Problem 4.1
In the coordinate system below, the following definitions hold:
\[
\vec{\rho} = x\hat{x} + y\hat{y} \\
\vec{r} = x\hat{x} + y\hat{y} + z\hat{z} \\
\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}
\]

Below are a set of functions that may or not be descriptive of elementary unit-amplitude electromagnetic waves:

(a) \( U(\vec{r}) = e^{j\vec{k}\cdot\vec{r}} \)
(b) \( U(\vec{r}) = e^{jkz}e^{-j\frac{kr^2}{2}} \)
(c) \( U(\vec{r}) = e^{j\frac{k}{2r}(x^2+y^2)} \)
(d) \( U(\vec{r}) = e^{j\frac{k}{2r}[2(x-x_0)^2 + 2(y-y_0)^2]} \)
(e) \( U(\vec{r}) = e^{jk(z^2+x^2+y^2)^{1/2}} \)
(f) \( U(\vec{r}) = e^{jk[z^2+(x-x_0)^2+(y-y_0)^2]^{1/2}} \)

(4.1a) Which of these functions do not satisfy the wave equation and why? Show your work!

(4.1b) For the ones that can describe electromagnetic waves, show the direction of propagation, and algebraically derive, sketch and describe the shape of their wavefronts.
Problem 4.2

The figure below is a 1-D crosssectional plot of a 2-D piecewise-continuous approximation to an actual wavefront exiting a certain device in the \( z = 0 \) plane. The actual 2-D wavefront (not shown) is smooth (no sharp corners). For simplicity, the normal to the wavefront segments all lie in the \( x - z \) plane, and the wavefront, of wavelength \( \lambda \), is travelling nominally in the \( +z \)-direction. Assume all segments of the 2-D piecewise continuous wavefront are of the same area (10\( \lambda \) in the x-z plane, 10\( \lambda \) in y) and they all carry the same power density of \( I \) Watts/m\(^2\).

(a) Write a frequency-domain expression, \( U(f_x, f_z) \), that describes the primary directions of power flow in this wavefront. (ignore any effective aperturing and, therefore, diffraction effects).

(b) Sketch the spatial-frequency content of this wavefront on a graph with the co-ordinate system shown below.

(c) Assume the above wavefront exists in the back focal plane of a lens of focal length \( F \) and is traveling nominally toward the lens. Sketch the intensity pattern that would be seen on a screen placed in the front focal plane of the lens, and label the positions and the sizes of any critical features that will be present on the screen.
Problem 4.3

Consider the following three, infinitely-long one-dimensional, deformable grating mirrors (mirror surface with variable $d$) with the surface profiles shown below. The mirrors are illuminated at normal incidence (along the $z$-axis) with a plane wave of wavelength $\lambda$.

(i) square-wave grating profile

(ii) right-triangular grating profile

(iii) isosceles-triangular grating profile

For cases (ii) and (iii) above only:

(a) Write expressions $\phi(x)$ for the phase imparted by the mirrors on the wave.

(b) Using inspection techniques (do not use Fourier transform approach), determine the minimum value of $d$ that will extinguish the zero-order diffracted light (justify your answers with physical arguments).
Problem 4.4

A plane-wave of amplitude $A$ and wavelength $\lambda$ is incident at normal incidence in the $\hat{z}$-direction on the transmission object $U_a(x, y)$ described below.

$$U_a(x, y) = \begin{cases} 
e^{j\frac{\pi}{2}} & 0 < x < a \\ e^{-j\frac{\pi}{2}} & -a < x < 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Draw a sketch of this object in the x-y plane

(b) Give an example of how you would fabricate such an object.

(c) Compute analytically the intensity of the Fraunhofer diffraction field owing to this object.

(d) Plot the Fraunhofer diffraction intensity of part (c) using your favorite software package.

Problem 4.5

A Fresnel zone plate with a opaque central zone is illuminated with on-axis collimated light of wavelength $\lambda$. For an on-axis observation point $P$ at a distance $h$ behind the plate,

(a) How does the number zones, $m$, that fit within each transparent ring of the zone plate depend on $h$?

(b) If there are $M$ zones in the first transparent ring, how many zones will there be in the 5th transparent ring?

(c) Large glass lenses are heavy. Design a large (about 30 cm diameter) thin (non-holographic) light-weight, diffractive-refractive plate that would be made out of transparent plastic material about 1 or 2 mm thick, and that uses Fresnel zone concepts to achieve lens-like behavior. Your resulting structure must be more optically efficient than a Fresnel zone plate (puts more light into a focused spot). Be sure to specify all dimensions in your design.