already concluded by finding a higher rate (3.22)) and that is since the WB and no intercell cooperation assumptions were imposed. It follows thus that WB and no ITS is not necessarily optimal when the system is considered as a whole, multiaccess/interference system. Had the capacity region of the general system been known, its features would have revealed also what is the optimum intercell sharing (division) protocol. It is already demonstrated (at least for the case $\alpha = 1$), that intercell cooperation is theoretically demanded. Note that in this case, $\alpha = 1$, with WB intracell channel accessing and no intercell time sharing $(\theta = 0)$ each cell site can decode, with no loss of optimality, all the users within this cell and the two adjacent cells. However, with full ICTS $\theta = 1/2$, the cell-site receiver is not able to decode the users in the adjacent cells as on their assigned time slots the adjacent cells interfere with each other while no active signals of the users within the cell are present. Thus for $\alpha = 1$, the maximum achievable information rate is $R = (1/8K)\log(1 + 4\ \mathrm{SNR}_T)$, which is smaller than the actual transmitted rate as in (3.22) and, therefore, the adjacent-cell users cannot be reliably decoded (and, of course, their decoding is not required at the cell site).

At this point, it should be emphasized that for an ultimate receiver which processes the information received in all cell sites as considered by [27], the optimal strategy is no intercell time-sharing $(\theta = 0)$, that is, in every cell the full resources are used for the cell users, resulting in interference to other cells. Here, when decoding is based only on the information received in the current cell, the conclusion is different and the usefulness of ICTS has, under certain conditions, been demonstrated. The significant role in an information-theoretic context played by the ICTS will further be highlighted when

$$\mu_1 = \begin{cases} \mathrm{SNR}_T, & \text{for } \theta = 0 \\ \mathrm{SNR}_T/(1-\theta), & \text{for } \alpha = 0 \\ -\dfrac{(1+4\alpha^2\theta-\theta) + \sqrt{(1+4\alpha^2\theta-\theta)^2 + 8(\mathrm{SNR}_T)\alpha^2\theta(2\alpha^2+1)}}{4\alpha^2(2\alpha^2+1)\theta} \end{cases} \tag{3.19a}$$

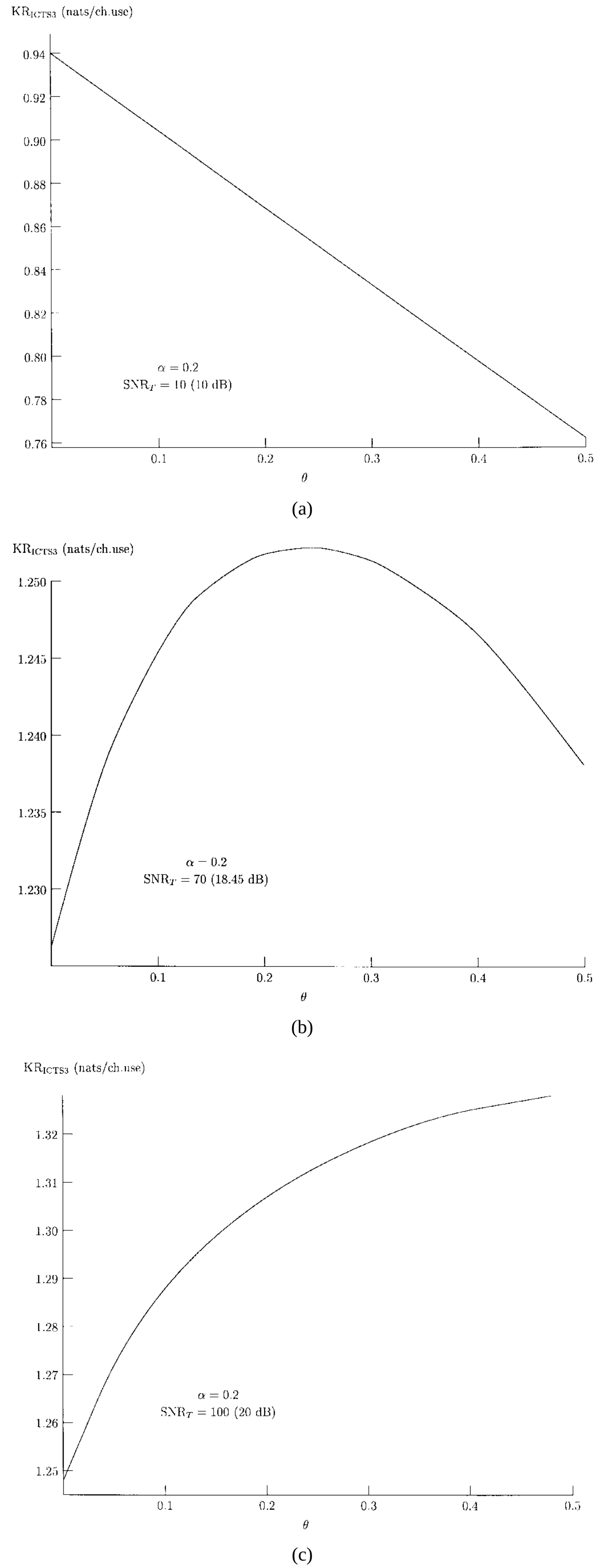

Fig. 2. The total cell capacity $KR_{\rm ICTS3}$ (nats/channel use) versus the fractional intercell time-sharing parameter $0 \le \theta \le 1/2$ for $\alpha = 0.2$ and (a) $\text{SNR}_T = 10$, (b) 70, and (c) 100.

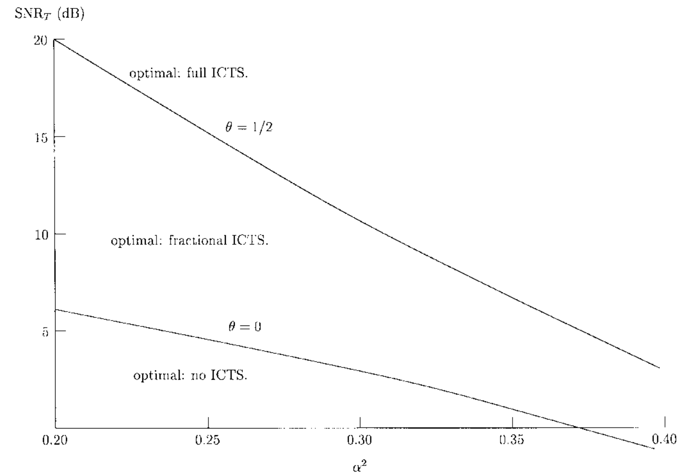

Fig. 3. The region of SNR_T and α^2 for which full ICTS ($\theta = 1/2$), fractional ICTS ($0 < \theta < 1/2$), and no ICTS ($\theta = 0$) are optimal for the ICTS3 (3.18), (3.19) strategy.

it emerges in a general formulation of an inner bound on the achievable rate region taking the form of a statistical relation of two auxiliary random variables.

We note here that because of the well-known duality between time and frequency, all of our TDMA results can be interpreted in the obvious way to apply to FDMA. Indeed, intercell time and frequency sharing is common in cellular systems and mainly termed as the frequency (or time) reuse factors [1]–[10]. Here we have demonstrated that from the Shannon theoretic standpoint, a reuse factor less than 1 is preferable in systems with high intercell interference, and where the cell-site receiver cannot access the received signals at other cell sites.

