## The Throughput and Delay Trade-off of Wireless Ad-hoc Networks

## 1. Introduction

In this report, we summarize the papers by Gupta and Kumar [GK2000], Grossglauser and Tse [GT2002], Gamal, Mamen, Prabhakar, and Shah [GMPS2004], and Neely and Modiano [NM2004] respectively. These papers consider the problem of the throughput and delay of the wireless ad-hoc networks: Gupta and Kumar studies the throughput of wireless networks where the nodes are randomly located but immobile. The main result shows that as the number of nodes $n$ increases, the throughput per source-to-destination (S-D) pair decreases approximately like $1 / \sqrt{n}$. Grossglauser and Tse address the throughput issue of mobile networks. They show that by using the mobility and a single relay, a S-D pair on average can have a constant throughput. [GMPS2004] and [NM2004] take different approaches to study the delay and the throughput-delay tradeoff. In [GMPS2004], the trade-off is achieved by varying the transmission range, while in [NM2004] it is obtained by using the concept of redundancy.

## 2. The Capacity and Delay Trade-off of a Fixed Wireless Network:

## A. Fixed Network, Successful Transmission, and Throughput

In [GK2000] nodes are fixed over time. Their positions are i.i.d. and uniformly distributed in the $S^{2}$ surface of a three-dimensional sphere (or a disk in the plane) of unit area. The destination for each source is a randomly chosen node in the network and the destinations are chosen independently. The packet from a source can reach its destination either in one hop or in multiple hops. That is, each node acts simultaneously as a source, a destination for some other node, as well as relays for others' packets.

To model what constitutes a successful reception of a transmission over a single hop, [GK2000] proposes two models - the Protocol Model and the Physical Model.
The Protocol Model: suppose $X_{i}$ transmit to node $X_{j}$. Then this transmission is successful received by node $X_{j}$ if

$$
\begin{equation*}
\left|X_{k}-X_{j}\right| \geq(1+\Delta)\left|X_{i}-X_{j}\right| \tag{1}
\end{equation*}
$$

for every other node $k$ simultaneously transmitting over the same frequency. The Physical Model: The transmission from $X_{i}$ to $X_{j}$ is successful if

$$
\begin{equation*}
\frac{\frac{P_{i}}{\left|X_{i}-X_{j}\right|^{\alpha}}}{N+\sum_{k \neq i, k \in \Psi} \frac{P_{k}}{\left|X_{k}-X_{j}\right|^{\alpha}}} \geq \beta, \tag{2}
\end{equation*}
$$

where $\Psi$ is the subset of the nodes that simultaneously transmit on the same frequency, $P_{k}$ is the power level chosen by node $X_{k}$, and the signal power decays with distance $r$ as $1 / r^{\alpha}$

In [GK2000] the notion of throughput is defined as the time average of the number of bits per second that can be transmitted by every node to its destination. Thus a feasible throughput of $\lambda(n)$ bits per second for each node means that there is a spatial and temporal scheme, such that by operating the network in multi-hop fashion and buffering at intermediate nodes, every node can send $\lambda(n)$ bits per second on average to its chosen destination node. That is, for time interval $[(i-1) t, i t]$, where $t<\infty$, every node can send $t \lambda(n)$ bits to its corresponding destination node.

