Cooperative Diversity for Wireless Networks

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Introduction

- Fading is a major limitation in wireless systems
- Different forms of diversity used to overcome fading
  - Spatial Diversity
  - Temporal Diversity
  - Frequency Diversity
- Focus on specific form of spatial diversity named cooperative diversity
  - Other terminals in network act as relays between source and dest.
  - Relays experience independent fading and act as a virtual antenna array

Background

- Scalar Fading Channel
  - Slow/Fast Fading
  - Transmitter CSI
- MIMO Communication System
  - Diversity Gain
  - Multiplexing Gain
- Connection to Cooperative-Diversity

Scalar Fading Channel

Input signal experiences Rayleigh fading h and additive Gaussian noise w

$$y = hx + w$$

where $x \sim CN(0, \sigma_x^2)$, $w \sim CN(0, \sigma_w^2)$

SNR:

$$\rho \stackrel{\triangle}{=} SNR = \sigma_x^2 / \sigma_w^2$$

Two major questions:

- How often does the realization of h change?
- Does the transmitter have knowledge of h?

Transmission Cases

Fading:

- Slow Fading - Channel does not change within time period of interest
- Fast Fading - Channel changes on the order of symbols/codewords

Channel State Information (CSI):

- Tx CSI - Transmitter has knowledge of h
- No Tx CSI - Transmitter does not know h, only its distribution

	Slow Fading	**Fast Fading**
Tx CSI	Deterministic Capacity $C = \log(1 + \|h\|^2\rho)$	Time Water-filling
No Tx CSI	Outage Capacity R, p^{out}	Ergodic Capacity $C = E[\log(1 + \|h\|^2\rho)]$

Slow Fading/No Tx CSI

- Most difficult case
- Any non-zero rate chosen by the transmitter may be above the supported rate of the channel
- Channel is not changing with time (non-ergodic), the transmission may always fail with some non-zero probability
- Shannon capacity is zero, i.e., system cannot guarantee that any amount of information can be transmitted reliably.
- Outage capacity: given R and SNR, there is a non-zero probability that the channel does not support this rate; this probability is known as the outage probability (p^{out}).
- Can use relays to improve transmission, act like a virtual MIMO system

Multiple Antenna Systems

- The discrete-time MIMO channel model for system with m transmit and n receive antennas is

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}.$$

- $\mathbf{H}$ is an $n \times m$ matrix of channel gains whose entries are i.i.d., $CN(0,1)$

- $\mathbf{H}$ drawn from such a distribution causes
  - Outages in the system
  - Increases degrees of freedom for communications

Diversity Gain

- High-SNR regime: the diversity gain d measures the rate at which the error probability decays with SNR ($1/\rho^d$)

$$d = -\lim_{\rho \to \infty} \frac{\log P_e(\rho)}{\log \rho}$$

- For a system with m transmit and n receive antennas, in the best case, the error can be made to decay with SNR as $1/\rho^{mn}$

- Diversity is a result of the mn independent paths between the transmit and receive antennas.

