MASSACHUSETTS INSTITUTE of TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.161 Modern Optics Project laboratory 6.637 Optical Signals, Devices & Systems

Problem Set No. 7	**Fourier Optics**	Issued Thr. 11/1/2018
Fall Term, 2018	**-**	Due Thr. 11/8/2018

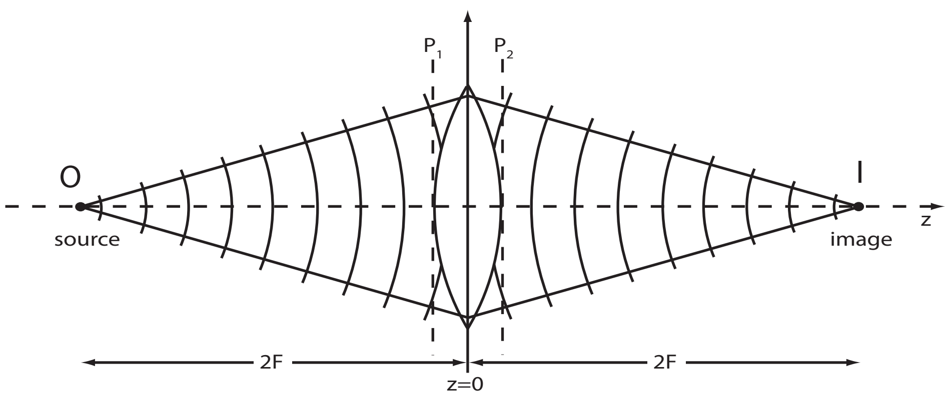
Reading recommendation: Class Notes, Chapter 8. Be neat in your work!

Problem 7.1

We know that if an object is placed in the front foal plane of a lens, the field in the back focal plane of the lens is related to the Fourier transform of the object. We also know that the far-field diffraction pattern of an object is related to the Fourier transform of that object. Make a list of the similarities and differences between the Fourier transforms formed by these two systems. Explain each of the differences with the aid of diagrams and an example such as a circular aperture, or a slit, or rectangular grid, etc.

Problem 7.2

In geometric optics, it is well known that a lens will image a point-source, O, at a distance $2F$ in front of the lens to a point, I, at a distance $2F$ in back of the lens as shown.



(a) Using the wave-optics approach, write an expression for the wavefront $\underline{U}_1(\overline{\rho}_1)$ incident on the plane, P_1, located at $z = 0_-$ (lens is thin).

(b) Using the paraxial approximation, simplify the expression obtained in part (a).

(c) Similarly, starting with the fact that light leaving the plane P_2 converges to a point, I, in the back image plane, write an expression for the wavefront $\underline{U}_2(\overline{\rho}_2)$ at the plane, P_2, located at $z = 0_+$ (lens is thin).

(d) Using the paraxial approximation, simplify the expression obtained in part (c).

(e) What then is the expression for the thin lens transformation $\underline{t}_l(\overline{\rho})$?

Problem 7.3

In a classical two-lens coherent optical processor with lenses of focal length F, two signal transparencies of transmittance $g_a(x_1, y_1)$ and $g_b(x_1, y_1)$ are inserted in the $\bar{\rho}_1$ (input) plane with centers at $(a, 0, 0)$ and $(-a, 0, 0)$ respectively. The Fourier-plane filter $\underline{t}_h(\bar{\rho})$ is a sinusoidal grating with amplitude transmittance

$$\underline{t}_h(\bar{\rho}) = \underline{t}_h(F\lambda \bar{f}) = \frac{1}{2} + \frac{1}{2}\sin(2\pi\, f_g x + \phi)$$

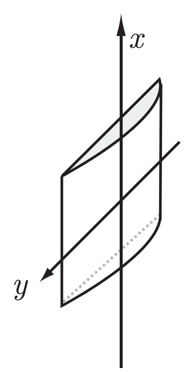
where f_g and ϕ represent the grating frequency and position respectively.

(a) Calculate the complex amplitude distribution at the output plane of the processor.

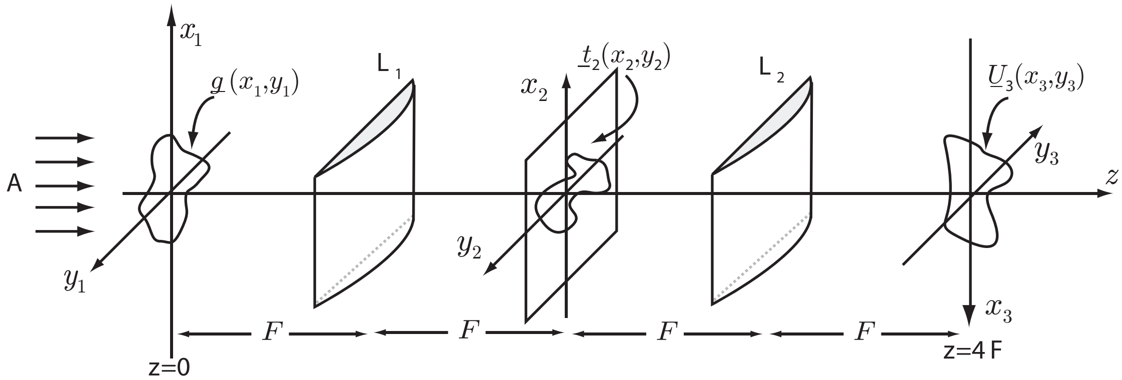
(b) For the special case where f_g= a/λF and $\phi = 0$, comment on the significance of the output. Repeat the process for $\phi = 90^\circ$.

