Cooperative-Diversity in Wireless Networks

Vijay Divi

November 17, 2004

1 Introduction

One major limitation of wireless systems is the fading channel that exists between users and the base station. When a user's channel realization is bad, it may be unable to communicate with the base station. To overcome this problem, many different forms of diversity have been studied including: spatial, temporal, and frequency diversity. Recently, there has been an increasing popularity in a specific form of spatial diversity, namely cooperative diversity.

Spatial diversity relies on the principle that multiple antennas located a sufficient distance apart experience independent fading channels. Unlike the traditional form of spatial diversity where a user has access to a physical array of different antennas, cooperative diversity is based off the relay channel model where different users share their individual antennas. In particular, information is transmitted from source to destination both directly and through relays in the network; thus the relays and source act as a virtual antenna array. Within this report, we present multiple cooperative-diversity protocols and analyze their performance via the diversity-multiplexing tradeoff curves.

2 Background

To begin, we will overview the scalar fading channel and introduce different channel modeling assumptions that will have a key impact on the performance. We then introduce the MIMO communication system in which the source and destination have multiple antennas and study the performance gains with spatial diversity. The analysis of MIMO systems is closely related to the analysis of cooperative-diversity protocols; in particular, the MIMO systems provide an upper bound for our the cooperative-diversity system performance.

2.1 Scalar Fading Channel

In a scalar fading channel, the input signal x experiences Rayleigh fading h and additive Gaussian noise w, modeled as

$$y = hx + w \tag{1}$$

where the input $x \sim CN(0, \sigma_x^2)$ and noise $w \sim CN(0, \sigma_w^2)$.

The analysis of system performance depends on the assumptions we make on the fading parameter h. There are two different properties of h that we focus on: how often this parameter changes with time and whether its value is known at the transmitter (Tx). These properties define four major cases which are studied in communications. The terms fast-fading and slow-fading refer to the coherence time for which a specific realization of h is valid. For fast-fading, we assume that the channel fading realization changes often, on the time order of symbols or codewords. For slow-fading, the channel realization does not change within the time period of interest. Channel State Information (CSI) is used to describe whether a terminal knows the value of h; in particular, we focus on whether CSI exists at the transmitter. In practice, CSI is attained at the transmitter via a feedback link or when there is 2-way communication. For our discussion, we make the assumption that CSI exists at the receiver, which is a valid assumption for most communication systems.

- **Slow Fading/Tx CSI** - The channel has one realization of the fading coefficient and this value is known to the transmitter. The capacity of the channel is a well defined random quantity since the transmitter can use a rate equal to the max rate supported by the fading realization (see table 1).

- **Fast Fading/Tx CSI** - Unlike the case above, the channel is changing with every symbol. Thus, an optimal strategy is for the transmitter to vary the amount of power used for each symbol based up the channel realization, i.e. a water-filling procedure in time. It is difficult to find a closed form expression for the capacity in this case.

- **Slow Fading/No Tx CSI** - Because the channel is unknown to the transmitter, any non-zero rate chosen by the transmitter may be above the supported rate of the channel. Note, since the channel is not changing with time (non-ergodic), the transmission may always fail with some non-zero probability. Thus, the Shannon capacity is zero, i.e., the system cannot guarantee that any amount of information can be transmitted reliably. Instead, the notion of outage capacity is introduced. For a specified rate R and SNR, there is a non-zero probability that the channel does not support this rate; this probability is known as the outage probability (p^{out}).

- **Fast Fading/No Tx CSI** - For this case, the notion of ergodic capacity is introduced. Although the channel may be bad for certain times, we can bound this amount of time and pick long codewords which experience enough good channels so that it can be decoded without error; the capacity is the average over many channel realizations. Rephrased, as we at longer periods of time, the average mutual information per symbol converges to the expectation of mutual information.

The cases are summarized in the table below:

	Slow Fading	**Fast Fading**
Tx CSI	Deterministic Capacity $C = \log(1 + \|h\|^2\rho)$	Time Water-filling
No Tx CSI	Outage Capacity R, p^{out}	Ergodic Capacity $C = E[\log(1 + \|h\|^2\rho)]$

Table 1: Transmission regimes for scalar fading channel

The variable ρ represents the SNR, σ_x^2/σ_w^2. For the sequel, we will focus on the case of slow fading with no CSI (Channel State Information) at the transmitter. Note, this is the most challenging case since we do not know the channel and it does not change.