In Part II, where the channel model is extended to account for interference from distant cells (and not only neighboring adjacent cells) it is demonstrated, in the linear cellular model, that ICTS plays a role even when interference emerges from all cells (assuming a forth-power decay of the interference power with the distance of the interfering cell).

E. Intercell Interference ($\alpha > 0$), Fading Signals

We proceed now to consider the effect of fading and start by examining an idealized situation where fading affects only the interfering signals.

1) Adjacent-Cell Faded Interfering Signals: This scenario where the desired signals are assumed not to fade while the interfering signals do fade serves as a naive model of ideal genie-aided power control where no power penalty (unpenalized) is associated with the absolute neutralization of fading.[2] This naive model, which we do not claim to be practical, demonstrates the interesting effect of the fading on the different intracell accessing strategies on one hand

[2] The average power needed to overcome the fading is increased by a factor of $E(a^{-2})$ which may not be even bounded (as for the Rayleigh fading case). Had the realizations of the fading values been revealed to the transmitter, the best strategy under average power constraint is to use time "water-pouring" that is distribute the power at the transmitter optimally according to the quality of the actual channel [48]. In an isolated cell scenario, a TDMA-like strategy, when only that user who enjoys the best channel is transmitting adapting its power accordingly, is shown to be optimal [32], [38].

and facilitates to better understand the results for the more interesting case treated in Section III-E2, where all signals undergo fading. In this idealized case following (3.1) and (3.2), the achievable rates with WB and TDMA intracell accessing strategies (with no ICTS, $\theta = 0$) are given, respectively, by

$$R_{\rm WB} = \frac{1}{2K} E_{b,c} \log \left(1 + \frac{{\rm SNR}_T}{1 + \alpha^2 \ {\rm SNR}_T \sum_{\ell=1}^{K} \frac{1}{K}(b_\ell^2 + c_\ell^2)} \right) \tag{3.24}$$

$$R_{\rm TDMA} = \frac{1}{2K} E_{b,c} \log \left(1 + \frac{{\rm SNR}_T}{1 + \alpha^2 \ {\rm SNR}_T (b^2 + c^2)} \right). \tag{3.25}$$

Using the convexity of $\log(1 + cx^{-1})$ as a function of $x \geq 0$ (for any $c \geq 0$) yields by Jensen's inequality

$$R_{\rm TDMA} \geq R_{\rm WB}. \tag{3.26}$$

For Rayleigh fading, the random variable

$$\sum_{\ell=1}^{K} (b_\ell^2 + c_\ell^2)$$

is chi-square-distributed with $4K$ degrees of freedom having the probability density function (pdf) as in Appendix III, (AIII.5) (where $M = 2K$ and $\sigma_b^2 = 1/2$) and the expectations in (3.24) and (3.25) are readily given as signal integrals expressed explicitly in terms of the exponential integral function and its derivatives (similar to (3.7)). For example, the expression for

$$\begin{aligned} R_{\rm TDMA} &= \frac{1}{2K} \int_0^\infty \nu e^{-\nu} \log \left(1 + \frac{{\rm SNR}_T}{1 + \alpha^2 \ {\rm SNR}_T \nu} \right) d\nu \\ &= \frac{1}{2K} \Bigg[\log(1 + {\rm SNR}_T) - \left(1 - \frac{1 + {\rm SNR}_T}{\alpha^2 \ {\rm SNR}_T} \right) \\ &\qquad \cdot e^{((1+{\rm SNR}_T)/\alpha^2 \ {\rm SNR}_T)} E_i \left(-\frac{1 + {\rm SNR}_T}{\alpha^2 \ {\rm SNR}_T} \right) \\ &\qquad + \left(1 - \frac{1}{\alpha^2 \ {\rm SNR}_T} \right) \\ &\qquad \cdot e^{(1/\alpha^2 \ {\rm SNR}_T)} E_i \left(-\frac{1}{\alpha^2 \ {\rm SNR}_T} \right) \Bigg] \end{aligned} \tag{3.27}$$

results straightforwardly using integration in parts and results of [107, sec. 3.352 and 4.337]. For $K \gg 1$, the asymptotic expressions are

$$R_{\rm WB} \underset{K \gg 1}{\approx} \frac{1}{2K} \log \left(1 + \frac{1}{2\alpha^2} \right), \qquad \alpha > 0 \tag{3.28}$$

while

$$R_{\rm TDMA} \underset{{\rm SNR}_T \gg 1}{\approx} \frac{1}{2K} \left[-\log \alpha^2 + \mathbb{C} - \left(1 - \frac{1}{\alpha^2} \right) e^{1/\alpha^2} E_i \left(\frac{-1}{\alpha^2} \right) \right], \quad \alpha > 0. \tag{3.29}$$

We proceed now to the case where both the desired signals of the cell users and the signals of the adjacent cell interfering users experience fading.

2) Intercell Interference ($\alpha > 0$) *and Fading:* In this case, all signals undergo fading, and the achievable rates with WB and TDMA intracell strategies and no ICTS ($\theta = 0$) are given by (3.1) and (3.2), respectively. In this case, the relation between $R_{\rm WB}$ and $R_{\rm TDMA}$ can go either way, depending on the actual parameters ${\rm SNR}_T$, α, and the pdf of the fading variable a. For Rayleigh fading $\{a_\ell, b_\ell, c_\ell\}$, the expectations in (3.1) and (3.2) can be expressed in terms of single integrals by writing the relevant logarithms in (3.1) as the difference

$$\log \left(1 + \alpha^2 \ {\rm SNR} \sum_{\ell=1}^{K} (b_\ell^2 + c_\ell^2) + {\rm SNR} \sum_{\ell=1}^{K} a_\ell^2 \right) - \log \left(1 + \alpha^2 \ {\rm SNR} \sum_{\ell=1}^{K} (b_\ell^2 + c_\ell^2) \right)$$

and recognizing in the first part of the expectation the random variable

$$\alpha^2 \ {\rm SNR} \sum_{\ell=1}^{K} (b_\ell^2 + c_\ell^2) + {\rm SNR} \sum_{\ell=1}^{K} a_\ell^2$$

is the random variable W in Appendix III, (AIII.5), (where $M = 2K$, $\sigma_a^2 = {\rm SNR}$, and $\sigma_b^2 = \alpha^2 \ {\rm SNR}$). The second part of the expectation is done with respect to the random variable T (in Appendix III, (AIII.3), where $M = 2K$ and $\sigma_b^2 = \alpha^2 \ {\rm SNR}$). The same arguments apply for $R_{\rm TDMA}$ (3.2) (where in Appendix III, $K = 1$ and $M = 2$ are used).

