Problem 1.1

The two-mirror imaging system shown on the next page consists of a large primary mirror, $M_1$, with radius of curvature, $R_1$, and a small secondary mirror, $M_2$, with a radius of curvature, $R_2$. Both mirrors are concave. In the system, $d_1$ is the distance of the object from the primary mirror, $d_2$ is the separation between the mirrors, and $d_3$ (not shown) is the distance of the final image from $M_2$.

(a) In the figure, you are given the special case where $d_2 = \frac{R_1}{2}$. Perform a geometric (ray-optics) construction (i.e., draw in the rays on the diagram) to show where the final image is formed. Assume that the desired image is formed after only one reflection from each mirror (ignore any possible multiple reflections between the mirrors).

(b) Is the final image real or virtual?

(c) Show the position and orientation of intermediate images, if any, and label them as real or virtual on the diagram.

(d) For the case where both the mirror separation, $d_2$, is arbitrary and $d_1 \gg \{d_2, R_1, R_2\}$, and with the help of the class notes, write down and simplify an expression for the final image distance, $d_3$, in terms of $d_1$, $d_2$, $R_1$, and $R_2$. 
Problem 1.1, Continued...

\[ d_2 = \frac{R_1}{2} \]
Problem 1.2

(a) Show that the ABCD matrix for a convex interface with radius of curvature, $R_1$, separating two media of refractive indices $n_0$ and $n_1$ when the light is traveling from medium 0 to medium 1 is

$$M_1 = \begin{pmatrix} \frac{1}{n_1-n_0} & 0 \\ \frac{n_0}{n_1} & \frac{n_1}{n_0} \end{pmatrix}$$

(b) Now let us make a thin biconvex lens out of this medium $n_1$ by carving out a second surface boundary which is concave and of radius $R_2$ (not shown) to the right of the first convex boundary of radius, $R_1$. Throw away the residual unused material. Derive an expression for the focal length of the lens thus formed, which is now fully embedded in medium $n_0$

(c) From your expression in (b) what is the focal length of this lens if $n_0 = 1$?

(d) The above lens is now placed in water of refractive index $4/3$. What is the focal length of the lens in water?
Problem 1.3 Matching FOV and entrance pupil of cameras

When creating synthetic scenes, projection optics are often used to manipulate the source image (typically formed on a real-time display). The figure below shows a 3-lens system that is designed to couple the image on a display into an imaging sensor (the eye or a camera). The display has a diameter $2D_m$ and emits light over a full cone angle of $2\theta_m$. The camera has a fixed full angular field of view (FOV) of $2\theta_s$ and a fixed entrance pupil of diameter $2D_s$. It is assumed the eye/camera has its own lens $F_4$ (shown as a dashed outline) and can therefore form an image of the object generated by the display.

The goal is to design the 3-lens projection optics so that no light leaving the display is wasted, and that maximum image size is achieved inside the eye/camera. That is, we want to incoming light to match the FOV and the pupil diameter of the camera.

(a) First write a user friendly computer program to compute $A$, $B$, $C$, $D$ in symbolic form for a 3 lens system (7 matrices). That is, Derive $A$, $B$, $C$, and $D$ of the ABCD ray-optics matrix (in terms of the focal lengths of the lenses and $d_1$, $d_2$, $d_3$ and $d_4$) for the system bounded by the given object and the input plane to the eye/camera. To help eliminate algebraic errors, you may want to use Mathematica, Maple or Matlab for this exercise (Note: These 4 equations are also very helpful for designing zoom lens systems, especially when one has constrains such as a limited choice of lenses with specific focal lengths and diameters, a maximum overall system design length, and require a specific angular or linear magnification.)

(b) Consider the special imaging case where we want to make the image on the display appear to the eye/camera as if the object was infinitely far away. That is, when the eye/camera is focused at infinity, the eye/camera output image must be sharp. In this case, the region between $F_3$ and the camera is often referred to as “collimated space”. What condition on the matrix elements $A$, $B$, $C$, and $D$ would have to hold so that this is the case?

(c) To match the FOV and the input aperture dimensions of the eye/camera, what constraints on the matrix elements $A$, $B$, $C$, and $D$ would have to hold to realize these two conditions?

(d) For the special case where $d_1 = F_1$, and $F_1$ and $F_2$ are identical, draw the ray-optics system corresponding to the matched projection system.

(e) Assuming Case (d), $d_2$, $d_3$ and $d_4$ are unconstrained. What are the conditions on these 3 variables so that matching FOV and entrance pupil occurs simultaneously?
Problem 1.4

(a) A ray of light enters a cylindrically symmetric bead of glass ("thick lens") of refractive index $n$ at a height $h$ above its principal axis. The lens has entrance and exit faces with radii of curvature $R_1$ and $R_2$ as shown. Assuming the usual small-angle and thin-lens approximations hold for each component of the thick lens, use the ray-matrix approach to determine the approximate distance $F^*$ at which the ray crosses the $z$-axis.

(b) Now run the ray backwards through the bead, keeping it still at a height $h$ above the principal axis, so it is incident of the $R_2$ facet first. What is the new value $F^{'*}$ of $F^*$?

(c) Does your result depend on $h$?

(d) When $d = 0$, show that your result for $F^*$ is in agreement with the lens maker's formula.

(e) When $d = 0$, do you get the expected result for two lenses in series?

(f) Based on your results for (a)-(e), comment on the use of such a bead as a lens. For example, does it have a well-defined focal length? What are its imaging properties?

A Note on Thick Lenses: The planes $P$ and $P'$ are often called principal planes. $F^*$ and $F^{'*}$ are called the Effective Focal Lengths (EFL). However, in practical use, the tendency is to measure the Front Focal Length (FFL), sometimes called the front focal distance (FFD), which is the distance from the vertex of the first optical surface to the front focal point. A similar definition holds for the Back Focal Length (BFL). The FFL and the BFL can be calculated from the EFLs and $R_1$ and $R_2$ and $d$. 