## B. Main Result:

Result1 (Main Result 4 in [GK2004]): there exist constants $c$ and $c^{\prime}$ such that,
$\lim _{n \rightarrow \infty} \operatorname{Pr}\left\{\lambda(n)=\frac{c R}{\sqrt{n \log n}}\right.$ is feasible $\}=1$ and $\lim _{n \rightarrow \infty} \operatorname{Pr}\left\{\lambda(n)=\frac{c^{\prime} R}{\sqrt{n}}\right.$ is feasible $\}=0$.
This main result demonstrates that as the number of nodes per unit area $n$ increases, the throughput per S-D pair decreases approximately like $1 / \sqrt{n}$. This is the best performance achievable even allowing for optimal scheduling, routing, and relaying of packets in the networks and is a somehow pessimistic result on the scalability of such networks. The intuition behind this phenomenon is that a transmission may travel either through a single direct transmission or through multiple hops via relay nodes. As shown in the protocol model, the successful reception of the transmission of a given S-D pair prohibits simultaneous transmission with in the disk of radius proportional to the transmission distance of the pair: a successful transmission on the range $r$ incurs a cost proportional to $r^{2}$ by excluding other transmissions in the vicinity of the sender. In order to maximize the transport throughput of a network, i.e., the total number of meters traveled by all bits per time unit, it is therefore beneficial to schedule a large number of short transmissions. The best we can do is to restrict transmissions to neighbors, which are at a typical distance of $1 / \sqrt{n}$. (Since the expected distance for each session is $\Theta(1)$, the number of relays a packet has to go through scales as $\sqrt{n}$ ). The transport capacity is then at most $\sqrt{n}$ bits $\bullet \mathrm{m} / \mathrm{s}$. As there are $n$ sessions, it follows that the throughput per session can at best be $O(1 / \sqrt{n})$.


Figure 1 The unit torus is divided into cells of size $a(n)$ for Scheme1.

## C. The Capacity and Delay Trade-off

In [GMPS2004] the authors recover the results in [GK2000] and analyze the throughput and delay trade-off using the following scheme:

## Scheme 1(Scheme 1 in [GMPS2004]):

- Divide the unit torus using a square grid into square cells, each of area $a(n)$.
- A TDMA scheme is used, in which each cell becomes active, i.e., the nodes in the given cell can transmit successfully to nodes in the cell or in neighboring cells, at regularly scheduled cell time slots.
- A source $S$ transmits data to its destination $D$ by hopping along the adjacent cells on its S-D line.
- When a cell becomes active, it transmits a single packet for each of the S-D lines passing through it.

Result 2 (Theorem 1 in [GMPS2004]): for Scheme 1, with $a(n) \geq 2 \log n / n$
$\lambda(n)=\Theta\left(\frac{1}{n \sqrt{a(n)}}\right)$ and $D(n)=\Theta\left(\frac{1}{\sqrt{a(n)}}\right)$, i.e., the achievable throughput-delay tradeoff is $\lambda(n)=\Theta\left(\frac{D(n)}{n}\right)$.


Figure 2 The throughput-delay trade-off curve.
We can interpret the above results as follows: the highest throughput per node achievable in a fixed node is $\Theta(1 / \sqrt{n \log n})$, consistent with the result by [GK2004]. At this throughput the average delay is $D=\Theta(\sqrt{n / \log n})$ (corresponding to the point Q in the throughput-delay trade-off curve plotted in Fig.2). The delay of a fixed network is proportional to the number of hops that a packet has to travel. By increasing the transmission radius the average number of hops (thus the delay) can be reduced. Since the interference is higher now, the throughput would decrease. If we only allow one-hop transmission $(a(n)=1)$, the throughput will go as low as $1 / n$ (corresponding to point S in Fig.2). Next we outline the proof of Result 2:

## Outline of the proof:

(1) Throughput:

To analyze the scaling of the throughput with the setup of Scheme 1, we use the following lamas:
Lemma 1 (Lemma 1 in [GMPS2004]): if $a(n) \geq 2 \log n / n$, then all cells have at least one node with high probability (whp).
Lemma 2 (Lemma 2 in [GMPS2004]): the number of cells that interfere with any given cell is bounded by a constant $c_{1}$, independent of $n$.
$\rightarrow$ Each cell can be active for a guaranteed fraction of time, i.e., it can have a constant throughput.
Lemma 3 (Lemma 3 in [GMPS2004]): the number of S-D lines passing through any cell is $O(n \sqrt{a(n)})$.
$\rightarrow$ If each cell divides its cell time-slot in to $O(n \sqrt{a(n)})$ packet time slots, each S-D pair hopping through it can use one packet time slot. Equivalently, each S-D pair can successfully transmit to $\Theta(1 / n \sqrt{a(n)})$ fraction of time. That is, the achievable throughput per S-D pair is $\lambda(n)=\Theta(1 / n \sqrt{a(n)})$.
(2) Delay:

Packet delay is the amount of time spent in each hop. Since each hop covers a distance of $\Theta(\sqrt{a(n)})$, the number of hops per packet for S-D pair $i$ is $\Theta\left(d_{i} / \sqrt{a(n)}\right)$, where $d_{i}$ is the length of the S-D line $i$. Thus the number of hops taken by a packet averaged over all S-D pairs is $\Theta\left(\frac{1}{n} \sum_{i=1}^{n} d_{i} / \sqrt{a(n)}\right)$. For large $n$, the average distance between a S-D pair is $\frac{1}{n} \sum_{i=1}^{n} d_{i}=\Theta(1)$, thus the average number of hops is $\Theta(1 / \sqrt{a(n)})$.

## 3. The Capacity and Delay Trade-off of a Mobile Wireless Network-Single Relay Case:

## A. The Mobile Wireless Network

The capacity of a mobile wireless network was first studied in [GT2002]. The network in [GT2002] consists of $n$ nodes all lying in the disk of unit area. The location of the $i$ th user at time $t$ is given by $X_{i}(t)$, which is modeled as a stationary and ergodic process with stationary distribution; moreover, the trajectories of different users are independent and i. i. d. Each node is the source for one session and the destination of another session, and the S-D association does not change with time. The central intuition of [GT2002] is that any two nodes can be expected to be close to each other from time to time so that we may improve the capacity of the network - the delay tolerance can be usefully exploited in a mobile wireless network.

## B. The Role of Relaying

In [GT2002] the authors first show that without relaying, there is no way to achieve a $\Theta(1)$ throughput per S-D pair. (The results are presented in Lemma III-2 and Theorem of III-3 of [GT2002]. For the brevity of this summary, they are not repeated here.) The explanation is that the number of simultaneous long-range communication is limited by interference. If transmissions over long distance are allowed, then there are many S-D pairs that are within the range - interference limits the number of concurrent transmissions overlong distances; the throughput is interference limited. On the other hand, if we constrain communication to neighboring nodes, then there is only a small fraction of S-D pairs that are sufficiently close for transmitting a packet. Hence, the throughput is distance limited.
To increase the throughput, one need to find a way to limit the transmission locally, while guaranteeing that there would be enough sender-receiver pairs that have packets to send. Thus [GT2002] proposes to spread the traffic stream between the source and the destination to a large number of intermediate relay nodes. The goal is that in the steadystate, the packet of every source node will be distributed across all the nodes in the network, ensuring that every node will have packets buffered destined to every other node. This ensures that a scheduled sender-receiver pair always has a packet to send, in contrast to the case of direct transmissions.

A question that naturally follows is that how many times a packet needs to be relayed. In fact, as the node location processes are independent, stationary, and ergodic, it is sufficient to relay only once. This is because the probability for an arbitrary node to be scheduled to receive a packet from a source node $S$ is equal for all nodes and independent of S. Each packet then makes two hops, one from the source to its randomly chosen relay node and one from the relay node to the destination as shown in Fig. 3.


Figure 3 The two-phase scheduling scheme viewed as a queuing system.

## C Throughput

[GT2002] proposes a scheduling algorithm that consists of two phases: the scheduling of packet transmissions from sources to relays and the scheduling of transmissions form relays to final destinations. Note that in both phases a transmission from a source directly to a destination is possible. This two-phased algorithm leads to the following result.

Result 3 (Theorem III-5 in [GT2002]): the two-phased algorithm achieves a throughput per S-D pair of $\Theta(1)$, i.e., there exists a constant $c>0$ such that

$$
\lim _{n \rightarrow \infty} \operatorname{Pr}\{\lambda(n)=c R \text { is feasible }\}=1
$$

## Outline of the proof:

The proof of the Result 3 depends on an important result that states that the probability that given two nodes are selected as a S-D pair is $\Theta(1 / n)$ (This result is formally stated as Theorem III-4 in [GT2002], the outline of the proof is not included here for brevity of this summary). For a given S-D pair, there is one direct route and $n-2$ two-hop routes. The throughput of the direct route is $\Theta(1 / n)$. Each of the two-hop route can be treated as a single server queue, each with arrival and service rate of $\Theta(1 / n)$. The total throughput is $\Theta(1 / n)$ by summing all the throughputs of $n-1$ routes.

