Problem 6.1

We know that if an object is placed in the front focal plane of a lens, the field in the back focal plane of the lens is related to the Fourier transform of the object. We also know that the far-field diffraction pattern of an object is related to the Fourier transform of that object. Make a list of the similarities and differences between the Fourier transforms formed by these two systems. Explain each of the differences with the aid of diagrams and an example such as a circular aperture, or a slit, or rectangular grid, etc.

Problem 6.2

In geometric optics, it is well known that a lens will image a point-source, $O$, at a distance $2F$ in front of the lens to a point, $I$, at a distance $2F$ in back of the lens as shown.

(a) Using the wave-optics approach, write an expression for the wavefront $U_1(\rho_1)$ incident on the plane, $P_1$, located at $z = 0_-$ (lens is thin).

(b) Using the paraxial approximation, simplify the expression obtained in part (a).

(c) Similarly, starting with the fact that light leaving the plane $P_2$ converges to a point, $I$, in the back image plane, write an expression for the wavefront $U_2(\rho_2)$ at the plane, $P_2$, located at $z = 0_+$ (lens is thin).

(d) Using the paraxial approximation, simplify the expression obtained in part (c).

(e) What then is the expression for the thin lens transformation $t_\mathcal{L}(\rho)$?
Problem 6.3

In a classical two-lens coherent optical processor with lenses of focal length $F$, two signal transparencies $g_a(x_1, y_1)$ and $g_b(x_1, y_1)$ are inserted in the $\bar{\rho}_1$ (input) plane with centers at $(a, 0, 0)$ and $(-a, 0, 0)$ respectively. The Fourier-plane filter $\mathcal{L}_h(\bar{\rho})$ is a sinusoidal grating with amplitude transmittance

$$\mathcal{L}_h(\bar{\rho}) = \mathcal{L}_h(F \lambda \bar{f}) = \frac{1}{2} + \frac{1}{2} \sin(2\pi f_g x + \phi)$$

where $f_g$ and $\phi$ represent the grating frequency and position respectively.

(a) Calculate the complex amplitude distribution at the output plane of the processor.

(b) For the special case where $f_g = a / \lambda F$ and $\phi = 0$, comment on the significance of the output. Repeat the process for $\phi = 90^\circ$.

Problem 6.4

You are given a thin convex lens of focal length $F$ and told to find the matched filter that “identifies” the given lens. So, following standard practice, you place the given lens in the input plane of a 4-F classical two-lens processor as shown in the Figure below, and you start trying various filters that you have found lying around the lab.

(a) Write down the equation for $U_1(\bar{\rho}_1)$ for the system as shown above, assuming collimated input light.

(b) What will the output, $U_3(\bar{\rho}_3)$, look like on a screen/detector in the $\bar{\rho}_3$ plane when you have found the perfect matched filter?

(c) What is the functional form (give equation) of the Fourier-plane filter, $\mathcal{F}(\bar{f})$, that will do the trick?

(d) Describe with words exactly what this matched filter is in real space, and write an expression for its transmission function, $h(\bar{\rho})$.

(e) Draw a ray optics picture for the complete system to justify/support your wave-optics answers in (a), (b) and (c).
Problem 6.5

In the polychromatic optical processor shown below, $S$ is a white-light point source, and $L_0$, $L_1$ and $L_2$ are achromatic lenses. Two three-color (each monochromatic) signal transparencies $g_a(x,y,\lambda)$ and $g_b(x,y,\lambda)$ are placed in the input plane $P_1$ at the points $(0, d)$ and $(0, -d)$ respectively and in contact with a high-efficiency diffraction grating $G(x,y)$. Assume the two input signal color transparencies, $g_a(x,y,\lambda)$ and $g_b(x,y,\lambda)$, each have a spatial frequency bandwidth limit of $B_x \times B_y$. The transmission function of the in-plane grating is given by

$$t_G(x,y) = \frac{1}{2} [1 + \cos(2\pi f_g x)]$$

(a) Write an expression for $U_{2i}(x,y,\lambda_n)$ where $n$ is the color subscript [$n=1$ (blue), 2 (green), 3 (red)].

(b) Explain/interpret each term in the equation you have derived in (a). For $n=1, 2, 3$ make a sketch of the pattern of the light falling on the $P_2$ plane, and label the position of the centroid of each of the components.

(c) Suppose the Fourier-plane composite filter in the $P_2$ plane is composed of a set of non-overlapping subfilters given by

$$\tilde{h}(x_2,y_2) = \hat{h}(f_x,f_y) = \Sigma_n(1/2) [1 + \sin(2\pi df_{yn})]$$

for the first-order spectra, but is opaque for the zero and -1 orders. Sketch the three subfilters, corresponding to the three values of $n$, in physical space.

(d) Write an expression for the output $U_{3i}(f_x,f_y,\lambda)$ in the $P_3$ plane for one of the colors using its corresponding Fourier-plane subfilter.

(e) To perform full color subtraction, we must synthesize the composite filter $\hat{h}(f_x,f_y)$. Make a sketch of the desired composite filter. On your diagram specify and give expressions for: (1) its periodicity, $\Lambda_{gy}$, (2) the centroids and (3) widths of the filter segments (if any), and (4) the location of the desired subtraction image, $g_a(x,y,\lambda) - g_b(x,y,\lambda)$, in the output plane.

(f) What are the conditions on $\lambda_n$, $f_{gy}$, $B_x$, $B_y$ and $F$ so that the desired output image is clearly separated from other images in the output plane?

(g) Comment on the practical limitations of this system.