Multiplexing Gain

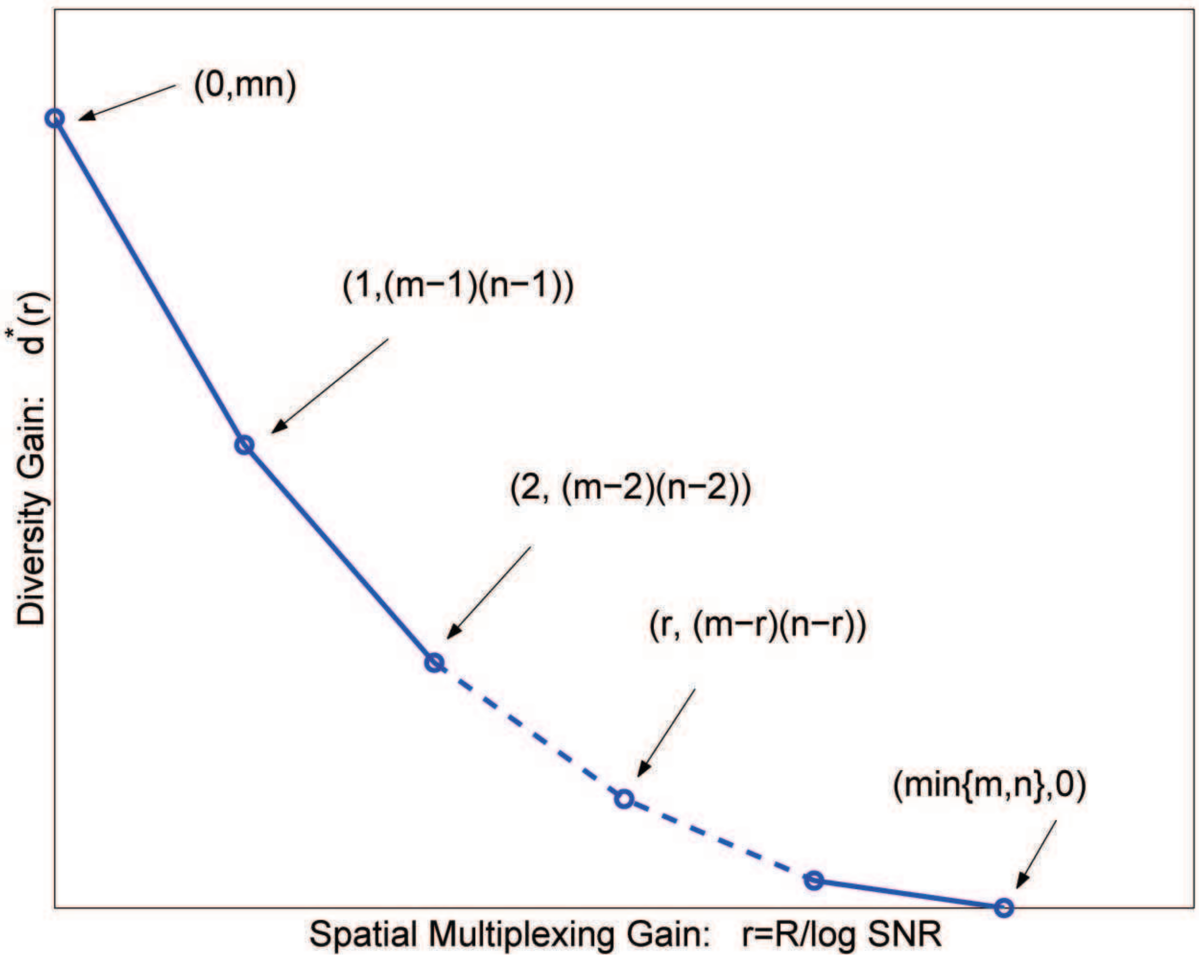
- Because the path gains are independent, the channel matrix is well-conditioned with high probability
- Can view the MIMO system as $\min(m,n)$ independent spatial channels between the transmitter and receiver.
- Fast fading ergodic capacity: $C(\rho) = \min(m,n)\log\rho + O(1)$
- For slow fading, spatial multiplexing gain r measures the rate at which R increases with respect $\log\rho$ and is formally defined as:

$$r = \lim_{\rho\to\infty} \frac{R(\rho)}{\log\rho}, \qquad R \doteq r\log\rho$$

- Multiplexing gain can be thought of as the number of independent spatial channels being used optimally, which is upper-bounded by $\min(m,n)$.

Diversity-Multiplexing Tradeoff

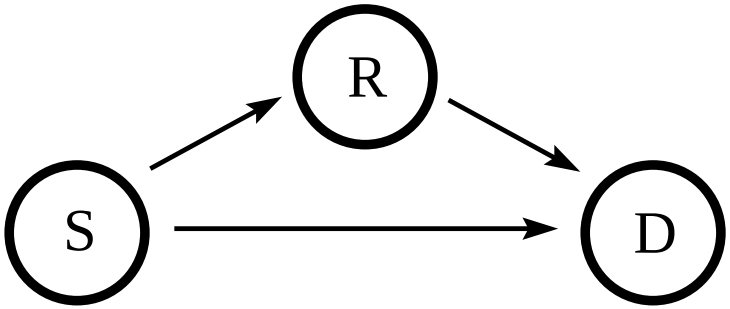
- Natural to talk about a tradeoff between the r and d



Connection to Cooperative-Diversity

- Scalar fading channel provides a lower bound for the performance of any valid cooperative-diversity scheme
  - Can always ignore use of the relay (non-cooperative protocol) and achieve scalar channel performance
- 2×1 MIMO system provides an upper bound
  - In MIMO system, the transmitter has full control over both antennas and can code across both of them, allowing more flexibility.
  - Often referred to as genie-aided protocol