Although transmission directly between the source and destination may be in outage for a certain rate, using relays increases diversity by providing alternate channels which may experience less fading. Cooperative diversity systems use relays as virtual antenna arrays which allow them to perform better than the single antenna system. However, virtual antennas may not be as helpful as having extra antennas at the transmitter. Similar to the lower bound provided by the scalar fading channel, we can attain an upper bound on performance of cooperative-diversity systems by looking at the MIMO (multiple input multiple output) setup where the transmitter has multiple antennas.

2.2 Multiple Antenna Systems

Multiple antennas at the transmitters and/or receivers help increase the performance of wireless systems. The diversity due to multiple paths between the transmitter and receiver results in higher data rates with lower outage probabilities. Recently, the diversity-multiplexing tradeoff has been widely studied as a metric for determining the performance of a MIMO schemes. This tradeoff measures the high-SNR asymptotic tradeoff between outage probability and rate.

The discrete-time MIMO channel model is

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}. \tag{2}$$

where $\mathbf{H}$ is an $n \times m$ matrix whose entries are i.i.d., $CN(0, 1)$. On one hand, having H drawn from such a distribution causes outages in the system and on the other hand, independent channel gains can be exploited to our advantage.

Diversity Gain

In the high-SNR regime, the diversity gain d measures the rate at which the error probability decays ($1/\rho^d$). It is well know that in a system with m transmit and n receive antennas, the average error probability can be made to decay as $1/\rho^{mn}$, yielding a maximum diversity gain of mn [1]. This diversity is a result of the mn independent paths between the transmit and receive antennas.

[1]This diversity gain can be achieved by repeating a single transmission across each transmitter/receiver pair

Formally written,

$$d = \lim_{\rho \to \infty} \frac{\log P_e(\rho)}{\log \rho} \tag{3}$$

where P_e is the probability of error in transmission.

Multiplexing Gain

Although path fading causes outages in the system, it also allows the system to increase the degrees of freedom for communication. For example, if all paths experience the same amount of fading, the rows of the channel matrix (mapping input to output) would be identical, decreasing the ability to discern between different transmitted signals. Because the path gains are independent, the channel matrix is well-conditioned with high probability. The well-conditioned property allows us to view the MIMO system as $\min(m, n)$ independent spatial channels between the transmitter and receiver.

In the fast fading case, the capacity of such a system would be

$$C(\rho) = \min(m, n) \log \rho + O(1). \tag{4}$$

Note, with high SNR, $E[\log(1 + |a_{s,d}|^2 \rho)]$ is approximated by $\log \rho$. In our slow fading scenario, we set a rate equal to a fraction of this capacity and determine our probability of outage. We know that for a fixed rate, as SNR tends to infinity, the probability of outage for a random code goes to zero. Thus, we would like to increase our rate with SNR as follows $R = r \log \rho$. The spatial multiplexing gain r measures the rate at which R increases with respect $\log \rho$ and is formally defined as:

$$r = \lim_{\rho \to \infty} \frac{R(\rho)}{\log \rho}. \tag{5}$$

The multiplexing gain can also be thought of as the number of independent spatial channels being used optimally, which is upper-bounded by $\min(m, n)$.

For higher values of a multiplexing gain r, the probability of outage increases, i.e. a decrease in diversity d. Thus, it is natural to talk about a tradeoff between the two values. This relationship between r and d is defined as the diversity-multiplexing tradeoff, made popular by Zheng and Tse [3].

To gain intuition into the diversity-multiplexing tradeoff, one can think of the multiplexing gain r as r parallel channels and having robustness against $\min(m, n) - r$ parallel channels [2]. However, these parallel channels can be ordered in terms of their statistical reliability. Assuming $m < n$ WLOG, the distribution of these channel realizations from weakest to strongest are $\chi^2_{2(n-m+1)}, \chi^2_{2(n-m+3)}, \ldots$. Allowing the weakest channel to fail is equivalent to decreasing r by one to $\min m, n-1$, which gives a diversity gain of $n-m+1$. This allows $n - m + 1$ of the mn channel realizations to be bad. Similarly, decreasing r by two from the full rate allows the two weakest channels to fail, giving a diversity gain of $(m - n + 1) + (m - n + 3) = 2(m - n + 2)$. These increases in multiplexing gains for increases in the diversity are seen within figure 1.

