Problem 1.1

The two-mirror imaging system shown on the next page consists of a large primary mirror, $M_1$, with radius of curvature, $R_1$, and a small secondary mirror, $M_2$, with a radius of curvature, $R_2$. Both mirrors are concave. In the system, $d_1$ is the distance of the object from the primary mirror, $d_2$ is the separation between the mirrors, and $d_3$ (not shown) is the distance of the final image from $M_2$.

(a) In the figure, you are given the special case where $d_2 = \frac{R_1}{2}$. Perform a geometric (ray-optics) construction (i.e., draw in the rays on the diagram) to show where the final image is formed.

(b) Is the final image real or virtual?

(c) Show the position and orientation of intermediate images, if any, and label them as real or virtual on the diagram.

(d) For the case where both the mirror separation, $d_2$, is arbitrary and $d_1 \gg \{d_2, R_1, R_2\}$, and with the help of the class notes, write down and simplify an expression for the final image distance, $d_3$, in terms of $d_1$, $d_2$, $R_1$, and $R_2$. 
Problem 1.1, Continued...

\[ d_2 = R_{1/2} \]

\[ d_1 \]

\[ R_1 \]

\[ R_2 \]

\[ M_1 \]

\[ M_2 \]

object
Problem 1.2

(a) A ray of light enters a cylindrically symmetric bead of glass ("thick lens") of refractive index \( n \) at a height \( h \) above its principal axis. The lens has entrance and exit faces with radii of curvature \( R_1 \) and \( R_2 \) as shown. Assuming the usual small-angle and thin-lens approximations hold for each component of the thick lens, use the ray-matrix approach to determine the approximate distance \( F^* \) at which the ray crosses the \( z \)-axis.

(b) Now run the ray backwards through the bead, keeping it still at a height \( h \) above the principal axis, so it is incident of the \( R_2 \) facet first. What is the new value \( F^{*'} \) of \( F^* \)?

(c) Does your result depend on \( h \)?

(d) When \( d = 0 \), show that your result for \( F^* \) is in agreement with the lens maker’s formula.

(e) When \( d = 0 \), do you get the expected result for two lenses in series?

(f) Based on your results for (a)-(e), comment on the use of such a bead as a lens. For example, does it have a well-defined focal length? What are its imaging properties?

The planes \( P \) and \( P' \) are often called principal planes. \( F^* \) and \( F^{*'} \) are called the Effective Focal Lengths (EFL). However, in practical use, the tendency is to measure the Front Focal Length (FFL), sometimes called the front focal distance (FFD), which is the distance from the vertex of the first optical surface to the front focal point. A similar definition holds for the Back Focal Length (BFL). The FFL and the BFL can be calculated from the EFLs and \( R_1 \) and \( R_2 \) and \( d \).
Problem 1.3 Matching FOV and entrance pupil of cameras

When creating synthetic scenes, projection optics are often used to manipulate the source image (typically formed on a real-time display). The figure below shows a 3-lens system that is designed to couple the image on a display into a camera. The display has a diameter $2D_m$ and emits light over a full cone angle of $2\theta_m$. The camera has a fixed full angular field of view (FOV) of $2\theta_s$ and a fixed entrance pupil of diameter $2D_s$. It is assumed the camera has its own lens $F_4$ (shown as a dashed outline) and can therefore form an image of the object generated by the display.

The goal is to design the 3-lens projection optics so that no light leaving the display is wasted, and that maximum image size is achieved inside the camera. That is, we want to incoming light to match the FOV and the pupil diameter of the camera.

(a) Derive $A$, $B$, $C$, and $D$ of the ABCD ray-optics matrix (in terms of the focal lengths of the lenses and $d_1$, $d_2$, $d_3$ and $d_4$) for the system bounded by the given object and the input plane to the camera. To help eliminate algebraic errors, you may want to use Mathematica, Maple or Matlab for this exercise.

(b) Consider the special imaging case where we want to make the image on the display appear to the camera as if the object was infinitely far away. That is, when the camera is focused at infinity, the camera output image must be sharp. In this case, the region between $F_3$ and the camera is often referred to as "collimated space". What condition on the matrix elements $A$, $B$, $C$, and $D$ would have to hold so that this is the case?

(c) To match the FOV and the input aperture dimensions of the camera, what constraints on the matrix elements $A$, $B$, $C$, and $D$ would have to hold to realize these two conditions?

(d) For the special case where $d_1 = F_1$, and $F_1$ and $F_2$ are identical, draw the ray-optics system corresponding to the matched projection system.

(e) Assuming Case (d), $d_2$, $d_3$ and $d_4$ are unconstrained. What are the conditions on these 3 variables so that matching FOV and entrance pupil occurs simultaneously?
Problem 1.4 - 6.637 only

Consider a microscope with the geometry shown below.

(a) Use the ABCD matrix method to show that the effective focal length of the two-lens combination (the distance behind $L_2$ that collimated input light comes to a focus) is approximately $d_2F_2/g$, where $d_2$ is the separation between the lenses. For positive values of $g$, what is your interpretation of this result?

(b) Use the $M_{system}$ equation in the notes for the two-lens system to calculate the exact (no approximations) angular magnification of the microscope. That is, assume $d_1 = F_1 + \epsilon$, the intermediate image is placed at a distance a little less than $F_2$ from the eyepiece, and $d_3 = -d_{min}$. 

Problem 1.5 - 6.637 only

An illumination point source is located at the back of a short cylindrical glass slab (coupler) which has a radius of curvature $R$ at its other end as shown. The coupler has a length $d$ and a refractive index $n$.

(a) What is the length, $d_1$, of the coupler which produces a collimated exiting beam (in air)?

(b) What is the length, $d_2$, of the coupler so that the point source is imaged at an equal distance $d_2$ in air away from the the convex end of the coupler?

The coupler is now butted axially against a long glass rod (light pipe) of the same material and of the same diameter. The goal is to efficiently transfer light from the source into the light pipe, but it turns out that the contiguous end of the light pipe also has a convex surface of radius of curvature $R$ as shown.

(c) Ignoring the outer boundaries of the coupler and the light pipe, what should be the length, $d_3$, of the coupler so that the light is collimated within the light pipe?

(d) Again ignoring the outer boundaries of the coupler and the light pipe, what should be the length, $d_4$, of the coupler so that the point source is imaged at an equal distance $L = 2d_4$ in the light pipe away from the convex side of the interface between the coupler and the light pipe?