We will focus on the case where $K \gg 1$ for which $R_{\rm WB}$ is given by

$$R_{\rm WB} \underset{K \gg 1}{\approx} \frac{1}{2K} \log \left(1 + \frac{{\rm SNR}_T}{1 + 2\alpha^2 \ {\rm SNR}_T} \right) \tag{3.30}$$

which again results by the law of large numbers, as detailed in Appendix II, averaging out thus the effect of fading. The rate $R_{\rm TDMA}$ (3.2) experiences no such averaging by the very nature of a TDMA protocol, and still when comparing the rates $R_{\rm WB}$ (3.1) and $R_{\rm TDMA}$ (3.2) the result can go either way depending on the system parameters. We demonstrate this interesting relation in the special case of the interference-limited regime which results by letting the additive Gaussian noise power $\sigma^2 = 0$, that is, ${\rm SNR} = P/\sigma^2 \to \infty$. For this special case and $K \gg 1$

$$R_{\rm WB} \approx \frac{1}{2K} \log \left(1 + \frac{1}{2\alpha^2} \right), \quad \alpha > 0, \ K \gg 1, \ {\rm SNR}_T \gg 1 \tag{3.31}$$

as follows from (3.30) with ${\rm SNR}_T \to \infty$ resembling the no-fading interference-limited case (3.11). In Appendix IV ((AIV.10) with $M = 2$ and $s = \alpha^2$), it is shown that for Rayleigh distributed i.i.d. fading random variables (a, b, c), the interference-limited $({\rm SNR} \to \infty)$ expression for $R_{\rm TDMA}$

is given by

$$R_{\rm TDMA} = \frac{1}{2K} E_{a,b,c} \log\left(1 + \frac{a^2}{\alpha^2(b^2+c^2)}\right) = \frac{1}{2K(1-\alpha^2)}\left(\frac{-\log\alpha^2}{1-\alpha^2} - 1\right), \quad \alpha > 0. \tag{3.32}$$

It follows that $R_{\rm TDMA} > R_{\rm WB}$ for $\alpha^2 > 0.17712$ while $R_{\rm WB} > R_{\rm TDMA}$ for $\alpha^2 < 0.17712$. This indicates, as expected, that with fading and relatively strong intercell interference, TDMA intracell accessing strategy is advantageous, while for low intercell interference, a WB approach which is capable of achieving an average effect on the fading phenomenon is preferable. The later conclusion conforms with the results of [28] where an isolated cell scenario $(\alpha = 0)$ was examined.

3) Intercell Time Sharing, Fading Present: As we have already concluded for the case of no fading, when intercell interference is present, a fractional intercell time-sharing strategy proves advantageous in certain range of parameters (SNR_T and α). Incorporating fractional ICTS in the presence of fading is straightforward and follows the arguments presented in Section III-D1. Note, however, as opposed to the case of no fading, the intracell WB and TDMA approaches are not equivalent. For reasons of conciseness, we shall only present the results for the case where the receiver exploits the knowledge available on the location of the interfered and uninterfered symbols but the transmitter does not adapt its power to optimize performance (analogous to ICTS2 in Section III-D1), rather, constant power is used. Also here as in Section III-D1, $0 \le \theta \le 1/2$ stands for the fractional time-sharing parameter, where $\theta = 0$ corresponds to no sharing while $\theta = 1/2$ corresponds to full sharing (division) which absolutely eliminates the intercell interference. Under an intracell WB strategy the rate is given by

$$R_{\rm WB(ICTS2)} = \frac{1-2\theta}{2K} E_{\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}} \cdot \log\left(1 + \frac{\mathrm{SNR}_T/(1-\theta)\,\frac{1}{K}\sum_{\ell=1}^{K} a_\ell^2}{1 + \frac{\alpha^2\,\mathrm{SNR}_T}{1-\theta}\,\frac{1}{K}\sum_{\ell=1}^{K}(b_\ell^2 + c_\ell^2)}\right) + \frac{\theta}{2K} E_{\boldsymbol{a}} \log\left(1 + \frac{\mathrm{SNR}_T}{(1-\theta)}\,\frac{1}{K}\sum_{\ell=1}^{K} a_\ell^2\right) \tag{3.33}$$

while for a TDMA intracell protocol, the rate equals

$$R_{\rm TDMA(ICTS2)} = \frac{1-2\theta}{2K} E_{a,b,c} \cdot \log\left(1 + \frac{\frac{\mathrm{SNR}_T}{(1-\theta)}\,a^2}{1 + \frac{\alpha^2\,\mathrm{SNR}_T}{(1-\theta)}\,(b^2+c^2)}\right) + \frac{\theta}{2K} E_a \log\left(1 + \frac{\mathrm{SNR}_T}{1-\theta}\,a^2\right) \tag{3.34}$$

where the parameter $0 \le \theta \le 1/2$ representing time sharing is to be optimized. For the special case of no fading (that is, $a_\ell = b_\ell = c_\ell = 1$, $\ell = 1, 2, \cdots, K$) with ICTS both (3.33) and (3.34) coincide with $R_{\rm ICTS2}$ in (3.17). The expressions for intercell time-sharing strategies that are associated with ICTS1 (3.16) (no use either at the receiver or transmitter of the available knowledge on interfered symbols) and ICTS3 (3.18) (where the transmitter exercises optimal power distribution) readily follow by the same arguments used in the nonfading case.

Again, we specialize to the case $K \gg 1$ and α^2 SNR $\to \infty$. Use of the WB system alleviates the effect of the fading and yields the result

$$R_{\rm WB(ICTS2)} \underset{\substack{K\gg 1\\ \mathrm{SNR}\gg 1}}{\approx} \frac{1-2\theta}{2K} \log\left(1 + \frac{1}{2\alpha^2}\right) + \frac{\theta}{2K}\log\left(1 + \frac{\mathrm{SNR}_T}{(1-\theta)}\right). \tag{3.35}$$

For the TDMA intracell strategy with Rayleigh fading and the assumption $\mathrm{SNR}_T \gg 1$, by the results in (3.9) and (3.32), it is concluded that

$$R_{\rm TDMA(ICTS2)} \underset{K\mathrm{SNR}_T\gg 1}{\approx} \frac{1-2\theta}{2K(1-\alpha^2)}\left(\frac{-\log\alpha^2}{1-\alpha^2} - 1\right) + \frac{\theta}{2K}\left(\log\left(1 + \frac{\mathrm{SNR}_T}{1-\theta}\right) - \mathbb{C}\right). \tag{3.36}$$

Obviously, since $\alpha^2\,\mathrm{SNR}_T \gg 1$, the optimized strategy in both cases is full intercell time sharing $\theta_{\rm opt} = 1/2$, and in this case $R_{\rm WB(ICTS2)} > R_{\rm TDMA(ICTS2)}$ as is evident by comparing the expressions (3.35) and (3.36). For nonasymptotic cases (3.33), (3.34) with optimized intercell time sharing, the advantage of TDMA over the WB strategies for large intercell interference diminishes or absolutely disappears (as demonstrated before for $\mathrm{SNR}_T \gg 1$) as now there is yet another mechanism (fractional intercell time sharing) to control the intercell interference.

This observation demonstrates the theoretical importance of intercell resource (time or bandwidth) sharing protocols even for an intracell WB strategy in the presence of fading. Combining thus the WB intracell channel access with a intercell time-sharing protocol may provide improved rates (in terms of Shannon theoretic predictions) in the presence of fading and intercell interference, restoring thus the advantage of WB strategies as was found in the presence of fading and isolated cells (no intercell interference, $\alpha = 0$) [28].