Connection to Cooperative-Diversity

For our one relay cooperative-diversity system, we know that the transmission protocol should do no worse than the scalar fading channel since we can just ignore use

[2]Joint coding over all channels is necessary to achieve the full tradeoff

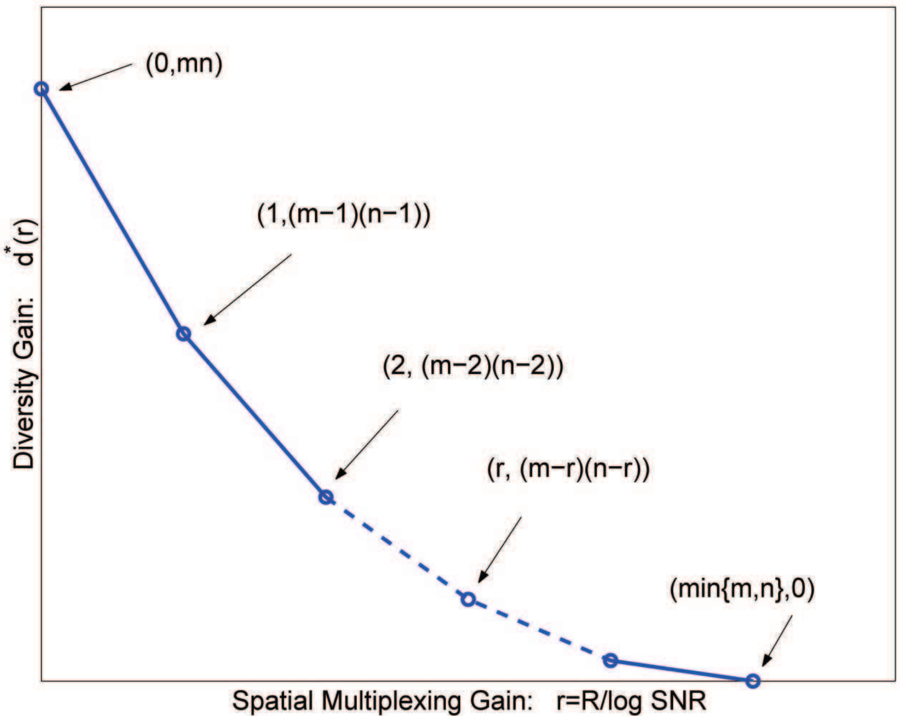

Figure 1: Outer Bound for Diversity Multiplexing Tradeoff from [3]

of the relay (non-cooperative protocol). Thus, the scalar channel provides a lower bound for the performance of any valid scheme. The schemes are also upper bounded by the 2×1 MIMO system. In the MIMO system, the transmitter has full control over both antennas and can code across both of them, allowing more flexibility. Using cooperative-diversity, we also have to worry about whether the virtual antennas (relays) receive the signal, which may limit performance.

3 System Model

Cooperative diversity involves the transmission of a message from a source to a destination with the help of relays; one direct path and other paths through the relays as seen in figure 2. In the single relay case, we let T_s, T_d and T_r represent the source, destination, and relay terminals respectively. Thus, $x_s[n]$ and $x_r[n]$ are the transmitted signals from the source and relay; $y_r[n]$ and $y_d[n]$ are the received signals of the relay and destination.

In direct transmission between the source and destination, the channel is modeled as:

$$y_d[n] = a_{s,d}x_s[n] + z_d[n]. \tag{6}$$

For cooperative diversity, we will use a variety of different transmission schemes, which will be discussed in future sections.

Throughout the paper, the variables $a_{i,j}$ are used to represent the path-loss/fading of the channel between terminals i and j. The $a_{i,j}$ are zero-mean, independent, circu-

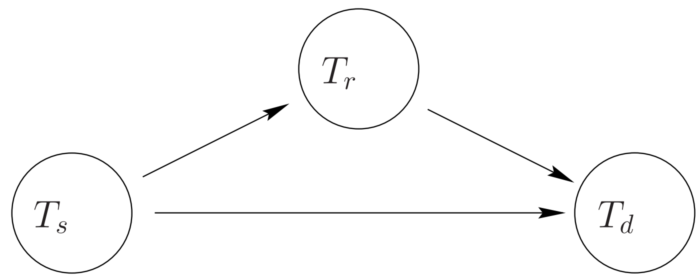

Figure 2: Diagram of terminals set-up with one relay. Arrows indicate radio signal transmission paths

larly symmetric, complex Gaussian random variables with variances $\sigma^2_{i,j}$. The channel noise $z_j[n]$ is modeled as zero-mean, mutually independent, circularly-symmetric, complex Gaussian random sequences with variance N_o.

- A half duplex constraint is imposed on the antennas; they cannot transmit and receive information at the same time.
- Channels experience flat Rayleigh-fading. The gains are independent and the additive noise is zero-mean, mutually independent, circularly symmetric and white complex Gaussian noise.
- The terminals exist in a slow fading environment where each path gain is constant for all time.
- Transmitters have no CSI. Receivers have full CSI.
- Each terminal has only one antenna and each has the same power constraint, P.

