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Seminar on Auditory Signal Processing and Perception

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Notes on Auditory Sensitivity.

Intensity Discrimination *

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3 Discrimination of Sound Differences

Two sounds that can be distinguished with a specified degree of reliability are said to be *discriminable.* If the sounds differ only in the value of a single physical parameter (e.g., intensity, frequency, duration) the difference in the values of the parameter across the two sounds is often called the *Just Noticeable Difference* or *JND* (sometimes the *Difference Limen* or *DL*) in the parameter. In current parlance, listener's are said to have a finite *resolution* for the parameter.

3.1 Intensity

The ability of listeners to discriminate differences in sound pressure, or equivalently sound intenstiy, has been studied systematically for at least 80 years. Studies of intensity discrination have ututilized different psychoacoutstic techniques. The "best method" is still simewhat elusive.

Consider two stimuli that differ only in rms pressure:

$$\begin{aligned} p_1(t) &= Pu(t) \\ p_2(t) &= (P + \Delta P)\, u(t) \end{aligned}$$

where $u(t)$ has unit rms pressure and $\Delta P > 0$. The equivalent intensities are proportional to the square of pressures:

$$\begin{aligned} I_1 &\propto P^2 \\ I_2 = I_1 + \Delta I &\propto (P + \Delta P)^2 . \end{aligned}$$

In the case where ΔP is small relative to P

$$\Delta I \propto 2P\Delta P.$$

Measurements of intensity discrimination are typically reported as

$$\frac{\Delta I}{I} = \frac{(P + \Delta P)^2 - P^2}{P^2} \approx 2\frac{\Delta P}{P},$$

or as

$$\frac{I + \Delta I}{I} = 1 + \frac{\Delta I}{I}$$

3.2 Riesz and the Method of Just-Noticeable Beats

The first systematic attempts to measure discrimination for sound pressure were made by R.R. Riesz (1928). Possibly for reasons of simplicity of generating the pressures, Riesz produced pressure variations by adding two sinusoidal voltages and then applying the sum to an earphone. Riesz's stimulus can thus be described as

$$p(t) = P_1 \cos(2\pi f_1 t) + P_2 \cos(2\pi f_2 t)$$

which Riesz expressed (in the case $P_1 \gg P_2$) as

$$p(t) = m(t) \cos(2\pi f_1 t + \Phi),$$

where

$$m^2(t) = P_1^2 + P_2^2 + 2P_1P_2 \cos(2\pi (f_2 - f_1) t)$$

In Riesz's experiments $\Delta f = f_2 - f_1$ was small compared to f_1 or f_2 so that the listener heard a tone-like stimulus having a frequency of very nearly f_1 whose amplitude fluctuates periodically between a maximum of $P_1 + P_2$ and a minimum of $P_1 - P_2$ at a rate of $\Delta f = f_2 - f_1$ beats per second.

Riesz had his subjects determine the minimum value of P_2 that caused the beats to be audible and repored his results in terms of the difference between the tone intensities[2] corresponding to the maximum and minimum tone pressure amplitudes:

$$\frac{\Delta I}{I} = \frac{(P_1 + P_2)^2 - (P_1 - P_2)^2}{(P_1 - P_2)^2} = 4\frac{P_1P_2}{(P_1 - P_2)^2}$$

which, in the case $P_2 \ll P_1$ is approximately

$$\frac{\Delta I}{I} \approx 4\frac{P_2}{P_1}$$

Riesz reported his results[3] in terms of

$$\Delta\alpha = 10 \log_{10} 1 + \frac{\Delta I}{I}.$$

As shown in Fig 6, Riesz found that for 1000 Hz tones, the minimum value of $\Delta\alpha$ corresponding to audible beats depended on Δf as well as on P_1 expressed in senation level (i.e. in dB relative to the threshold SPL for 1000 Hz tones). Clearly, while $\Delta\alpha$ depends on P_1, there is a minimum value of $\Delta\alpha \approx 1$ dB that occurs at $\Delta f \approx 3$ Hz, roughly indpendent of the value of P_1.

While holding the rate of beating constant at 3 beats/sec, Riesz determined the differential sensitivity, the value of $\Delta\alpha$, systematically as a function of the intensity (expressed in terms of senation level) and the frequency of the more intense tone (Fig. 7). Generally, $\Delta\alpha$ decreases as the value of intensity increases, with most of the decrease occurring in the first 20 dB above threshold. The function relating the value of $\Delta\alpha$ to intensity is not a strong function of frequency f when intensity is expressed in sensation level.