System Model



- $x_s[n]$ and $x_r[n]$ are the transmitted signals from the source and relay
- $y_r[n]$ and $y_d[n]$ are the received signals of the relay and destination
- Direct source to destination transmission modeled as:

$$y_d[n] = a_{s,d} x_s[n] + z_d[n]$$

- Fading $a_{i,j} \sim CN(0, \sigma^2_{i,j})$ and independent
- Noise $z_j[n] \sim CN(0, N_0)$, mutually independent

Model Assumptions

- Half duplex: antennas cannot transmit and receive simultaneously
- Channels experience independent flat Rayleigh-fading and white complex Gaussian noise.
- Slow fading environment
- Transmitters have no CSI. Receivers have full CSI.
- Each terminal has one antenna with power constraint, P.

Case Analysis

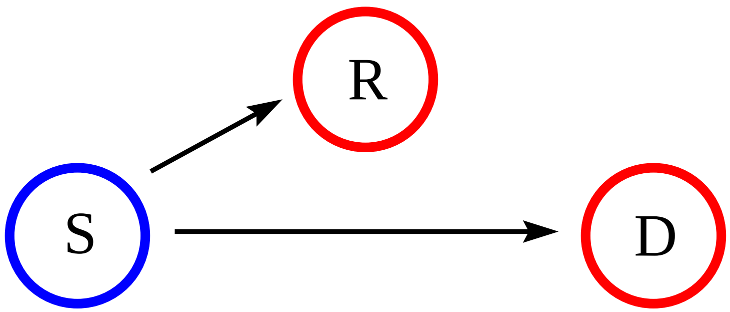
1. Single Relay Systems
   - Amplify-and-Forward
   - Decode-and-Forward
2. Multiple Terminal System
   - AF Revisited
   - DF Revisited
   - Cooperative Broadcast Channel
   - Cooperative Multiple-Access Channel

Single Relay Systems

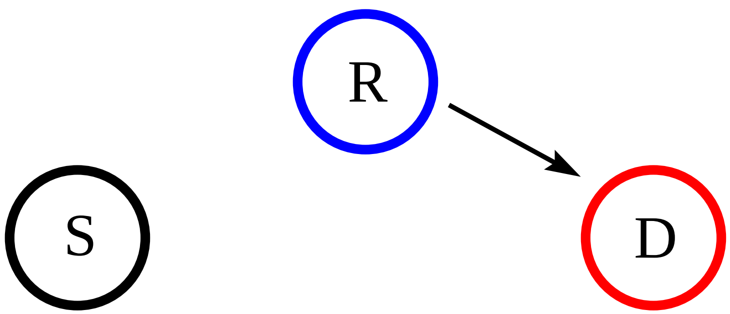
- Three terminals: source, relay, destination
- Relay can either amplify or decode, re-encode, and transmit
- Work by Laneman-Wornell-Tse
  - Additional constraint that only one terminal is transmitting at a time
- Work by Azarian-El Gamal-Schniter
  - Allow multiple terminals to transmit simultaneously
  - Protocols optimal in certain regimes.

Basic Amplify-and-Forward

- Relay can only transmit amplified version of received signal



	$n = 1, \ldots, N/2$
R	$a_{s,r} x_s[n] + z_r[n]$
D	$a_{s,d} x_s[n] + z_d[n]$



	$n = N/2+1, \ldots, N$
R	
D	$a_{r,d}(\beta y_r[n - N/2]) + z_d[n]$

- Relay power constraint: $\beta \leq \sqrt{\frac{P}{|a_{s,r}|^2 P + N_o}}$

Basic AF Performance

- Mutual information between the input and 2 outputs as

$$I_{AF} = \frac{1}{2}\log(1 + \rho|a_{s,d}|^2 + f(\rho|a_{s,r}|^2, \rho|a_{r,d}|^2))$$

  where

$$f(x,y) = \frac{xy}{x+y+1}.$$

- For an outage event (small fading coefficients), $f(x,y)$ can be approximated as $\min(x,y) \stackrel{\triangle}{=} \gamma$.

Basic AF Performance cont.