Comments:

- There are two different errors in the system: outage errors and decoding errors. As we have described, outage errors occur if the rate chosen by the transmitter is not supported for the channel realization. Decoding errors occur when a codeword is decoded incorrectly due to the noise in transmission. For long codeword lengths, this error is much smaller than the outage error, thus we omit discussion of decoding errors from our analysis.

4 Case Analysis

We now analyze multiple cooperative diversity protocols. We start with methods applicable to the one relay case, then generalize to more complicated cases of multiple relays/sources/destinations.

1. Single Relay Systems

- Amplify-and-Forward
- Decode-and-Forward

2. Multiple Terminal System
   - AF Revisited
   - DF Revisited
   - Cooperative Broadcast Channel
   - Cooperative Multiple-Access Channel

4.1 Single Relay Systems

Within this section, we will investigate one-relay systems where the relay can either amplify the received signal or decode, re-encode, and transmit it. For each method, we first present protocols developed by Laneman-Wornell-Tse [2] which apply the additional constraint that only one terminal is transmitting at a given time. We subsequently provide strategies by Azarian-El Gamal-Schniter [1] which allow multiple terminals to transmit simultaneously and are shown to be optimal in certain regimes.

4.1.1 Amplify-and-Forward

Using the Amplify-and-Forward (AF) method, the relay can only transmit an amplified version of its received signal to the destination. We will now investigate different protocols which can be used to exploit the spatial diversity of the relay.

Let us start with a basic strategy in which the source transmits one codeword in the first time slot ($n = 1, ..., N/2$). The receiver listens to the source signal in the first time slot, amplifies it, and transmits the amplified signal in the second time slot. In this case, the source is only transmitting half of the time.

Thus, for time $n = 1, ..., N/2$, the destination and relay receive the following signals

$$y_d[n] = a_{s,d}x_s[n] + z_d[n], \tag{7}$$

$$y_r[n] = a_{s,r}x_s[n] + z_r[n]. \tag{8}$$

The relay chooses its amplifier gain

$$\beta \leq \sqrt{\frac{P}{|a_{s,r}|^2 P + N_o}}, \tag{9}$$

so that it remains within its transmit power constraint.

In the second time slot, $n = N/2 + 1, ..., N$, the relay transmits its received signal amplified by β and the source does not transmit. Thus, the destination receives

$$y_d[n] = a_{r,d}(\beta y_r[n - N/2]) + z_d[n]. \tag{10}$$

After two time slots, the destination has received the output of a single input through two complex Gaussian noise channels, each with a different level of noise.

We now turn to an analysis of the performance of this scheme. For the AF protocol described previously, we can compute the mutual information between the input and 2 outputs as

$$I_{AF} = \frac{1}{2}\log(1+\rho|a_{s,d}|^2 + f(\rho|a_{s,r}|^2, \rho|a_{r,d}|^2)) \tag{11}$$

where

$$f(x,y) = \frac{xy}{x+y+1}. \tag{12}$$

The outage probability for a rate R can computed as the probability that $I_{AF} < R$. For an outage event, we expect only one term out of our arguments of $f(x,y)$ to be small; thus $f(x,y)$ can be approximated as $\min(x,y) \triangleq \gamma$. The outage event can be approximated as

$$\begin{aligned}
\frac{1}{2}\log(\rho|a_{s,d}|^2 + \rho\gamma) &< R & (13)\\
\log(\rho) + \log(|a_{s,d}|^2 + \gamma) &< 2r\log\rho & (14)\\
\log(|a_{s,d}|^2 + \gamma) &< (2r-1)\log\rho & (15)\\
|a_{s,d}|^2 + \gamma &< \rho^{(2r-1)} & (16)
\end{aligned}$$

The variables γ and $|a_{s,d}|^2$ are both exponential. An outage occurs if the sum of these are less than ρ^{2r-1}, which occurs with probability $(\rho^{2r-1})^2$. Thus, the diversity multiplexing tradeoff is given by $d(r) = 2(1-2r)$. Although this system is relatively simple to implement, it does not attain the optimum diversity-multiplexing tradeoff for AF systems. We now present a more complicated scheme which is shown to be optimal.