[2] Intensity is proportional to the square of pressure.

[3] Riesz used E for intensity I.

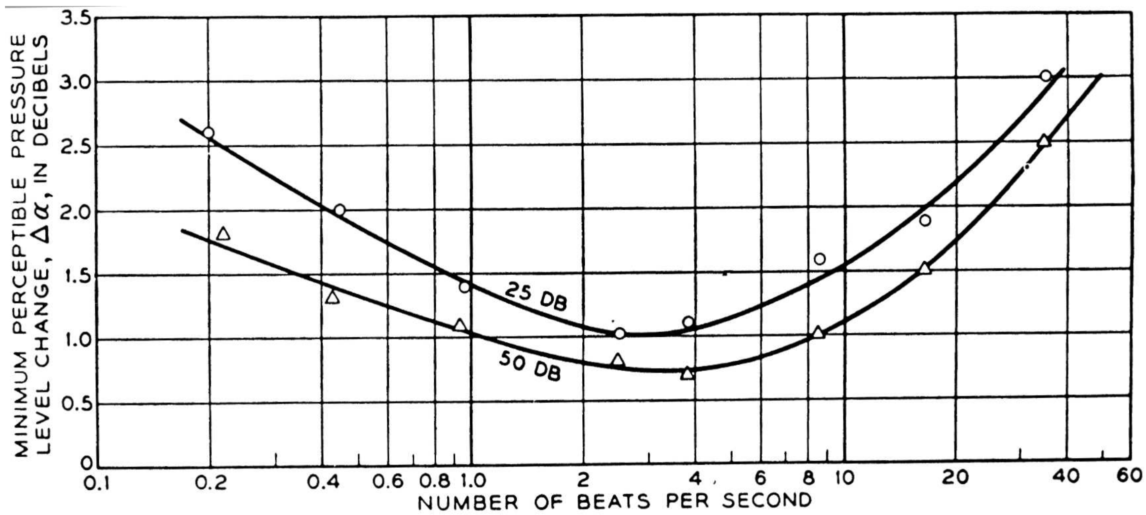

Figure 6: Dependence of the minimum perceptible change in the intensity of 1000 Hz tones, $\Delta\alpha = 10\log_{10}(1 + \Delta I/I)$, as measured by beat detection, on beat rate at P_1 levels of 25 and 50 dB SL. From Riesz (1928).

Controversy

Riesz's results are not without controversy, however. He determined the conditions under which a listener was able to distinguish between a pure tone

$$p_1(t) = A\cos\omega t$$

where $\omega = 2\pi f$ and a tone to which a second tone of different frequency was added:

$$p_2(t) = A\cos\omega t + B\cos(\omega + \delta)t$$

where $\delta = 2\pi\Delta f$. While $p_2(t)$ can exhibit beats for small δ it is not a purely amplitude modulated tone, such as $p_3(t)$:

$$p_3(t) = A\cos\omega t + \frac{B}{2}\cos(\omega - \delta)t + \frac{B}{2}\cos(\omega + \delta)t$$

In particular, $p_3(t)$ has zero crossings that occur with the same period as $p_1(t)$,

$$p_3(t) = A\left(1 + \frac{B}{A}\cos\delta t\right)\cos\omega t$$

whereas $p_2(t)$ does not.

In the case $B \ll A$ it is possible to develop a useful approximation to $p_2(t)$. Elementary trigonometry allows us to represent the added tone as