## D. The Capacity and Delay Trade-off

[GMPS2004] also assumes that $n$ nodes forming $n$ S-D pairs in a torus of unit area and assumed slotted transmission time. Each node moves independently and uniformly on the unit torus. The authors recover the results by [GT2002] and analyze the throughput and delay trade-off using the following scheme:

## Scheme 2 (Scheme 2 of [GMPS2004])

- Divide the unit torus into $n$ square cells, each of area $1 / n$.
- Each cell becomes active once in every $1+c_{1}$ cell time slots.
- Transmission is limited with in an active cell.
- In an active cell, if two or more nodes are present pick one at random. Each cell time-slot is divided into two subslots A and B.
A. Transmit to destination if it is in the same cell. Otherwise, it transmits to a randomly chosen node in the same cell, which acts as a relay,
B. The randomly chosen node picks another node at random form the same cell and transmits to it a packet that is destined to it.

Result 4 (Theorem 3 and 4 of [GMPS2004]): the throughput and delay using Scheme 2 is given by $\lambda(n)=\Theta(1)$ and $D(n)=\Theta(n)$.
This point is shown as R in Fig.2.

## Outline of the proof:

(1) Throughput

Each packet is transmitted directly to its destination or relayed at most once and hence the net traffic is at most twice the original traffic. Since: (a) a relay node is chosen randomly; (b) the nodes have independently and uniformly distributed motion, each source's traffic gets spread uniformly among all other nodes. As a result, in steady state, each node has packets for every other node for a constant fraction of time $c_{2}$. $1-2 e^{-1}$ fraction of the cells contain at least 2 nodes. $0.26 c_{2} /\left(1+c_{1}\right)$ fraction of cells can execute the scheme successfully. Since each cell has a throughput of $\Theta(1)$, the net
throughput in any time-slot is $\Theta(n)$ whp. $\Theta(n)$ is divided equally among $n$ pairs, we have a throughput $T(n)=\Theta(1)$.
(2) Delay

Since the torus has $n$ square cells, the area of each cell is $1 / n$, the side of each cell is $1 / \sqrt{n}$. The movement of the nodes is modeled as a random walk on a 2-D torus of size $\sqrt{n} \times \sqrt{n}$, where each move occurs every $t(n)$ time slots, where $t(n)=\Theta(1 / v(n) \sqrt{n})$.

Since most of the S-D pairs have to go through two hops, the delay for this scheme has two components: hop-delay and mobile delay. Since node velocity is much lower than the speed of electromagnetic propagation, the delay is dominated by the mobile delay.
[GMPS2004] first models the queues formed at a relay node for each S-D pair as GI/GI/1-FCFS, then characterizes the inter-arrival and inter-departure times of the queue to obtain the average mobile delay. By bounding the first and the second moments of the inter-arrival and departure process and using the following average delay bound (Lemma 8 of [GMPS2004]), one can upper bound the number of random walks required as

$$
\begin{equation*}
\mathrm{E}[\# \text { of rand. wlk }] \leq \max \left\{\mu, \frac{\sigma_{\mathrm{a}}^{2}+\sigma_{s}^{2}}{2 \mu \varepsilon}\right\}=\Theta(n) \tag{3}
\end{equation*}
$$

The delay can be derived as

$$
\begin{equation*}
D(n)=\Theta(n) \Theta\left(\frac{1}{\sqrt{n} v(n)}\right)=\Theta\left(\frac{\sqrt{n}}{v(n)}\right) \tag{4}
\end{equation*}
$$