F. General Formulation of an Achievable Rate Region

In the preceding subsections of this section, we have examined a variety of cases with different intracell access protocols as WB and TDMA and fractional ICTS, in a faded and unfaded environments.

In the following, a general formulation of an achievable rate region (inner bound) is given encompassing all the cases discussed previously as special cases. Here, we relax the demand that all users signal at the same rate R and possess the same normalized power SNR. Let the ℓth user (out of the K)

in each cell signals at an average information rate R_ℓ and uses average power P_ℓ, where the normalized signal-to-noise of the ℓth user is $\text{SNR}_\ell \triangleq P_\ell/\sigma^2$. The rates $R_1, R_2, \cdots, R_K$ specified by the following theorem are achievable (inner bound) in each cell.

Theorem 1: An inner bound on the achievable set of rates $R_1, R_2, \cdots, R_K$ is given by

$$\bigcup_{P(u_0,\, u_1)} \bigcup_{\sigma_\ell(\,)} \sum_{\ell \in \mathbb{L}} R_\ell \le E_{u_0,\, u_1} \tfrac{1}{2} E_{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}} \cdot \log \left(1 + \frac{\displaystyle\sum_{\ell \in \mathbb{L}} a_\ell^2 \sigma_\ell^2(u_0,\, u_1)}{1 + \alpha^2 \displaystyle\sum_{\ell=1}^{K} (b_\ell^2 + c_\ell^2) \sigma_\ell^2(u_1,\, u_0)} \right) \quad \forall \mathbb{L} \in \{1, 2, \cdots, K\} \tag{3.37}$$

where the equation should be satisfied for any subset $\mathbb{L}$ of the K users and where the union is taken over all functions $\sigma_\ell(u_0,\, u_1) \ge 0$ such that the normalized average power constraints

$$E_{u_0,\, u_1} \sigma_\ell^2(u_0,\, u_1) = \text{SNR}_\ell, \qquad \ell = 1, 2, \cdots, K \tag{3.37a}$$

and over all joint distributions $P(u_0,\, u_1)$ of the *simple* auxiliary random variables u_0 and u_1 which satisfy the symmetry condition

$$P(u_0,\, u_1) = P(u_1,\, u_0). \tag{3.37b}$$

The cardinality of the simple random variables u_0 and u_1 individually is no larger than $K+2$. □

The detailed proof of the theorem and further comments appear in Appendix I. Here we outline the basic intuition behind this result, indicating the central role, played by the auxiliary random variables u_0 and u_1, and demonstrating that all the cases discussed so far are included within this formulation.

The basic idea of the proof in Appendix I is the interpretation of the values taken by the auxiliary random variables u_0 and u_1 individually as the intracell access strategies of the even- and odd-numbered cells while the statistical dependence between u_0, u_1 reveals the intercell cooperation (if such exists). The usual time-sharing interpretation is associated with the auxiliary simple random variables u_0, u_1. The probability that u_0 and u_1 take on certain values is associated with the fraction of time the inter- and intracell access strategy, which is uniquely determined by the value of u_0 and u_1, is used. All cells with the same "parity" (even or odd) employ exactly the same strategy. The power assigned to user ℓ in a certain cell m is $\sigma_\ell^2(x,\, y)$ where x, y designate, respectively, the strategy of the cells having the same and different parity as that of m. The standard constrained multiple-access achievable region [27], [108], [109] assuming random Gaussian codebooks which are redrawn per message and the fact that a convex combination of achievable rates is also achievable [95] are the steps which facilitate the derivation of (3.37). The symmetry condition (3.37b) along with the specific power assignment provide the necessary invariance to the specific cell considered, yielding thus exactly the same rate region in all cells.

For the symmetric case in focus in this work, i.e., $R_\ell = R$, $\text{SNR}_\ell = \text{SNR}$, $\ell = 1, 2, \cdots, K$ it is straightforwardly verified in Appendix I, that the only equation in (3.37) to be considered (which provides the tightest constraint) is the one which corresponds to the set of all users, i.e., the cardinality of $\mathbb{L}$ designated by $\|\mathbb{L}\|$ equals K.

We turn now to some special cases which were treated before and examine them under the general setting here.

First consider *intracell TDMA and no ICTS* ($\theta = 0$). The result in (3.2) for equal rates and powers is reproduced by taking $u_0, u_1 \in (1, 2, \cdots, K)$ equiprobable and independent

$$P(u_0,\, u_1) = P(u_0)P(u_1) = 1/K^2, \quad u_0,\, u_1 = 1, 2, \cdots, K \tag{3.38a}$$

and the signal-to-noise ratio assignment

$$\sigma_\ell^2(j,\, m) = \begin{cases} \text{SNR}_T, & \ell = j \\ 0, & \text{otherwise.} \end{cases} \tag{3.38b}$$

Another assignment which yields the same result is taking u_0, u_1 to be dependent and governed by the joint distribution

$$P(u_0 = \ell,\, u_1 = k) = \begin{cases} \dfrac{1}{K}, & \ell = k,\, \ell, k = 1, 2, \cdots, K \\ 0, & \text{otherwise} \end{cases} \tag{3.39a}$$

and the signal-to-noise ratio assignment

$$\sigma_\ell^2(j,\, m) = \begin{cases} \text{SNR}_T, & \ell = j = m \\ 0, & \text{otherwise.} \end{cases} \tag{3.39b}$$

In the latter case (3.39), there exists a kind of ICTS in terms that the identically numbered users operate simultaneously in all cells while the first selection (3.38) dictates that only one user operates in each cell, not necessarily the identically numbered users. The role played by the auxiliary random variables u_0, u_1 is clear here. Their values identify, respectively, the number of the active users in the even- and odd-numbered cells and their statistical relation corresponds to the intercell cooperation protocol. Note that the power assignments as in (3.38b) and (3.39b) guarantee that only one active user per cell can operate at the same time.

The second and last example is that considered in Section III-E3, where the intracell access protocol is WB but fractional ICTS with parameter $0 \le \theta \le 1/2$ is exercised. Now u_0, u_1 take on individually three possible values, say 0, 1, and 2 with the joint pdf specified by

$$P(u_0 = k,\, u_1 = m) = \begin{cases} 1 - 2\theta, & k = m = 1 \\ \theta, & k = 2,\, m = 0 \\ \theta, & k = 0,\, m = 2 \\ 0, & \text{otherwise} \end{cases} \tag{3.40a}$$

while the power assignment is

$$\sigma_\ell^2(k,\, m) = \begin{cases} \text{SNR}_1, & k = 2,\, m = 0 \\ \text{SNR}_2, & k = m = 1 \\ 0, & \text{otherwise} \end{cases}, \qquad \forall \ell. \tag{3.40b}$$

Here, the value 0 of the auxiliary random values indicates no transmission, the value 1, designates interfered (by adjacent cell users) transmission, while the value 2 denotes uninterfered transmission. The statistical dependence between u_0 and u_1 as implied by (3.40a) rules out all possibilities but those of interference, that is, signals in adjacent cells are active simultaneously, or no interference, which takes place when signals in a given cell are active and in an adjacent cell no active transmission is allowed. The power assignment by (3.40b) which does not depend on ℓ specifies the intracell WB approach. Now SNR_1 and SNR_2 can either be optimized satisfying the average power constraint

$$(1-2\theta)\ \mathrm{SNR}_2 + \theta\ \mathrm{SNR}_1 = \mathrm{SNR} \tag{3.40c}$$

(which leads to the strategy ICTS3) or chosen equal $\mathrm{SNR}_1 = \mathrm{SNR}_2 = \mathrm{SNR}/(1-\theta)$ which implies strategy ICTS2 (3.33).