$$B\cos\omega t\cos\delta t - B\sin\omega t\sin\delta t$$

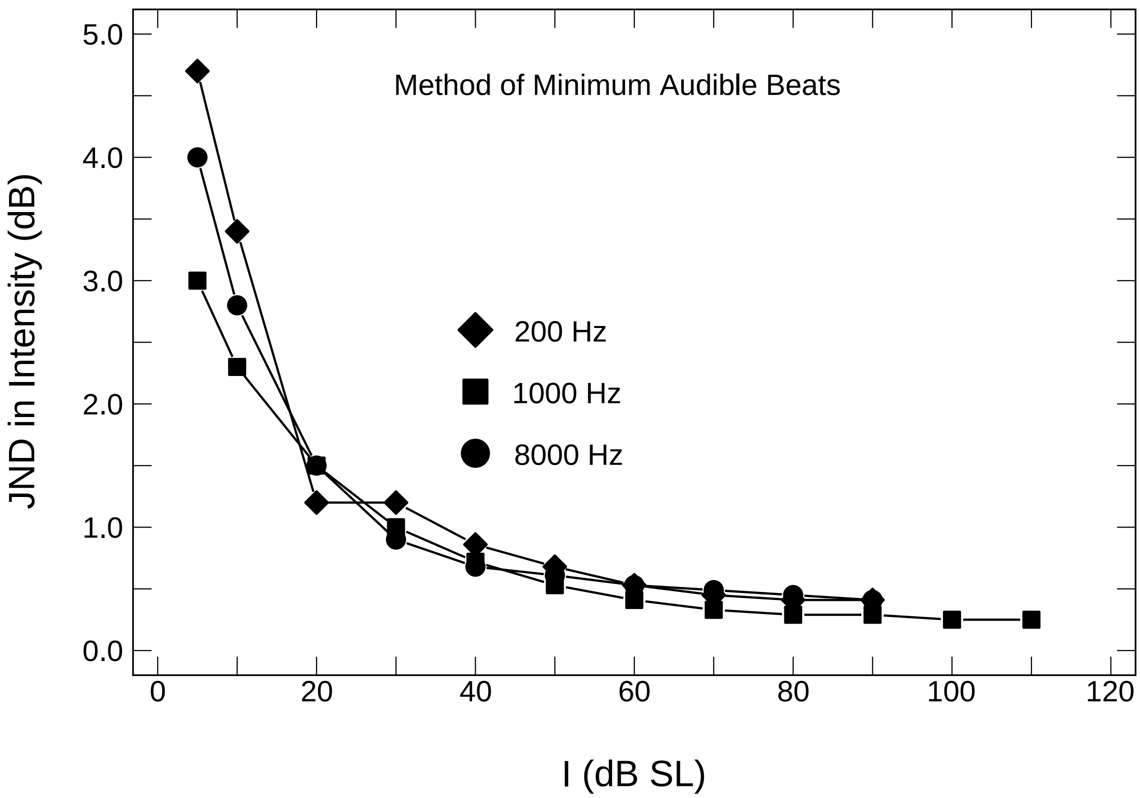

Figure 7: The JND ($\Delta\alpha$) for intensity discrimination of tones as a function of frequency and intensity (expressed in sensation level) as measured by beat detection. From Riesz (1928).

so that

$$p_2(t) = A\left[1 + \frac{B}{A}\cos\delta t\right]\cos\omega t - B\sin\omega t\sin\delta t$$

This expression for $p_2(t)$ has a fairly simpleinterpretation when $B/A < 1$. The first term in the sum is a simple amplitude modulated tone, with fluctuations that occur at a rate $\delta/2\pi$ at a modulation index of B/A, but has a constant zero-crossing rate of $\omega/2\pi$. The second term is in quadrature phase with $\cos\omega t$, with an amplitude that varies sinusoidally at a rate $\delta/4\pi$ and a phase that increments by 2π at the same rate.

Noting that

$$C\cos(\omega t + \theta) = C\cos\theta\cos\omega t - C\sin\theta\sin\omega t,$$

one can interpret the second expression for $p_2(t)$ by making the associations

$$C\cos\theta = A[1 + \frac{B}{A}\cos\delta t] \qquad (2)$$

$$C\sin\theta = A\frac{B}{A}\sin\delta t \qquad (3)$$

Evidently

$$C = A\sqrt{1 + \left(\frac{B}{A}\right)^2 + 2\frac{B}{A}\cos\delta t}$$

and

$$\tan\theta = \frac{B}{A}\frac{\sin\delta t}{1 + \frac{B}{A}\cos\delta t}$$

These expressions can be considerably simplified when $B/A \ll 1$. In particular, since if $\epsilon \ll 1$, $\sqrt{1+\epsilon} \approx 1 + \epsilon/2$ and

$$C \approx A[1 + \frac{B}{A}\cos\delta t]$$

and since $\tan\epsilon \approx \epsilon$

$$\theta(t) \approx \frac{B}{A}\sin\delta t$$

The signal $x(t) = X\cos\alpha(t)$ is said to have the "instantaneous frequency"

$$\frac{d\alpha(t)}{dt}$$

so that $p_2(t)$ has a time varying amplitude

$$A[1 + \frac{B}{A}\cos\delta t]$$

and a time varying frequency

$$\omega + \frac{B}{A}\delta\cos\delta t.$$

As a result we have the remarkable result

$$\begin{aligned} p_2(t) &= A\cos\omega t + B\cos(\omega + \delta t) \\ &\approx A\left[1 + \frac{B}{A}\cos\delta t\right]\cos\left(\omega\left[1 + \frac{B}{A}\frac{\delta}{\omega}\cos\delta t\right]t + \phi\right) \end{aligned}$$

where ϕ is an unspecified, but fixed, phase.