- Thus, our outage event becomes (I¡R):

$$
\begin{aligned}
\frac{1}{2}\log(\rho|a_{s,d}|^2+\rho\gamma) &< R \\
\log(\rho)+\log(|a_{s,d}|^2+\gamma) &< 2r\log\rho \\
\log(|a_{s,d}|^2+\gamma) &< (2r-1)\log\rho \\
|a_{s,d}|^2+\gamma &< \rho^{(2r-1)}
\end{aligned}
$$

- Since γ and $|a_{s,d}|^2$ are both exponential, the probability of outage becomes $(\rho^{2r-1})^2$

- Diversity-multiplexing tradeoff of $d(r) = 2(1-2r)$

Non-Orthogonal Amplify-and-Forward

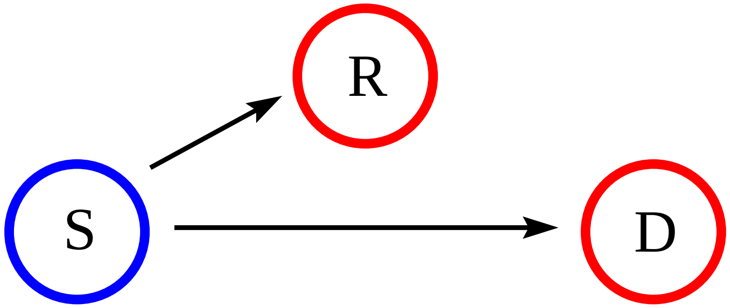
AF general form:

$$\mathbf{y} = \begin{bmatrix} a_{s,d}A_1 & 0 \\ a_{s,r}a_{r,d}\beta B & a_{s,d}A_2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ a_{r,d}B \end{bmatrix} \mathbf{z_r} + \mathbf{z_d}$$

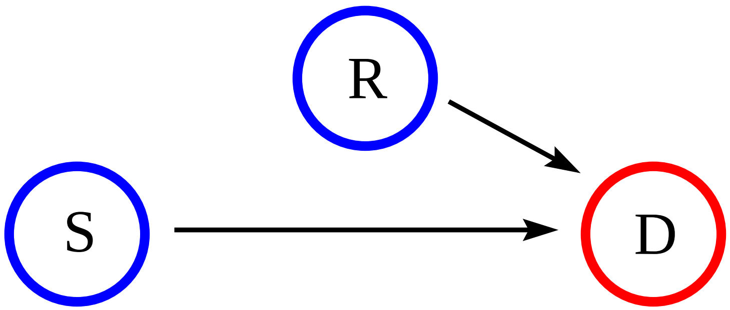
where $\mathbf{y}, \mathbf{x}, \mathbf{z_r}, \mathbf{z_d} \in \mathbb{C}^{\mathbf{N}}$, $\mathbf{A_1} \in \mathbb{C}^{\mathbf{N'},\mathbf{N'}}$, $\mathbf{B} \in \mathbb{C}^{\mathbf{N}-\mathbf{N'},\mathbf{N'}}$, $\mathbf{A_2} \in \mathbb{C}^{\mathbf{N}-\mathbf{N'},\mathbf{N}-\mathbf{N'}}$.

- Initial protocol: $N' = N/2$, $A_1 = B = I_{N/2}$, and $A_2 = 0$.
- NAF protocol: $N' = N/2$ and $A_1 = A_2 = B = I_{N/2}$
  - $A_1 = A_2 = I_{N/2}$ maximizes mutual information between x and y
  - B diagonal to prevent ISI, size $N/2$ for max symbol repetition

NAF cont.