The information-theoretic significance of the fractional ICTS as reflected by the statistical dependence of the auxiliary random variables u_0 and u_1, is highlighted again by the "converse" proved in Appendix I. The "converse" which agrees with the achievable part (3.37) for the symmetric case ($\|L\| = K$) is proved in Appendix I under the Gaussian approximation, i.e., the active interfering signals are treated as Gaussian random variables having the same power. Definitely, this limits considerably the general scope and significance of this result, which cannot be viewed as an ultimate converse theorem. Yet this result indicates that fractional ICTS emerges naturally by information-theoretic considerations, even if it is not pre-imposed as was done in the derivation of the achievable part.

Appendix I
Achievable Information Rates

Towards proving the achievability of the rates specified by (3.37) and the special symmetric case of equal rates and powers ((3.37) with $\|L\| = K$), we first consider the direct part (achievability). Then, for the symmetric case in focus here, we provide a "converse," which holds under the *underlying Gaussian* approximation for the interfering users assigned to other cells.

Consider now the channel described by (2.1) and let the channel outputs be $\{y_j^m\}$, $\{a_{\ell,j}^m\}$, $\{b_{\ell,j}^m\}$, and $\{c_{\ell,j}^m\}$,[3] $\forall\, \ell = 1, \cdots, K, m, j$. Assume further that *ideal interleaving* is employed by all users at all cells, which under the current strict stationarity and ergodicity assumptions implies[4] that all $a_{\ell,j}^m$, $b_{\ell,j}^m$, and $c_{\ell,j}^m$ can be treated as i.i.d. random variables in all indices m, ℓ, and also in the time index j abusing somewhat notations (as to save the introduction of alternative notations for independent fading variables).

[3]It is shown that the realization of all the fading random variables $\{a_{\ell,j}^m\}$, $\{b_{\ell,j}^m\}$, and $\{c_{\ell,j}^m\}$ is not needed and only instantaneous signal-to-noise ratio values for all the specific cell-assigned users are required.

[4]Throughout, we have assumed that $\{a_{\ell,j}^m\}$ are i.i.d. processes in indexes m and ℓ (that is, for different users and cells) and strictly stationary and ergodic in the discrete time index j. The ideal interleaving assumption used here for the sake of simplicity only in proving the achievable part (a rigorous treatment that avoids interleaving is based on the ergodicity property (or even a relaxed assumption of asymptotic mean stationarity) yields the same results and follows directly using the approach in [110]).

Partition now the cells into even- and odd-numbered cells where the counting reference point is of no relevance. Let u_0 and u_1 be two simple integer-valued variables with finite and equal cardinality $\|u_0\| = \|u_1\|$. The values of these variables are revealed both to the transmitter and the receiver and, in general, the transmitters of all users in each cell select the assigned powers of the users and their respective rates according to the values of these variables. In particular, we fix exactly the same strategies to be used in all even and odd cells separately and let $\sigma_\ell^2(u_{par(m)}, u_{par(m+1)})$ stand for the power assigned to user ℓ (out of K) in a certain cell m, where $\mathrm{par}\,(m) = 0, 1$ denotes the parity of m. All the cells with the same parity as m employs strategy $u_{\mathrm{par}\,(m)}$, and the adjacent cells (of different parity $\mathrm{par}\,(m+1)$) use strategy $u_{\mathrm{par}\,(m+1)}$. Let $R_\ell(u_{\mathrm{par}\,(m)}, u_{\mathrm{par}\,(m+1)})$ be the corresponding rate of user ℓ of cell m where the notations explicitly indicates the dependence of strategies $u_{\mathrm{par}\,(m)}$ (in cells of parity $\mathrm{par}\,(m)$) and $u_{\mathrm{par}\,(m+1)}$ (in cells of parity $\mathrm{par}\,(m+1)$). Note that under the same strategy user ℓ in all cells of parity $\mathrm{par}\,(m+1)$ signals at rate $R_\ell(u_{\mathrm{par}\,(m+1)}, u_{\mathrm{par}\,(m)})$ and employs power $\sigma_\ell^2(u_{\mathrm{par}\,(m+1)}, u_{\mathrm{par}\,(m)})$.

All users in all cells employ randomly generated codebooks [95] which are regenerated independently for every transmitted message in synchronism with the cell-site receiver (which, of course, knows for every message what is the current codebook used by his assigned cell users and is absolutely ignorant of the codebooks employed by other cells users). The random codebooks use Gaussian symbol distribution (in the sense of [95, ch. 10])[5] and the codebooks (of code lengths $N \to \infty$) for the ℓth user satisfy, of course, the rate constraint $R_\ell(u_{\mathrm{par}\,(m)}, u_{\mathrm{par}\,(m+1)})$ and power constraint $\sigma_\ell^2\ (u_{\mathrm{par}\,(m)}, u_{\mathrm{par}\,(m+1)})$. Since all transmitted symbols by the users of adjacent cell to m are viewed (under Gaussian random coding which is independently chosen per message) as Gaussian i.i.d. random variables.

The channel as described by (2.1) (focusing on cell m) is thus memoryless and by the standard results for multiaccess channels, all rates

$$\{R_1(u_{\mathrm{par}\,(m)}, u_{\mathrm{par}\,(m+1)}), \cdots, R_K(u_{\mathrm{par}\,(m)}, u_{\mathrm{par}\,(m+1)})\}$$

satisfying

$$\sum_{\ell \in \mathbb{L}} R_\ell(u_{\mathrm{par}\,(m)}, u_{\mathrm{par}\,(m+1)}) \le I\left(y^m, \bigcup_{\ell=1}^{K} \{a_\ell^m, b_\ell^m, c_\ell^m\};\ \bigcup_{\ell\in\mathbb{L}} x_\ell^m \,\middle|\, \bigcup_{\ell\in\mathbb{L}^c} x_\ell^m\right), \quad \forall \mathbb{L} \subseteq (1, 2, \cdots, K) \tag{AI.1}$$

are achievable, where $\mathbb{L}$ stands for a subset of the K users and $\mathbb{L}^c$ for the complement set. The time index j is omitted as the relevant channel is memoryless as indicated before and hence the achievable rate region assumes a single letter characterization. Note now that since the fading variables

[5]No deletion of codebooks with average small normalized power as compared to $\mathrm{SNR}_\ell - \epsilon$ or large average power is done (in contrast to [99]) rather in those atypical events (for $N \to \infty$) error is declared as in [95, ch. 10].