The ratio of the maximum to minimum amplitudes is

$$\frac{A+B}{A-B} = \frac{1+B/A}{1-B/A} \approx 1 + 2\frac{B}{A}$$

and the ratio of the maximum to minimum instantaneous frequencies is

$$\frac{\omega + B\delta/A}{\omega - B\delta/A} \approx 1 + 2\frac{B}{A}\frac{\delta}{\omega}.$$

Weber's Law and Deviations

Weber, a 19th century German physicist, measured the ability to distinguish between lifted weights. He concluded that there was a range of weights $W_{\rm min} \leq W \leq W_{\rm max}$ for which the "just noticeable difference" (JND, $\widehat{\Delta W}$) between two weights W and $W + \Delta W$ was proportional to the base weight, W, i.e.

$$\frac{\widehat{\Delta W}}{W} = k_W \qquad \text{independent of W}$$

A similar relationship has been found for many "intensive" physical properties (vibration amplitude, light intensity, ...). Provided $W > W_{\rm min}$, where $W_{\rm min}$ is typically well (e.g., 20 dB) above the absolute threshold for the stimulus.

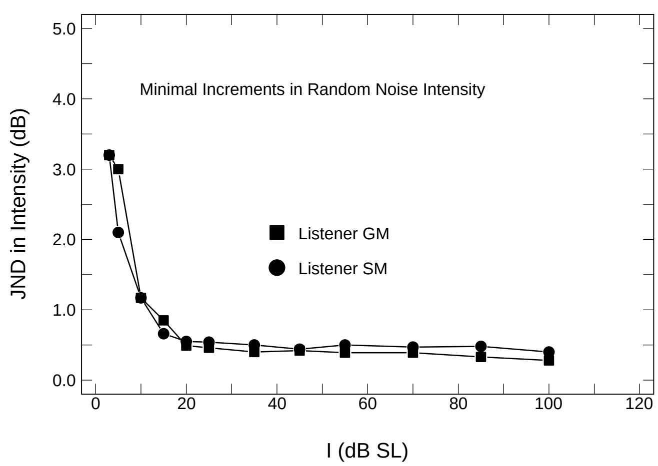

Figure 8: JNDs for intensity discrimination of white noise (flat spectrum within ±5 dB between 150 and 7000 Hz), from Miller (1947).

The JND in intensity for *wideband* stimuli such as broadband noise has been found to be reasonably well described by Weber's Law.

Miller (1947) presented two listeners with a continuous flat spectrum noise to which 25 increments (1.5 s duration) were added every 4.5 s. The subjects were asked whether they could hear the increment and the percentage "heard" was tabulated. Four such series were performed for each intnesity increment and 5–8 increments were measured for each intensity. The DL was obtained by linear interpolation as that increment that listener could hear 50

Miller (1947) determined that his measurements of the DL for the intensity of white noise (Fig. 8) could be well described by

$$\frac{I + \Delta I}{I} = 1 + C + D\frac{I_0}{I}$$

where I is the intensity of the noise, I_0 is the detection threshold of the noise, and ΔI is the DL for intensity. Clearly, at sufficiently high intensities,