The AF scenario can be modeled in its most general form [1] as

$$\mathbf{y} = \begin{bmatrix} a_{s,d}A_1 & 0 \\ a_{s,r}a_{r,d}\beta B & a_{s,d}A_2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ a_{r,d}B \end{bmatrix}\mathbf{w} + \mathbf{v} \tag{17}$$

where $\mathbf{y}, \mathbf{x}, \mathbf{w}, \mathbf{v} \in \mathbb{C}^{\mathbf{N}}$, $\mathbf{A_1} \in \mathbb{C}^{\mathbf{N',N'}}$, $\mathbf{B} \in \mathbb{C}^{\mathbf{N-N',N'}}$, $\mathbf{A_2} \in \mathbb{C}^{\mathbf{N-N',N-N'}}$. Thus, for every N symbols transmitted, the source transmits for the first N'. The relay can then begin transmitting amplified copies of these symbols; meanwhile, the source may still transmit symbols which the relay will never retransmit. The protocol considered previously is a specific case of this general setup, where $N' = N/2$, $A_1 = B = I_{N/2}$, and $A_2 = 0$.

The optimal diversity-multiplexing tradeoff of AF strategies can be reached by setting $N' = N/2$ and $A_1 = A_2 = B = I_{N/2}$, known as the Non-orthogonal Amplify Forward (NAF) protocol [1]. Due to the average energy constraint, the authors assert that $A_1 = A_2 = I$ in order to maximize the mutual information between x and y. The matrix B must be diagonal to prevent ISI (inter-symbol interference) and is of size $N/2$, as this is the maximum number of symbols the system can repeat.

This scheme is similar to the initial scheme except that the source transmits a new codeword while the relay is retransmitting the initial codeword. Through considerable algebraic manipulations, the outage event of $(I < R)$ yields the following diversity order

$$d^*(r) = (1-r) + (1-2r)^+ \tag{18}$$

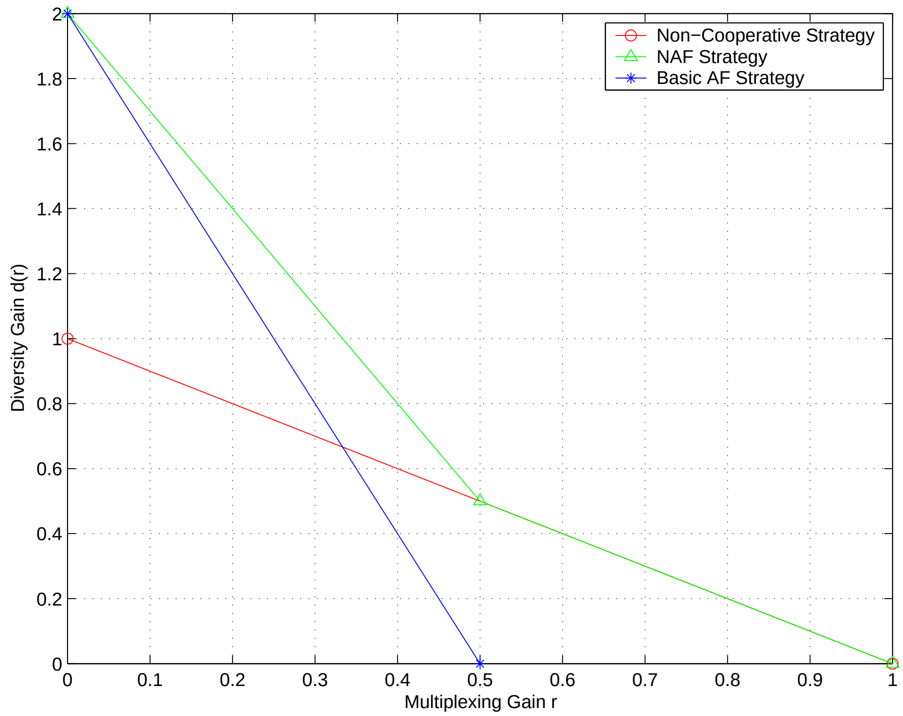

Figure 3: Diversity-Multiplexing curve for one-relay AF protocols

where x^+ is defined as $\max(x, 0)$. We will overview a similar proof in greater detail for the Decode-and-Forward strategy in section 4.1.2 since the algebra is less cumbersome in that case. As explained above, this NAF strategy achieves the optimal diversity-multiplexing tradeoff for schemes with a single AF relay. The comparison of AF protocols is given in figure 3.
Comments:

- The NAF method adds more complexity to the system since it is necessary to separate transmissions of the source and relay.
- For multiplexing gains greater than 0.5, the NAF is equivalent to the non-cooperative method. Thus, the relay provides no benefit in this regime.