and the channel inputs are statistically independent, as the transmitters are not aware of the fading parameters

$$I\left(\bigcup_{\ell=1}^{K}\{a_\ell^m, b_\ell^m, c_\ell^m\}; \bigcup_{\ell=1}^{K}\{x_\ell^m\}\right) = 0$$

and therefore we get (AI.2) (at the bottom of this page). The expression on the right-hand side is easily evaluated noting that given the realization of a_ℓ^m, b_ℓ^m, c_ℓ^m, the channel has Gaussian inputs x_ℓ^m, $l \in \mathbb{L}$, and Gaussian noises x_ℓ^{m-1}, x_ℓ^{m+1}, and n, where

$$E(x_\ell^m)^2 = \sigma_\ell^2(u_{par(m)}, u_{par(m+1)})$$
$$E((x_\ell^{m-1})^2) = E((x_\ell^{m+1})^2) = \sigma_\ell^2(u_{par(m+1)}, u_{par(m)}).$$

Inspection of (AI.2) reveals that no degradation in achievable rates is incurred if the instantaneous signal-to-noise ratio

$$\frac{a_\ell^2\sigma_\ell^2(u_{\mathrm{par}\,(m)}, u_{\mathrm{par}\,(m+1)})}{\left(1 + \sum_{\ell=1}^{K}(b_\ell^2 + c_\ell^2)\sigma_\ell^2(u_{\mathrm{par}\,(m)}, u_{\mathrm{par}\,(m+1)})\right)}$$

for all users $\ell = 1, 2, \cdots, K$ is available to the cell-site receiver rather than the actual realizations of all fading parameters individually. The rates for all cells having $\mathrm{par}\,(m)$ parity is the same while the rate in all cells having $\mathrm{par}\,(m+1)$ parity is given by (AI.2) where $u_{\mathrm{par}\,(m,)}$ and $u_{\mathrm{par}\,(m+1)}$ are permuted. Assign now a probability distribution to u_0, u_1, invariant under permutation $p(u_0, u_1) = p(u_1, u_0)$. Since any convex combination of achievable rates is achievable, $E_{u_0, u_1}\{R_\ell(u_0, u_1)\}_{\ell=1}^K$ is achievable where, with no loss of generality, it is assumed that $\mathrm{par}\,(m) = 0$. To satisfy the power constraints it is required that

$$E_{u_0, u_1}\sigma_\ell^2(u_0, u_1) = E_{u_1, u_0}\sigma_\ell^2(u_1, u_0) \le \mathrm{SNR}_\ell, \quad \forall\,\ell = 1, 2, \cdots, K.$$

The proof of the rate region in (3.37) is completed by noting the full symmetry of u_0 and u_1 which allows their permutation with no effect on the rate region and thus the same rate is achievable for cells of parity $\mathrm{par}\,(m) = 0$ and $\mathrm{par}\,(m) = 1$. As indicated before, the rates in (3.37) are achievable provided the instantaneous signal-to-noise ratio (at any time j) is known to the receiver and the knowledge of the individual fading parameters is not required. The operating strategy designated by u_0 and u_1 for cells of different parity is available also to the transmitter.

An upper bound on the cardinality of the random variables u_0 and u_1 is found by first fixing strategy u_1 in $\mathrm{par}\,(m+1)$ (again assuming $\mathrm{par}\,(m) = 0$, with no loss of generality) cells and noting that the capacity region of $\mathrm{par}\,(m)$ cells is given by the characterization in [108] and [109], that is, the multiple-access channel with input constraints. Similar arguments as used in Han [111] for the latter case, which rely on the Eggleston Theorem [95, ref. 95] yield that $||u_0|| \le K+2$. The same argument applies to upper-bound $||u_1||$.

We proceed now to examine the symmetric conditions $\mathrm{SNR}_\ell = \mathrm{SNR}$, $R_\ell = R\ \forall\,\ell = 1, 2, \cdots, K$. Under these conditions due to the full symmetry among all K users, the expression in (3.37) does not depend on the specific set of users $\mathbb{L}$ but only on the cardinality of that set $||\mathbb{L}||$. Assume now that the inequality for $||\mathbb{L}|| = q$ is satisfied with equality in (3.37) (a “tight constraint set” in the terminology of [112]). Consider now $\mathbb{L}_1$ and $\mathbb{L}_2$ which differ in a single element (user) and have the same cardinality $q = ||\mathbb{L}_1|| = ||\mathbb{L}_2||$ (for example, $\mathbb{L}_1 = (1, 2, 3)$ and $\mathbb{L}_2 = (2, 3, 4)$). By [112], the set of equation corresponding to $\mathbb{L} = \mathbb{L}_1 \cup \mathbb{L}_2$, the cardinality of which is $q+1$, is also a tight constraint set. Hence, by repeating the arguments it follows that under the symmetric conditions here, $||\mathbb{L}|| = K$ is also a tightest constraint set, proving thus the claim. This follows also directly by the results in [113].

The Gaussian approximation can be considered as a worst case noise in terms of a saddle-point argument in the sense of [103]. Taking the desired users at cell m to be Gaussian it follows directly by similar arguments as in [103], that under given signal-to-noise ratios of all interfering signals, the Gaussian noises yield the minimum value for the average mutual information expression in (AI.1). This is readily verified noting that under the realization of the fading parameter the corresponding channel is additive with a given power for the noise. The complementary part of the saddle point holds as well, that is, if the interfering noises are Gaussian, the optimizing inputs with given signal-to-noise ratios are also Gaussian.

We specialize now to the symmetric case of equal SNR and rates for all users. Consider the sum-rate $\sum_{\ell=1}^K R_\ell = KR$ and by the Fano inequality

$$KR \le \frac{1}{N}\sum_{j=1}^{N} I\left(y_j^m, \bigcup_{\ell=1}^{K}\{a_{\ell,j}^m, b_{\ell,j}^m, c_{\ell,j}^m\}; \bigcup_{\ell=1}^{K}\{x_{\ell,j}^m\}\right) \tag{AI.3}$$

where N stands for the length of the codewords used. The average power constraint to which all users in all cells are

$$I\left(y^m, \bigcup_{\ell=1}^{K}\{a_\ell^m, b_\ell^m, c_\ell^m\}; \bigcup_{\ell\in\mathbb{L}} x_\ell^m \,\middle|\, \bigcup_{\ell\in\mathbb{L}^c} x_\ell^m\right) = I\left(y^m; \bigcup_{\ell\in\mathbb{L}} x_\ell^m \,\middle|\, \bigcup_{\ell\in\mathbb{L}^c} x_\ell^m, \bigcup_{\ell=1}^{K}\{a_\ell^m, b_\ell^m, c_\ell^m\}\right)$$
$$= E_{\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}}\,\frac{1}{2}\log\left(1 + \frac{\sum_{\ell\in\mathbb{L}} a_\ell^2\sigma_\ell^2(u_{\mathrm{par}\,(m)}, u_{\mathrm{par}\,(m+1)})}{1 + \sum_{\ell=1}^{K}(b_\ell^2 + c_\ell^2)\sigma_\ell^2(u_{\mathrm{par}\,(m+1)}, u_{\mathrm{par}\,(m)})}\right). \tag{AI.2}$$

subjected indicates that

$$E(1/N)\sum_{j=1}^{N}(x_{\ell,j}^{m})^2 \le \text{SNR}, \qquad \text{for all } \ell = 1, 2, \cdots, K$$