$$\frac{I + \Delta I}{I} \to 1 + C$$

or

$$\frac{\Delta I}{I} \to C.$$

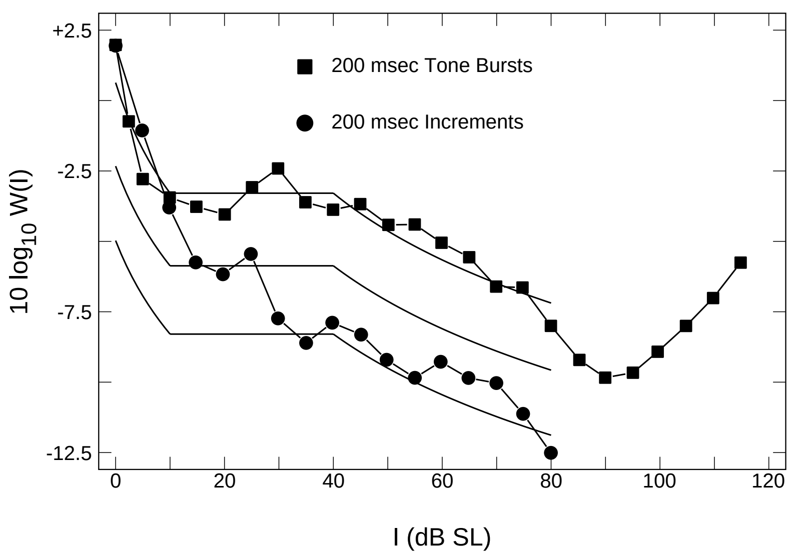

Figure 9: Measurements of the Weber fraction for intensity ($W(I) = \Delta I / I$ plotted on a logarithmic scale. From Viemeister (1988).

Modern methods of measuring the JND for sound pressure or intensity typically require the listener to compare two short pulses of sound presented with a brief interpulse interval.

For *narrowband* stimuli, in particular tones, the range over which Weber's law applies to intensity discrimination is fairly restricted (Fig. 9).

Dependence on Frequency and Level

Jesteadt, Wier, and Green (1977) determined the dependence of tone intensity discrimination on the intensity and frequency of the tone. They measured intensity discrimination for 500 ms bursts of tone using an adaptive two-interval forced-choice procedure that converged on the 70.7% correct point. each of three listeners were tested in four or five consecutive 100 trial adaptive blocks. Stimuli were presented in a low level (0 dB Spectrum Level) noise that was lowpass-filtered to 10 kHz. The noise raised thresholds by 5–15 dB relative to the ISO standard. Estimates of the intensity DL were obtained at distinct frequencies between 200 and 8000 Hz and over the range 5–80 dB SL.

When measured by the pulse comparison method, JND for intensity exhibits essentially the same dependence on intensity (specified in sensation level) for a wide range of frequencies (Fig. 10).

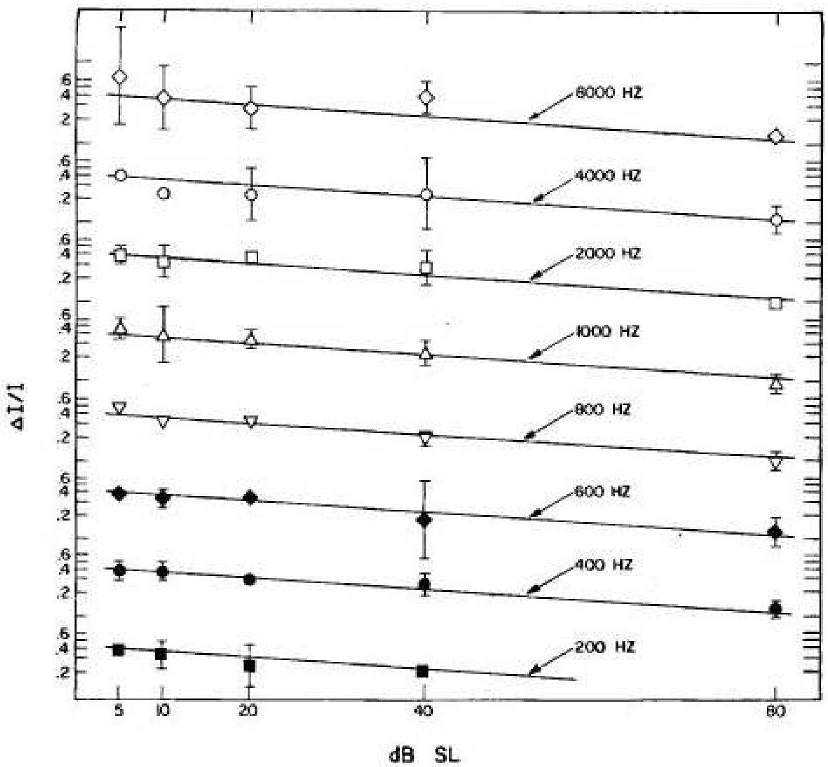

FIG. 1. Mean values of log($\Delta I/I$) across replications and subjects. The error bars indicate ±3 standard errors of the mean based on variability among subjects. Where error bars are absent, they are less than the size of the symbol. The same function has been filled to the data in all eight panels. The sensation level reference is the masked threshold in 0-dB spectrum-level noise.