4.1.2 Decode-and-Forward

In the Decode-and-Forward (DF) transmission scheme, the relay decodes the message from the source, then re-encodes and transmits it. Again, let us first examine a basic transmission scheme where the source transmits in the first slot and the relay transmits in the second slot.

Thus, for $n = 1, ..., N/2$,

$$y_d[n] = a_{s,d}x_s[n] + z_d[n], \tag{19}$$
$$y_r[n] = a_{s,r}x_s[n] + z_r[n]. \tag{20}$$

The relay performs full codeword decoding of $y_r[n]$ to produce an estimate $\hat{x}_s[n]$ of the source transmission. In the second time slot, $n = N/2 + 1, ..., N$, the destination receives

$$y_d[n] = a_{r,d}\hat{x}_s[n - N/2] + z_d[n]. \tag{21}$$

Using this scheme, we require the relay and destination to decode the codeword without error. Note, the assumption that relay must decode decreases the performance of this system greatly. The mutual information is given by

$$I_{DF} = \frac{1}{2}\min\left(\log(1 + \rho|a_{s,r}|^2), \log(1 + \rho|a_{s,d}|^2 + \rho|a_{r,d}|^2)\right) \tag{22}$$

An outage (either source or destination can't decode) occurs for a transmission rate R if

$$\min\left(|a_{s,r}|^2, |a_{s,d}|^2 + |a_{r,d}|^2\right) < \frac{2^{2R} - 1}{\rho}. \tag{23}$$

This occurs with probability

$$p_{DF}^{out}(\rho, R) \quad \sim \quad \frac{1}{\rho^{(1-2r)}}. \tag{24}$$

Thus, the diversity multiplexing tradeoff is

$$d(r) = 1 - 2r \tag{25}$$

Similar to the initial example of AF, this scheme is easy to implement however, it does not reach the information theoretic limit for our setup. We now examine a more complex method, known as Dynamic Decode and Forward (DDF). In this scheme, the source transmits a codeword through time $n = 1, ..., N$. The relay subsequently listens to the source until it collects sufficient energy to decode the message. It then begins retransmitting to the destination using an independent Gaussian codebook. Thus, if the channel between the source and receiver is good, the relay can decode quickly and begin transmitting.

The number of symbols that the relay must listen for N' is equal to

$$N' = \min\left[N, \lceil N\frac{R}{\log_2(1 + |a_{s,r}|^2\rho)}\rceil\right] \tag{26}$$

where R is the data rate of the source,

It can be shown that the diversity-multiplexing tradeoff achieved via this DDF scheme is:

$$d(r) = \begin{cases} 2(1-r) & \text{for } \frac{1}{2} \geq r \geq 0, \\ (1-r)/r & \text{for } 1 \geq r \geq \frac{1}{2} \end{cases} \tag{27}$$

Outline of Proof:

- For a Gaussian random variable g, we define the exponential order of $1/|g|^2$ as v, where

$$v = \lim_{\rho \to \infty} \frac{\log(|g|^2)}{\log \rho}. \tag{28}$$

This implies that in the high-SNR regime, $|g|^2 = \rho^{-v}$. The resulting pdf for v is computed to be ρ^{-v} for $v \geq 0$ and 0 for $v < 0$. Let us define v_1, v_2, and u as the exponential orders of $a_{s,d}$, $a_{r,d}$, and $a_{s,r}$ respectively. The exponential random variables (v_1, v_2, u) allow us to simplify the algebra.

- An outage occurs if the mutual information is less than the chosen rate

$$\begin{aligned}
Pr(I < R) &= Pr(N' \log(1 + |a_{s,d}|^2 \rho) + (N - N') \log(1 + (|a_{s,d}|^2 + |a_{r,d}|^2)\rho) < Nr \log(\rho)) && (29)\\
&= Pr(N' \log(1 + \rho^{-v_1} \rho) + (N - N') \log(1 + (\rho^{-v_1} + \rho^{-v_2})\rho) < Nr \log(\rho)) && (30)\\
&\approx Pr(N'(1 - v_1) \log(\rho) + (N - N')(1 - \min(v_1, v_2)) \log(\rho) < Nr \log(\rho)) && (31)\\
&= Pr((N'/N)(1 - v_1) + ((N - N')/N)(1 - \min(v_1, v_2)) < r) && (32)
\end{aligned}$$

- Thus the outage region O^+ is

$$O^+ = \{(v_1, v_2, u) \mid (N'/N)(1 - v_1) + ((N - N')/N)(1 - \min(v_1, v_2)) < r\}. \tag{33}$$

The probability of outage is

$$\begin{aligned}
p_{out} &= \int_{(v_1, v_2) \in O^+} \rho^{-(v_1 + v_2)} dv_1 dv_2 && (34)\\
&\approx \rho^{-d} && (35)
\end{aligned}$$

where

$$d = \inf_{(v_1, v_2) \in O^+} (v_1 + v_2). \tag{36}$$

Thus, each value of r defines a new outage region, and we can compute the minimum value of $(v_1 + v_2)$ in this region to get our diversity gain. Note, the variable u is incorporated in the N' term. The resulting diversity-multiplexing curve can be computed through simple algebra. The comparison between the one-relay protocols is given in figure 4.