and m (where E here stands for the expectation over the codebooks). Assume (as indicated by the worst case assumptions) that $x_{\ell,j}^{m-1}$ and $x_{\ell,j}^{m+1}$, the signals that correspond to the other-cell interfering users, are Gaussian (*Gaussian Approximation*). It follows by the left—hand side equality in (AI.2) and the fact that under a given realization of the fading variables $\{a_{\ell,j}^m\}$, $\{b_{\ell,j}^m\}$, and $\{c_{\ell,j}^m\}$ the channel is Gaussian, that Gaussian inputs with the *a priori* prescribed signal-to-noise ratios maximize the corresponding average mutual information expression (the other part of the saddle point argument). In conclusion, for all m, we must have (AI.4) (shown at the bottom of this page). Note now that $E(x_{\ell,j}^m)^2$, $E(x_{\ell,j}^{m-1})^2$, and $E(x_{\ell,j}^{m+1})^2$ are independent of the fading variables. In the expression (AI.4), there should be an absolute symmetry with respect to m. Further note that this expression is invariant under user permutation within the cell and also with user permutation within the adjacent interfering cells (having a different parity than $\text{par}\,(m)$). That is when cell m is considered there is full invariance to which cell a subset of interfering users are assigned. Since we examine the case of infinite linear cellular model, only the parity value of the cell matters and not its nominal (infinite) enumeration, thus there is no loss in generality as far as maximization of (AI.4) is considered for *all* m, in assuming that cells of the same parity employ the same access strategy. Consider a discrete random variable u, then it follows as shown in (AI.5) (see the second expression at the bottom of this page), where

$$E_u\mu_\ell^{\text{par}\,(m)}(u) = E_u\mu_\ell^{\text{par}\,(m+1)}(u) \le \text{SNR}.$$

This is since (AI.4) is a special case of (AI.5), where $u = 1, 2, \cdots, N$ with equal probability N^{-1}. Now note that the full symmetry with respect to $\text{par}\,(m)$, that is, when even- and odd-numbered cells change roles, facilitates to specify the power of user ℓ of cell m for a given u as

$$\mu_\ell^{\text{par}\,(m)}(u) = \sigma_\ell^2(u_{\text{par}\,(m)}, u_{\text{par}\,(m+1)}) \qquad \text{(AI.6)}$$

which means that the power assigned to user ℓ in cell m depends on two simple auxiliary random variables $u_{\text{par}\,(m)}, u_{\text{par}\,(m+1)}$ which can be viewed as the relevant decomposition of the single random variable u. The full symmetry between odd- and even-numbered cells, that is, invariance under parity permutations, implies $P(u_0, u_1) = P(u_1, u_0)$ which concludes the proof of the "converse" (under the Gaussian approximation) for the symmetric case. The fact that the cardinality of $\|u_0\| = \|u_1\|$ is finite and depends only on the number of users, stems readily from the results for a standard input-constrained multiple-access channel [108], [109], as established by Han [111]. The invariance to user permutation within a cell dictates further symmetries in (3.37) which imply that the values taken on by $\sigma_\ell^2(u_{\text{par}\,(m)}, u_{\text{par}\,(m+1)})$ are permutations of the values taken on by $\sigma_1^2(u_{\text{par}\,(m)}, u_{\text{par}\,(m+1)})$ as is demonstrated in the specific examples treated in Section III-F.

Appendix II

Equations (3.5) and (3.30) follow immediately from the following proposition. Let $\{a_k\}$, $1 \le k < \infty$, be a sequence of independent r.v.s with $Ea_k^2 = 1$. Define, for $1 \le K < \infty$

$$U_K = \sigma^2 + P\sum_{k=1}^{K} a_k^2 \qquad \text{(AII.1)}$$

and

$$U_K^* = EU_K = \sigma^2 + PK. \qquad \text{(AII.2)}$$

Proposition:

$$\lim_{K\to\infty} E\,\log\,(U_K/U_K^*) = 0.$$

Proof: By Jensen

$$E\,\log\,U_K \le \log\,EU_K = \log\,U_K^* \qquad \text{(AII.3)}$$

so that

$$E\,\log\,(U_K/U_K^*) \le 0. \qquad \text{(AII.4)}$$

$$KR \le \frac{1}{N}\sum_{j=1}^{N} E_{\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}}\,\frac{1}{2}\,\log\left(1 + \frac{\sum_{\ell=1}^{K}(a_{\ell,j}^m)^2E(x_{\ell,j}^m)^2}{1+\alpha^2\sum_{\ell=1}^{K}[(b_{\ell,j}^m)^2E(x_{\ell,j}^{m-1})^2 + (c_{\ell,j}^m)^2E(x_{\ell,j}^{m+1})^2]}\right), \qquad \forall\, m. \qquad \text{(AI.4)}$$

$$KR \le E_uE_{\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}}\,\frac{1}{2}\,\log\left(1 + \frac{\sum_{\ell=1}^{K}(a_{\ell,j}^m)^2\mu_\ell^{\text{par}\,(m)}(u)}{1+\alpha^2\sum_{\ell=1}^{K}[(b_{\ell^m})^2\mu_\ell^{\text{par}\,(m+1)}(u) + (c_{\ell^m})^2\mu_\ell^{\text{par}\,(m+1)}(u)]}\right) \qquad \text{(AI.5)}$$

On the other hand, by the Strong Law of Large Numbers

$$U_K/U_K^* = \left[\sigma^2 + PK\left(\frac{1}{K}\sum_{k=1}^{K} a_k^2\right)\right]/[\sigma^2+PK] \to 1 \tag{AII.5}$$

with probability 1, as $K \to \infty$.

Thus defining A_K by

$$A_K = \left\{\frac{U_K}{U_K^*} \geq 1-\epsilon\right\} \tag{AII.6a}$$

we have

$$\begin{aligned} E\,\log\frac{U_K}{U_K^*} =& \;\Pr(A_K)E\left[\log\frac{U_K}{U_K^*}\middle| A_K\right] \\ &+ \Pr(A_K^c)E\left[\log\frac{U_K}{U_K^*}\middle| A_K^c\right] \\ &\overset{a)}{\geq} P(A_K)\log(1-\epsilon) + P(A_K^c)\log\frac{\sigma^2}{(\sigma^2+PK)} \\ &\xrightarrow[K\to\infty]{b)} \log(1-\epsilon) \xrightarrow[\epsilon\to 0]{} 0. \end{aligned} \tag{AII.6b}$$

Step a) in (AII.6b) follows from the definition of A_K, and step b) from (AII.5) invoking Chernoff which guarantees that $\Pr(A_K^c) \leq e^{-bK}$ where $b > 0$. The proposition now follows from (AII.4) and (AII.6b).