	$n=1,\ldots,N/2$
R	$a_{s,r}x_s[n]+z_r[n]$
D	$a_{s,d}x_s[n]+z_d[n]$

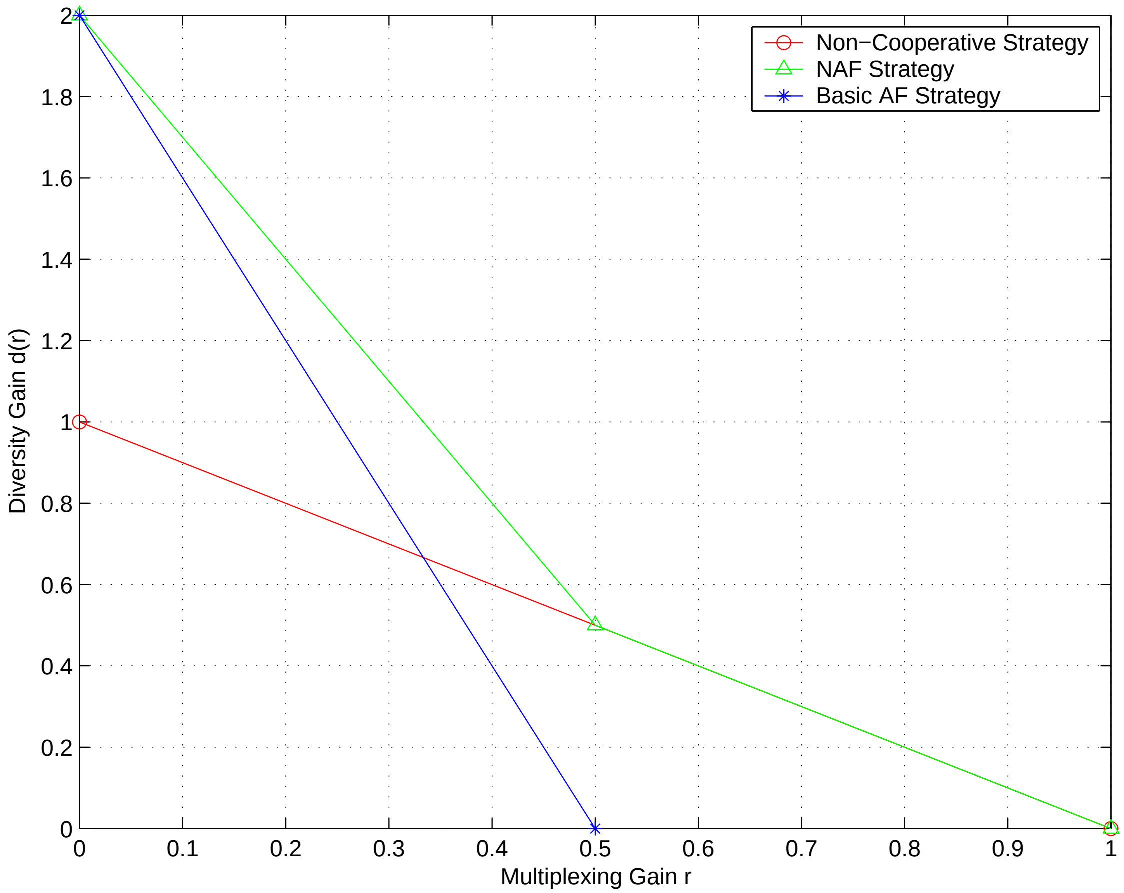


	$n=N/2+1,\ldots,N$
R	
D	$(a_{r,d}(\beta y_r[n-N/2]) + a_{s,d}x[n]+z_d[n])$

- Achieves optimal diversity-multiplexing curve for single AF systems

$$d(r)=(1-r)+(1-2r)^+$$

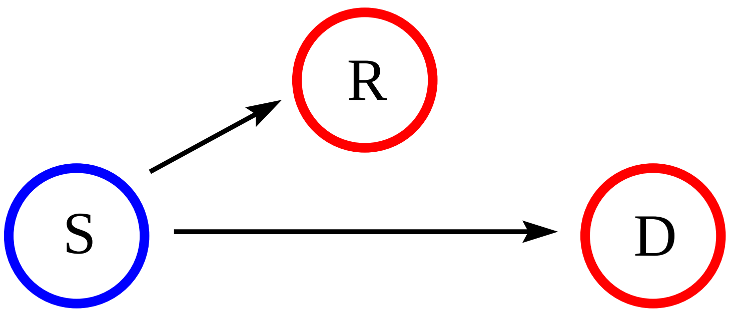
NAF cont.



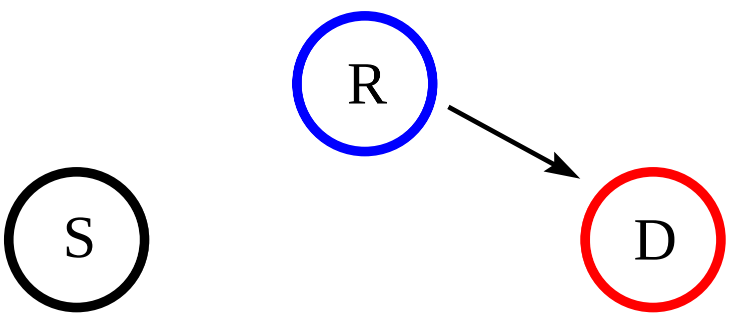
- For $r > 0.5$, the NAF is equivalent to the non-cooperative method. Relay provides no benefit in this regime.