Comments:

- It is unclear what the optimal diversity-multiplexing tradeoff is for a system with a single DF relay. As we see here, for multiplexing gains r less than 0.5, the diversity gain is equal to that of the upper bound (2×1 MIMO system); thus it is optimal. For $r > 0.5$, the relay is only able to help for a small portion of the codeword; it is an open problem whether a protocol exists which can get closer to the upper bound.

- Although this system provides an improvement over the initial decode-and-forward protocol, it is extremely difficult to implement. It requires a high level of synchronization between terminals since the destination does not know exactly when the relay will begin transmitting.

- A mixed AF/DF strategy cannot improve performance. Any event outage event for the DDF strategy will remain in outage through mixed strategies.

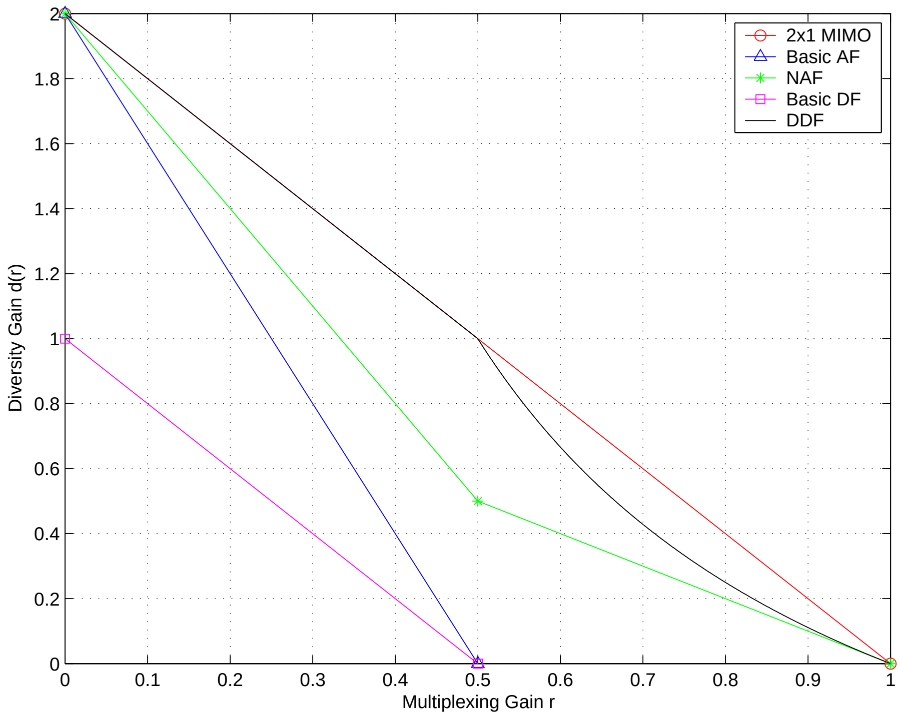

Figure 4: Diversity-Multiplexing curve for single relay AF and DF protocols

4.2 Multiple Terminals

In this section, we present cooperative-diversity protocols for systems with multiple terminals. First, we give a generalization of the AF and DF protocols for systems with $M-1$ relays. We then consider the broadcast channel, where a single source is transmitting to multiple destinations, and the multiple-access channel, where all terminals have an independent message to transmit to a single destination.

4.2.1 AF Revisited

The NAF protocol can be generalized to exploit the diversity of $M-1$ relays. In these multiple relay systems, each relay takes turns helping the source node. During the first time slots $(n = 1, ..., N)$, the first relay will help the source in a manner identical to the single-relay NAF scheme. Thus, the relay amplifies and transmits the first $N/2$ symbols it receives during the second $N/2$ time slots. During the second pair of time slots $(n = N+1, ..., 2N)$, the second relay supports the source. Within $(M-1)N$ symbols, each relay will have been active and no two relays operate at the same time. The subsequent diversity multiplexing tradeoff of such a scheme is

$$d(r) = (1-r) + (N-1)(1-2r)^{+} \tag{37}$$

- Multiple relays do not operate simultaneously. Due to power constraints on the relays, nothing can be gained by having multiple symbols repeat the same symbol.