Appendix III
Relevant Probability Density Functions

Let the random variable S and T be defined by

$$S = \sum_{k=1}^{K} a_k^2 \qquad T = \sum_{k=1}^{M} b_k^2 \tag{AIII.1}$$

where $\{a_k\}$ and $\{b_k\}$ are independent Rayleigh random variables and individually i.i.d. satisfying $E(a^2) = 2\sigma_a^2$ and $E(b^2) = 2\sigma_b^2$. The corresponding pdf's are $2K$ and $2M$ chi-squared specified by

$$f_S(\nu) = \frac{1}{2^K(\sigma_a^2)^K(K-1)!}\,\nu^{(K-1)}e^{-(\nu/2\sigma_a^2)}, \qquad \nu \geq 0 \tag{AIII.2}$$

$$f_T(\nu) = \frac{1}{2^M(\sigma_b^2)^M(M-1)!}\,\nu^{(M-1)}e^{-(\nu/2\sigma_b^2)}, \qquad \nu \geq 0. \tag{AIII.3}$$

Consider the random variable

$$W = S + T \tag{AIII.4}$$

the pdf of which is given by

$$\begin{aligned} f_W(\nu) &= \int_0^\nu f_S(\tau)f_T(\nu-\tau)\,d\tau \\ &= \nu^{M+K-1}e^{-(\nu/2\sigma_b^2)}2^{K+M}(\sigma_a^2)^K(\sigma_b^2)^M \\ &\quad\cdot (M+K-1)!\,{}_1F_1 \\ &\quad\cdot \left(K;\, K+M;\, \left(\frac{1}{2\sigma_b^2} - \frac{1}{2\sigma^2}\right)\nu\right), \qquad \nu \geq 0 \end{aligned} \tag{AIII.5}$$

where the right—hand side equality in (AIII.5) follows [107, Sec. 3.383], and where ${}_1F_1(\,;\,;\,)$ stands for the degenerate hypergeometric function [107, Sec. 9.9]. Consider a random variable Q which when conditioned on M is given by T (AIII.1) and let M be a Poisson random variable with parameter λ, that is,

$$\text{prob}(M = m) = \frac{\lambda^m e^{-\lambda}}{m!}, \qquad m = 0, 1, 2, \cdots. \tag{AIII.6}$$

The unconditioned distribution of Q is

$$\begin{aligned} f_q(\nu) &= e^{-\lambda}\delta_c(\nu) + \sum_{m=1}^{\infty}\frac{1}{\nu}\left(\frac{\nu}{2\sigma_b^2}\right)^m \frac{e^{-(\nu/(2\sigma_b^2))}}{(m-1)!}\,\frac{e^{-\lambda}\lambda^m}{m!} \\ &= e^{-\lambda}\delta_c(\nu) + \frac{1}{\nu}\,e^{-\lambda}e^{-(\nu/2\sigma_b^2)}\sum_{\ell=0}^{\infty}\left(\sqrt{\frac{\nu\lambda}{2\sigma_b^2}}\right)^{2\ell+2} \\ &\quad\cdot \frac{1}{\ell!(\ell+1)!} \\ &= e^{-\lambda}\delta_c(\nu) + e^{-(\lambda+(\nu/2\sigma_b^2))}\sqrt{\frac{\lambda}{2\nu\sigma_b^2}}\,I_1\left(\sqrt{\frac{2\nu\lambda}{\sigma_b^2}}\right), \\ &\qquad \nu \geq 0 \end{aligned} \tag{AIII.7}$$

where $\delta_c(\nu)$ and $I_1(z)$ are, respectively, the Kronecker delta function and the modified Bessel function of the first kind and first order. The last equality in the right-hand side of (AIII.7) follows by [107, Sec. 8.447]

$$I_1(z) = I_0'(z) = \sum_{k=0}^{\infty}[(z/2)^{2k+1}/k!(k+1)!].$$

Appendix IV

Consider first the random variable

$$\xi_M = \frac{a^2}{\sum_{\ell=1}^{M} a_\ell^2} \tag{AIV.1}$$

where all a, a_ℓ, $\ell = 1, \cdots, M$ are i.i.d. The cumulative distribution function of ξ_M is given by

$$\text{prob}(\xi_M \leq \nu) = \text{prob}\left(a^2 - \nu\sum_{\ell=1}^{M} a_\ell^2 \leq 0\right). \tag{AIV.2}$$

Since a^2 is chi-square distributed with two degrees of freedom and $\sum_{\ell=1}^{M} a_\ell^2$ is a chi-square random variable with $2M$ degrees of freedom (see (AIII.2) with $K = 1$ and $K = M$, respectively), the pdf $f_\tau(\alpha)$ of the random variable

$$\tau = a^2 - \nu\sum_{\ell=1}^{M} a_\ell^2 \tag{AIV.3}$$

is given by the convolution integral

$$f_\tau(\alpha) = \int_{-\infty}^{\min(\alpha,\,0)} \frac{(-1)^{M-1}x^{M-1}}{(N-1)!\nu^M}\,e^{(x/\nu)}e^{-(\alpha-x)}\,dx \tag{AIV.4}$$

which for $\alpha \geq 0$ yields

$$f_\tau(\alpha) = \frac{e^{-\alpha}}{(\nu+1)^M}, \qquad \alpha \geq 0. \tag{AIV.5}$$

Thus by (AIV.2)

$$\text{prob}\,(\xi_M < \nu) = 1 - \int_0^\infty f_\tau(\alpha)\,d\alpha = 1 - \frac{1}{(\nu+1)^M} \tag{AIV.6}$$

and the corresponding pdf is

$$f_\xi(\nu) = \frac{\partial}{\partial \nu}\,\text{prob}\,(\xi \leq \nu) = \frac{M}{(\nu+1)^{M+1}}, \qquad \nu \geq 0. \tag{AIV.7}$$

Consider now the expectation (where s is a positive parameter)

$$E_\xi \log\left(1 + \frac{\xi_M}{s}\right) = \int_0^\infty \frac{M}{(1+\xi)^{M+1}}\,(\log\,(s+\xi) - \log\,s)\,d\xi \tag{AIV.8}$$

for which, integration by part yields

$$E_\xi \log\left(1 + \frac{\xi_M}{s}\right) = \int_0^\infty \frac{d\xi}{(1+\xi)^M(s+\xi)} = \frac{1}{M}\,F(1;\,M;\,M+1;\,1-s) \tag{AIV.9}$$

(see [107, Sec. 3.197, eq. 9], where $F(\cdot;\cdot;\cdot;\cdot)$ is the hypergeometric function [107, Sec. 9.10]). By equation [107, Sec. 9.137, eq. 4], the following recursion is readily found:

$$M^{-1}F(1;\,M;\,M+1,\,Z) = \frac{F(1,\,1,\,2,\,Z)}{Z^{M-1}} - \sum_{\ell=1}^{M-1} \frac{1}{\ell + Z^{M-\ell}}. \tag{AIV.10}$$

Observing that [107, Sec. 9.121, eq. 6]

$$\Gamma(1;\,1;\,2;\,1-s) = -[\ln\,s/(1-s)]$$

and substituting this in (AIV.10) yields the solution

$$E_\xi \log\left(1 + \frac{\xi_M}{s}\right) = \frac{-\ln\,s}{(1-s)^M} - \sum_{\ell=1}^{M-1} \frac{1}{\ell(1-s)^{M-\ell}}. \tag{AIV.11}$$

For $M \to \infty$ the expectation yields

$$-[\ln\,s/(1-s)^M] + [\ln\,s/(1-s)^M] = 0$$

as expected. It can also be checked that

$$\lim_{s\to 1} E_\xi \log\left(1 + \frac{\xi_M}{s}\right) = \frac{1}{M}. \tag{AIV.12}$$

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