We have $D(n)=\Theta(n)$ when $v(n)=\Theta(1 / \sqrt{n})$.
In [NM2004], the authors propose a "Cell Partitioned Relay Algorithm" and obtain the same capacity-delay trade-off relationship for the mobile and single relay case. The problem setup assumes that the network is partitioned into $C$ non-overlapping cells of equal size. There are $n$ mobile users independently roaming from cell to cell over the network: for a given time slot users remain in their current cells for a timeslot and potentially move to a new cell at the end of the slot. As in Scheme 2, the transmission is also limited in the cell. The main difference of the setup in [NM2004] is that the authors assume infinite mobility - the network topology dramatically changes over timeslot, so that network behavior cannot be predicted. The other difference of the setup is that queuing delay at the source node is considered in [NM2004], while this is not taken into account in [GMPS2004].

Result 5a (Theorem 1 of [NM2004]): let $p_{0}$ denote the probability that there are at least 2 nodes in a cell, $q$ denote the probability that there is a S-D pair in a cell, and $d$ denote the node density (the number of nodes per cell). The capacity of the network is bounded
by $\lambda(n) \leq \frac{C p_{0}+C q+2 \varepsilon}{2 n}$ and $\lim _{n \rightarrow \infty} \frac{C p_{0}+C q+2 \varepsilon}{2 n}=\frac{1-e^{-d}-d e^{-d}}{2 d}$

Comments: note that

$$
\begin{equation*}
\lim _{\substack{d \rightarrow \infty \\ d \rightarrow 0}} \frac{1-e^{-d}-d e^{-d}}{2 d}=0 \tag{5}
\end{equation*}
$$

That is, if $d$ is too large, there will be many users in each cell, most of which will be idle as a single transmitter and receiver are selected. If $d$ is too small, the probability of two users being in a given cell vanishes. In both cases, the capacity diminishes. [NM2004] calculates the optimal node density and throughput: $d^{*}=1.7933$ and $\mu^{*}=0.1492$.


Figure 4 A two-stage queue, the first stage are Bernoulli with rate $\lambda(n)$. Service at the second stage (relay) queues is Bernoulli with rate $(p-q) /(2 d(N-2))$.

Result 5b (Theorem 3 of [NM2004]) Consider a cell partitioned network (with $n$ users and $C$ cells) under the 2-hop relay algorithm, and assume that users change cells i. i. d. and uniformly over each cell every timeslot. If the exogenous input stream to user $i$ is a Bernoulli stream of rate $\lambda_{i}$ (where $\lambda_{i}<\lambda(n)$, then the total network delay $T_{i}$ for user $i$ traffic satisfies:

$$
E\left\{T_{i}\right\}=\frac{N-1-\lambda_{i}}{\lambda(n)-\lambda_{i}}
$$

Comments:
(1) The randomized nature of the cell partitioned relay scheme admits a nice decoupling between the sessions, where individual users see the network as a twostage queues, the first stage are Bernoulli with rate $\lambda(n)$. Service at the second stage (relay) queues is Bernoulli with rate $(p-q) /(2 d(N-2))$. This is different from the single-stage queue treatment in [GMPS2004], in which the queuing delay at the source node is not considered.
(2) Because of the infinite mobility assumption, the delay is not modeled explicitly as a function of node velocity.
For the detail of the proof for Result 5a and Result 5b, please see [NM2004], the proof is not outlined in this summary.

## 4. The Capacity and Delay Trade-off of a Mobile Wireless Network - Transmission Range and Redundancy:

## A. Varying Transmission Range

[GMPS2004] proposes the following schemes to achieve the capacity and delay tradeoff through varying transmission range and multi-hopping.

## Scheme 3a (Scheme 3a of [GMPS2004])

- Divide the unit torus using a square grid into square cells, each of area $a(n)$.
- A TDMA scheme is used, in which each cell becomes active, i.e., its nodes can transmit successfully to nodes in the cell or in neighboring cells, at regularly scheduled cell time slots.
- A source $S$ transmits data to its destination $D$ either by direct transmission of by relaying to the nodes in the adjacent cells along its S-D line.