- No analysis is made to determine whether this protocol provides the optimal diversity-multiplexing tradeoff for $M-1$ AF relays.

4.2.2 DF Revisited

We now describe a generalization of the DDF scheme provided in section 4.1.2. In the $M-1$ relay system, all relays actively listen to each codeword that the source transmits. As soon as one relay can properly decode, it begins transmitting the codeword using an independent codebook (as in the DDF scheme). Thus, the relay with the least amount of path fading will be the first to begin transmitting to the destination. Other relays jointly decode using the source and transmitting relay messages. In order to solve complications of having relays know when another relay has begun transmitting, a beacon signal can be sent out by the transmitting relay which tells all others when they have begun. This protocol achieves a diversity multiplexing tradeoff of

$$d(r) = \begin{cases} M(1-r) & \text{for } \frac{1}{M} \geq r \geq 0, \\ 1 + \frac{(M-1)(1-2r)}{1-r} & \text{for } \frac{1}{2} \geq r \geq \frac{1}{M} \\ \frac{1-r}{r} & \text{for } 1 \geq r \geq \frac{1}{2} \end{cases} . \tag{38}$$

Note that this tradeoff is not known to be optimal for $r > 1/M$.

4.3 Cooperative Broadcast Channel

In the cooperative broadcast scenario, a single source broadcasts information to M different destinations. Let us assume that the source uses rate R_c to transmit a common signal that is needed by all destinations and uses rate R_i for messages specific to destination i, where $i = 1, ..., M$, yielding a total rate $R = R_c + \sum_i R_i$. For each rate in $(R_c, R_1, ..., R_M)$, we can define multiplexing gains $(r_c, r_1, ..., r_M)$ and an overall diversity gain $d = \min_i d_i$. For a fixed total rate, we focus on the worse case scenario where all users want to decode the full rate $R = R_c$. A strategy similar to DDF can be used, where the destination which decodes first begins transmitting to all other destinations. Destinations jointly decode the messages of the source and other destinations in order to decode and then begin transmitting. In this case, although we have M destinations, the tradeoff obtained is the same as the relay channel with M destinations.

4.4 Multiple-Access Channel

The cooperative multiple-access (CMA) scenario contains M sources, each wanting to transmit an independent message to the destination. In the CMA-NAF protocol, sources alternate turns to transmit. In each interval, a single source transmits a linear combination of its own signal and the signal it received from the source before it. Thus, sources work cooperatively to help transmit each others information. By cycling through source orders, each source is helped equally by all other sources. This protocol achieves the optimal diversity-multiplexing tradeoff $d^*(r) = M(1-r)$.

5 Summary of Results

In this section, we will summarize the results of the single relay protocols.

Protocol	D-M Tradeoff d(r)
Non-Cooperative	$1 - r$
Basic AF	$2(1 - 2r)$
NAF	$(1 - r) + (1 - 2r)^+$
Basic DF	$1 - 2r$
DDF	$\begin{cases} 2(1-r) & \text{for } \frac{1}{2} \geq r \geq 0, \\ (1-r)/r & \text{for } 1 \geq r \geq \frac{1}{2} \end{cases}$
2×1 MIMO	$2(1 - r)$

Table 2: Diversity-Multiplexing tradeoff for one relay transmission schemes

Although the DDF strategy offers the best performance result, it is also important to consider the complexity of the schemes. The basic AF and DF strategies only require the decoding of a single message at the receiving terminals; they are easy strategies to implement. However, we see that the DF strategy does strictly worse than the non-cooperative case. The NAF strategy requires the destination to jointly decode two codewords, increasing the complexity of the system greatly. Even more complex is the DDF strategy which requires the system to detect when the relay is transmitting, determine which codebook the relay is using, and jointly decode the messages.

In this report, we have presented both basic and complex cooperative-diversity protocols. In many cases, these protocols can help exploit the spatial diversity of having a relay in the network. However, in designing a system, one must understand not only the tradeoff between diversity and multiplexing, but also between performance and complexity.

References

[1] K. Azarian, H. El Gamal, and P. Schniter. On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels. *submitted to IEEE Trans. on Information Theory*, 2004.

[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell. Cooperative diversity in wireless networks: Efficient protocols and outage behavior. *to appear IEEE Trans. on Information Theory*, 2003.

[3] L. Zheng and D. N. C. Tse. Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels. *IEEE Trans. on Information Theory*, 49:1073–1096, May 2003.