Figure 10: Dependence of $\Delta I/I$ on I for 200–8000 Hz tones as a function of sensation Level. From Jesteadt et al., (1977).

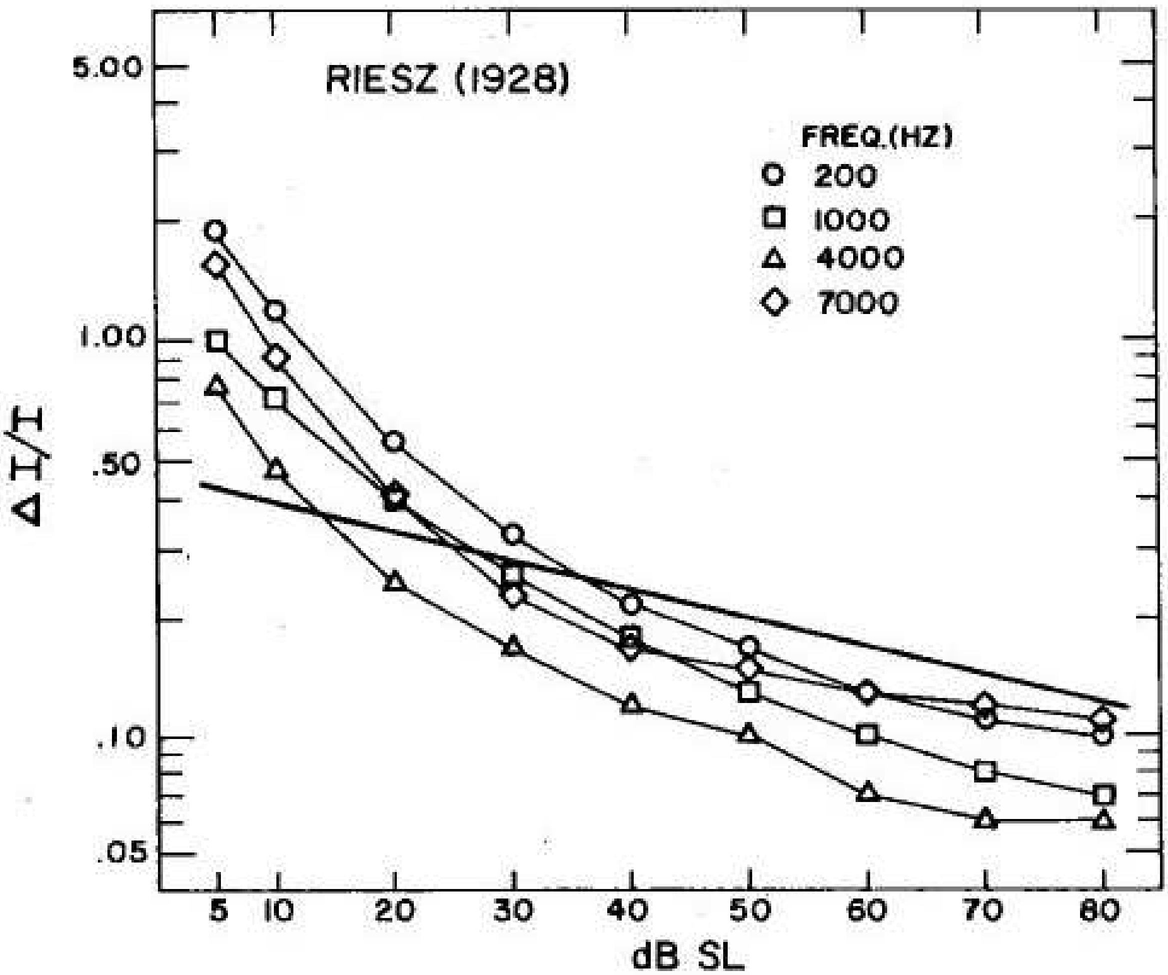

FIG. 4. A comparison of the data from the present study, represented by the linear function, with the data for comparable frequencies from Riesz (1928).

Figure 11: Dependence of $\Delta I/I$ on I for 200–8000 Hz tones as a function of Sensation Level. From Jesteadt et al., (1977).

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