Comments: this scheme is basically the same as the Scheme 1, except that now the nodes are moving around, so the S-D and R-D lines are changing, as illustrated by Fig. 4


Figure 5 Scheme3a is basically the same as Scheme 1

## Result 6 (Theorem 5 of [GMPS2004])

If $v(n)=o(\sqrt{\log n / n})$ is satisfied, achieves the following trade-off:
$T(n)=\Theta\left(\frac{D(n)}{n}\right)$, for $T(n)=O\left(\frac{1}{\sqrt{n \log n}}\right)$
Comments: this trade-off corresponds to the point Q of Fig.2.

## Outline of the proof:

The proof follows the same rationale as the proof for Result 1.
(1) The condition that $v(n)=o(\sqrt{\log n / n})$ is necessary for every packet to be eventually delivered. Note that $\lim _{n \rightarrow \infty} v(n)=0$, the mobile network behaves more like a fixed network as the number of nodes increases. The proof follows the same rationale as the proof for Result 1.
(2) Delay: the average number of times a packet has to be relayed in order to reach its destination is of order $\Theta(1 / \sqrt{a(n)})$, which is same as in Scheme 1 for fixed networks. Hence the delay scales as $D(n)=\Theta(1 / \sqrt{a(n)})$.
(3) Throughput: the proof for throughput is the same as that for Scheme 1, the number of S-D paths passing through any cell at any given time-slot is $\Theta(1 / n \sqrt{a(n)})$, thus the throughput per S-D pair is at least $\Theta(1 / n \sqrt{a(n)})$.

In [GMPS2004], the following scheme is also proposed to achieve the points on the trade-off segment QR.

## Scheme 3b (Scheme 3b of [GMPS2004])

- Divide the unit torus using a square grid into square cells, each of area $a(n)$. Divide each cell into sub-cells, each of area $b(n)$.
- A cellular TDMA scheme is used, with each cell slot contains $\Theta(n a(n))$ packet slots.
- Each active packet time-slot is divided into two sub-slots:
A. Transmit to destination if it is in the same cell. Otherwise, it transmits to a randomly chosen node in the same cell, which acts as a relay. The packet is sent using hops along sub-cells as in Scheme 3a.
B. The randomly chosen node picks another node at random form the same cell and transmits to it a packet that is destined to it. The packet is sent using hops along sub-cells as in Scheme 3a.


Figure 6 The unit torus is divided into cells of size $a(n)$ and sub-cells of $b(n)$ for Scheme3b.
This scheme, as illustrated in Fig.6, consists of "inter-cell travel" and "intra-cell transmission". The inter-cell travel, which enable the packets eventually to be delivered, is mostly similar to that of Scheme2, with the difference that the cell size $a(n)$ is adjustable, instead of $1 / n$ in Scheme 2. The intra-cell transmission is mostly similar to that of Scheme 3a, with the difference that the hop size is $b(n)$, instead of $a(n)$ in Scheme 3a.

## Result 7 (Theorem 6 of [GMPS2004])

If $v(n)=o(\sqrt{\log n / n})$ is satisfied, Scheme 3 b achieves the following trade-off:

$$
T(n)=\Theta\left(\frac{1}{\sqrt{n a(n) \log n}}\right) \text { and } D(n)=O\left(\frac{1}{\sqrt{a(n) v(n)}}\right)
$$

where $a(n)=O(1)$ and $a(n)=\Omega(\log n / n)$

## Outline of the proof:

The intuition behind the Scheme 3 b and the proof is that intra-cell transmission would provide us the desired throughput and inter-cell travel provides desired delay. The points on the segment QR is achieved by varying the size of the cell $a(n)$.
(1) In steady state, each node has packets for every other node for a constant fraction of time and the traffic between each source destination pair is spread uniformly across all other node, as for Scheme 2.
(2) Throughput: since the transmission is limited within the cell (size of $a(n)$ ), the throughput can be derived in the similar fashion as in that for Scheme 1, with the following difference, $n$ is replaced by $n a(n)$ and $a(n)$ is replace by $b(n)$.
(3) Delay: the mobile delay dominates the total delay. The mobile delay can be analyzed in the same fashion as for Scheme 2, albeit with the following differences:

- The random walk is of discrete size of $\sqrt{1 / a(n)} \times \sqrt{1 / a(n)}$, instead of $1 / n \times 1 / n$.
- A node travels to its neighboring slots every $t(n)=\Theta\left(\frac{a(n)}{v(n)}\right)$ time-slots, instead of

$$
t(n)=\Theta\left(\frac{1}{v(n) \sqrt{n}}\right)
$$

## B. Redundancy and Single Relay

In [NM2004] the authors adopt a different approach to achieve the throughput-delay trade-off - sending duplicates of the same packet to different users. The throughput will decrease but the delay can be improved.

In-Cell Feedback Scheme with $\sqrt{n}$ redundancy: in every cell with at least two users, a random sender and a random receiver are selected, with uniform probability over all users in the cell. With probability $1 / 2$, the sender is scheduled to operate in either 'source-torelay' mode, or 'relay-to-destination' mode, described as follows:

1) Source-to-Relay Mode: The sender transmits packet SN, and does so upon every transmission opportunity until $\sqrt{n}$ replicas have been delivered to distinct users, or until the sender transmits SN directly to the destination. After such a time, the send number is incremented to $\mathrm{SN}+1$. If the sender does not have a new packet to send, remain idle.
2) Relay-to-Destination Mode: When a user is scheduled to transmit a relay packet to its destination, the following handshake is performed:

- The receiver delivers its current RN number for the packet it desires.
- The transmitter deletes all packets in its buffer destined for this receiver that have SN numbers lower than RN .
- The transmitter sends packet RN to the receiver. If the transmitter does not have the requested packet RN , it remains idle for that slot.


## Result 8 (Theorem 6 in [NM 2004])

The In-Cell Feedback Scheme achieves the $O(\sqrt{n})$ delay bound and the throughput of $O(1 / \sqrt{n})$.

## Comments:

(1) The handshakes as described in the above scheme are necessary so that the old versions of packets already delivered are removed from the network to avoid unnecessary congestion.
(2) The delay can't be better than $O(\sqrt{n})$, if only single relay is allowed.

For the detail of the proof for Result 8, please see [NM2004], the proof is not outlined in this summary.

## 5. Summary

The schemes used in the report are summarized in the following table. And Fig. 8 plots the main results.

|  | Scheme 1 <br> $[$ GK] \& <br> [GMPS $]$ | Scheme 2 <br> $[$ [GT], [GMPS $]$ <br> $\&[$ NM $]$ | Scheme 3a <br> $[$ GMPS $]$ | Scheme 3b <br> $[$ GMPS $]$ |
| :--- | :--- | :--- | :--- | :--- |
| Node Mobility | Fixed | Mobile | Mobile | Mobile |
| Cell size | $a(n)$ | $1 / n$ | $a(n)$ | $a(n)$ |
| Sub-cell size | N.A. | N.A. | N.A. | $b(n)$ |
| Multiple <br> Access Scheme | TDMA <br> $1+c_{1}$ slots | TDMA <br> $1+c_{1}$ slots with <br> sub-slots | TDMA <br> $1+c_{1}$ slots | TDMA <br> $1+c_{1}$ slots with <br> sub-slots |
| Transmission <br> Range | To adjacent <br> cells | Within the cells | To adjacent <br> cells | With in the <br> cells; to <br> adjacent sub- <br> cells |
| Delivering Data <br> to Destination | S-D line | S-D or S-R and <br> R-D | S-D line | Inter Cells <br> using 2, intra <br> cells using 3(a) |
| Multiple hops | $>2$ hops <br> possible | 2 hops <br> maximum | $>2$ hops <br> possible | $>2$ hops <br> possible |




Figure 8 The throughput-delay trade-off curve. Left: the results from [GMPS2004]. Right: the results from [NM2004] .

## Reference:

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