Decode-and-Forward

- Relay decodes message from source, re-encodes and transmits it



	$n = 1, \ldots, N/2$
R	$a_{s,r} x_s[n] + z_r[n]$
D	$a_{s,d} x_s[n] + z_d[n]$



	$n = N/2+1, \ldots, N$
R	
D	$a_{r,d} \hat{x}_s[n - N/2] + z_d[n]$

- Relay required to decode correctly

Decode-and-Forward cont.

Mutual information is given by

$$I_{DF} = \frac{1}{2}\min\left(\log(1+\rho|a_{s,r}|^2), \log(1+\rho|a_{s,d}|^2+\rho|a_{r,d}|^2)\right)$$

An outage (either relay or destination cannot decode) occurs for a transmission rate R if

$$\min\left(|a_{s,r}|^2, |a_{s,d}|^2+|a_{r,d}|^2\right) < \frac{2^{2R}-1}{\rho}.$$

This occurs with probability

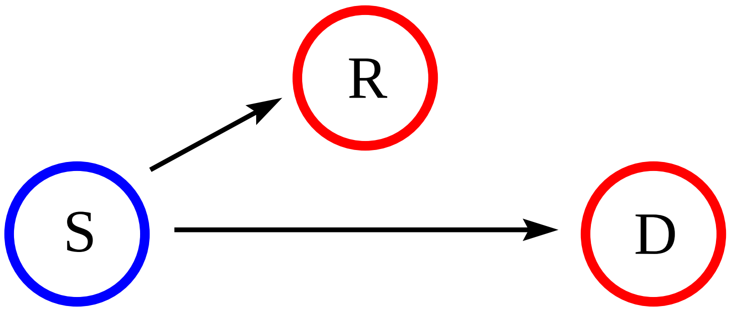
$$p_{DF}^{out}(\rho, R) \quad \sim \quad \frac{1}{\rho^{(1-2r)}}.$$

Thus, the diversity multiplexing tradeoff is

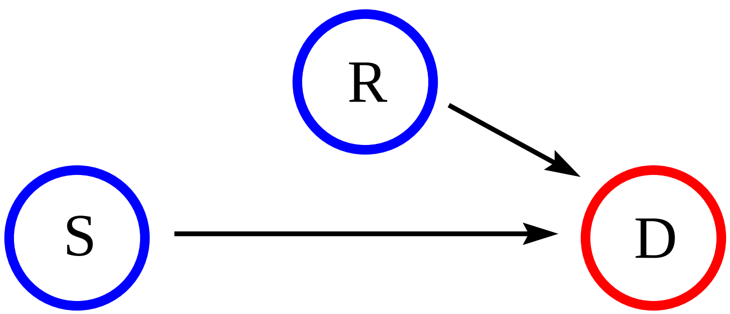
$$d(r) = 1-2r$$

Dynamic Decode and Forward

- Relay listens to the source until it collects sufficient energy to decode, then begins retransmitting to the destination using an independent codebook



	$n = 1, \ldots, N'$
R	$a_{s,r} x_s[n] + z_r[n]$
D	$a_{s,d} x_s[n] + z_d[n]$



	$n = N' + 1, \ldots, N$
R	
D	$a_{s,d} x_s[n] + a_{r,d} \hat{x}_r[n] + z_d[n]$

$$N' = \min \left[N, \left\lceil N \frac{R}{\log_2(1 + |a_{s,r}|^2 \rho)} \right\rceil \right]$$

DDF Analysis

It can be shown that the diversity-multiplexing tradeoff achieved via this DDF scheme is:

$$d(r) = \begin{cases} 2(1-r) & \text{for } \frac{1}{2} \geq r \geq 0, \\ (1-r)/r & \text{for } 1 \geq r \geq \frac{1}{2} \end{cases}$$

Outline of Proof:

- For a Gaussian random variable g, we define the exponential order of $1/|g|^2$ as v, where

$$v = \lim_{\rho \to \infty} \frac{\log(|g|^2)}{\log \rho}.$$

This implies that in the high-SNR regime, $|g|^2 \doteq \rho^{-v}$. The resulting pdf for v is equal to ρ^{-v} for $v \geq 0$ and 0 for $v < 0$. Let us define v_1, v_2, and u as the exponential orders of $a_{s,d}$, $a_{r,d}$, and $a_{s,r}$ respectively.

DDF Analysis cont.

- An outage occurs if the mutual information is less than the chosen rate $(I < R)$:

$$
\begin{aligned}
& Pr(N' \log(1+|a_{s,d}|^2\rho) + (N-N')\log(1+(|a_{s,d}|^2+|a_{r,d}|^2)\rho) < Nr\log(\rho)) \\
= \; & Pr(N' \log(1+\rho^{-v_1}\rho) + (N-N')\log(1+(\rho^{-v_1}+\rho^{-v_2})\rho) < Nr\log(\rho)) \\
\approx \; & Pr(N'(1-v_1)\log(\rho) + (N-N')(1-\min(v_1,v_2))\log(\rho) < Nr\log(\rho)) \\
= \; & Pr((N'/N)(1-v_1) + ((N-N')/N)(1-\min(v_1,v_2)) < r)
\end{aligned}
$$

- Outage region O^+:

$$O^+ = \{(v_1,v_2,u) \mid (N'/N)(1-v_1) + ((N-N')/N)(1-\min(v_1,v_2)) < r\}.$$

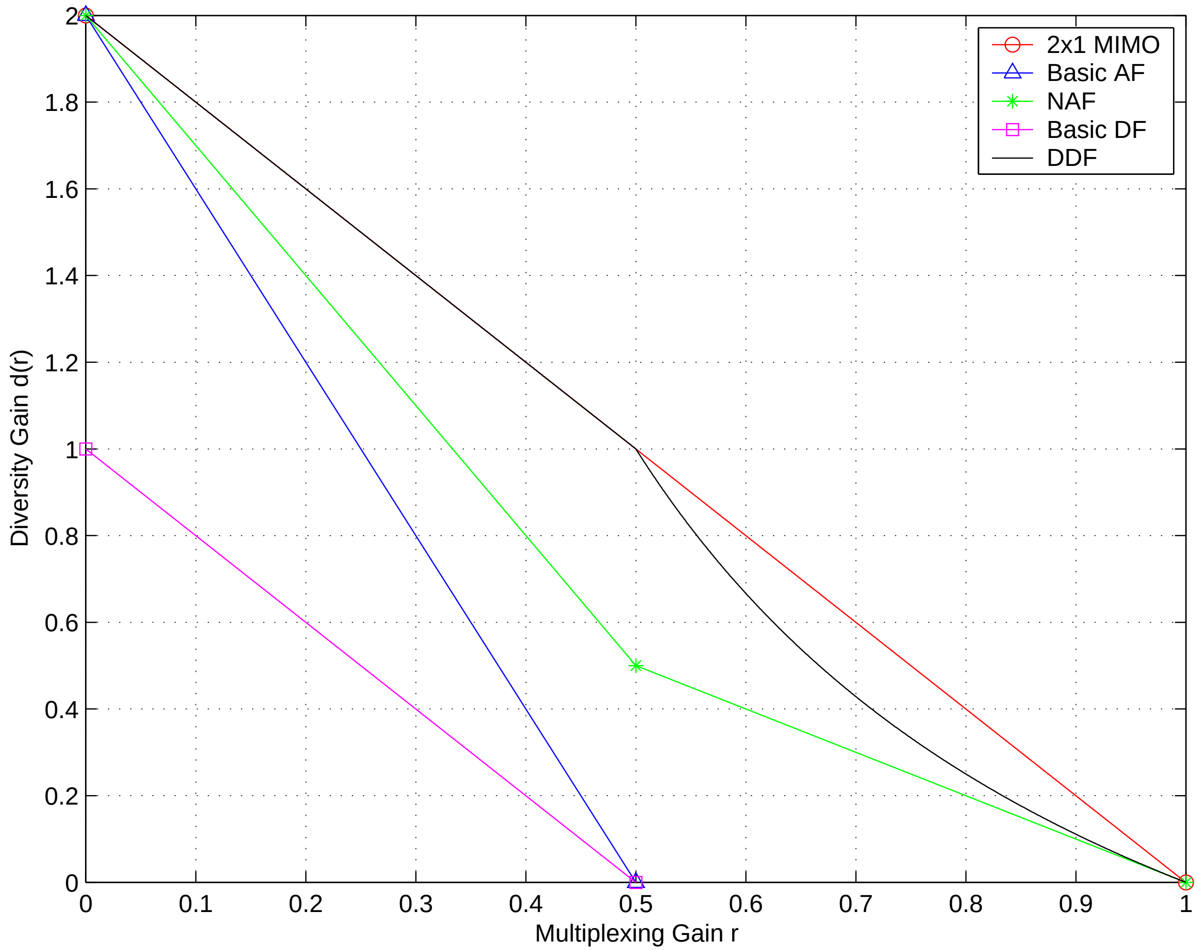
Probability of outage:

$$
\begin{aligned}
p_{out} &= \int_{(v_1,v_2)\in O^+} \rho^{-(v_1+v_2)} dv_1 dv_2 \\
&\approx \rho^{-d}
\end{aligned}
$$

where

$$d = \inf_{(v_1,v_2)\in O^+} (v_1+v_2).$$

DDF Analysis cont.



DDF Remarks

Comments:

- Optimal diversity-multiplexing tradeoff for single DF relay is unknown for $r > 0.5$
- Extremely difficult to implement, requires a high level of synchronization and coordination among terminals
- Mixed AF/DF strategy cannot improve performance. Any event outage event for the DDF strategy will remain in outage through mixed strategies.

Multiple Terminals

- AF Revisited
- DF Revisited
- Cooperative Broadcast Channel
- Cooperative Multiple-Access Channel

AF Revisited

- NAF protocol generalized to $M-1$ relays
  - Relay takes turns helping the source node (first relay is active during first time slots, second relay is active during second time slots, ...)
  - Due to power constraints on the relays, nothing can be gained by having multiple relays repeat the same symbol
- Diversity-multiplexing tradeoff

$$d(r) = (1-r) + (M-1)(1-2r)^+$$

DF Revisited

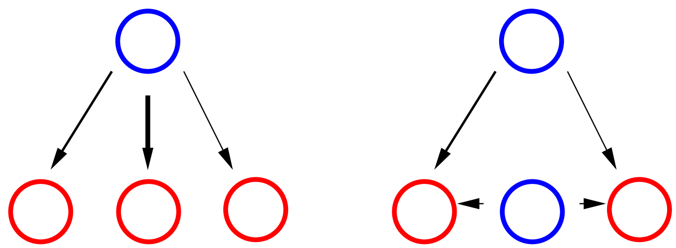
- DDF protocol generalized to $M-1$ relay system
- When a relay can properly decode, it begins transmitting codeword using an independent codebook (DDF)
- Relays use beacon signal used to inform other relays of transmission starting time
- Diversity Multiplexing tradeoff:

$$d(r) = \begin{cases} M(1-r) & \text{for } \frac{1}{M} \geq r \geq 0, \\ 1 + \frac{(M-1)(1-2r)}{1-r} & \text{for } \frac{1}{2} \geq r \geq \frac{1}{M} \\ \frac{1-r}{r} & \text{for } 1 \geq r \geq \frac{1}{2} \end{cases}.$$

Note that this tradeoff is not known to be optimal for $r > 1/M$.

Cooperative Broadcast Channel

- Single source broadcasts information to M different destinations
- Worse case scenario: all users want to decode the full rate $R = R_c$



- Can use protocol similar to DDF
  - Source transmits codeword
  - After a destination decodes, it begins transmitting to the rest
  - D-M tradeoff same as relay channel with M relays

Multiple-Access Channel

- M sources each with independent message to the destination
- CMA-NAF protocol:
  - Sources alternate turns to transmit
  - In each interval, a single source transmits a linear combination of its own signal and the signal it received from the source before it
  - Reaches optimal diversity-multiplexing tradeoff for CMA systems:

$$d(r) = M(1 - r)$$

Summary of Results

Protocol	**D-M Tradeoff d(r)**
Non-Cooperative	$1-r$
Basic AF	$2(1-2r)$
NAF	$(1-r)+(1-2r)^+$
Basic DF	$1-2r$
DDF	$\begin{cases} 2(1-r) & \text{for } \frac{1}{2} \geq r \geq 0, \\ (1-r)/r & \text{for } 1 \geq r \geq \frac{1}{2} \end{cases}$
2×1 MIMO	$2(1-r)$

- Diversity-Multiplexing Tradeoff
- Performance-Complexity Tradeoff