Problem 7.4. - Cylindrical-lens Fourier Optics

(a) Consider the thin cylindrical lens shown below. Write an equation for the thin cylindrical lens transformation $\underline{t}_{cl}(x, y)$.

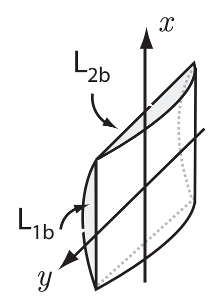


(b) A classical two-cylindrical lens coherent processor is built as shown below. The lenses L_1 and L_2 each have focal length F. Such a system can used to process 1-D objects $\underline{g}(x, y)$ using 1-D Fourier plane filters, $\underline{t}(x, y)$. Indicate on the diagram how the 1-D object, $\underline{g}(x, y)$ and the filter $\underline{t}(x, y)$ must be oriented to properly function in this system.



(c) Write an equation for the input-output characteristics of the system. That is, express $\underline{U}_3(x_3, y_3)$ in terms of $\underline{U}_1(x_1, y_1)$.

(d) Two square in cross section plano-convex cylindrical lenses are crossed and fused together to form a compound lens as shown below.

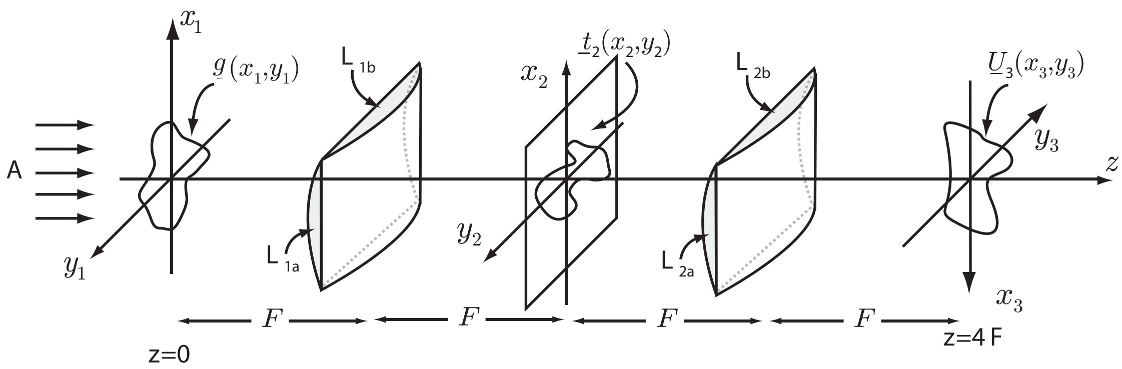


d(1) Write the lens transformation for this compound lens assuming the two original cylindrical lenses have the same focal length and the same square cross sectional dimensions.

d(2) What is the focal length of the compound lens?

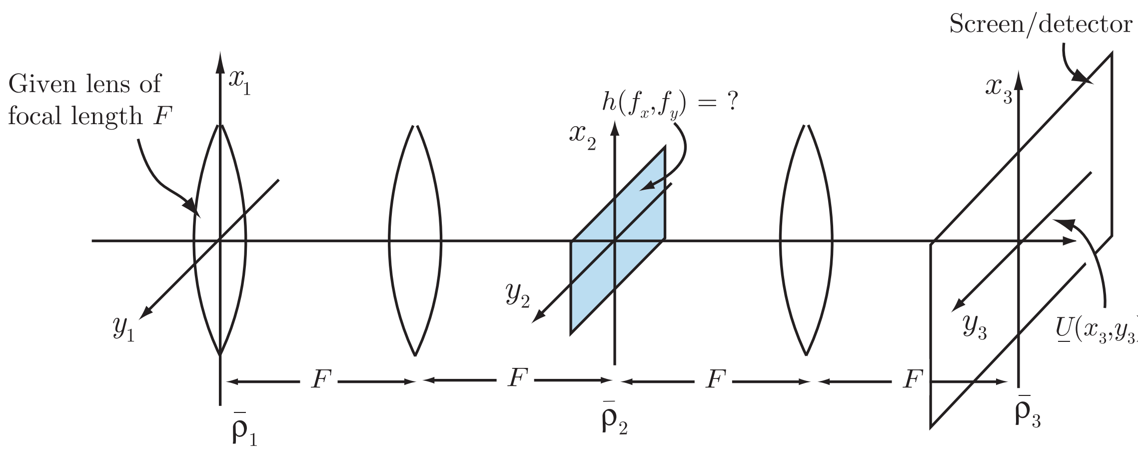
d(2) How does this lens differ from the conventional bi-convex lens in shape and lens transformation?

(e) A classical two-compound lens coherent processor is built as shown below. Write an equation for the input-output characteristic of the system. That is, express $\underline{U}_3(x_3, y_3)$ in terms of $\underline{U}_1(x_1, y_1)$



Problem 7.5 - 6.637 only

You are given a thin convex lens of focal length F and told to find the matched filter that "identifies" the given lens. So, following standard practice, you place the given lens in the input plane of a 4-F classical two-lens processor as shown in the Figure below, and you start trying various filters that you have found lying around the lab.



(a) Write down the equation for $\underline{U}_1(\overline{\rho_1})$ for the system as shown above, assuming collimated input light.

(b) What will the output, $\underline{U}_3(\overline{\rho}_3)$, look like on a screen/detector in the $\overline{\rho}_3$ plane when you have found the perfect matched filter?

(c) What is the functional form (give equation) of the Fourier-plane filter, $\underline{\tilde{h}}(\bar{f})$, that will do the trick?

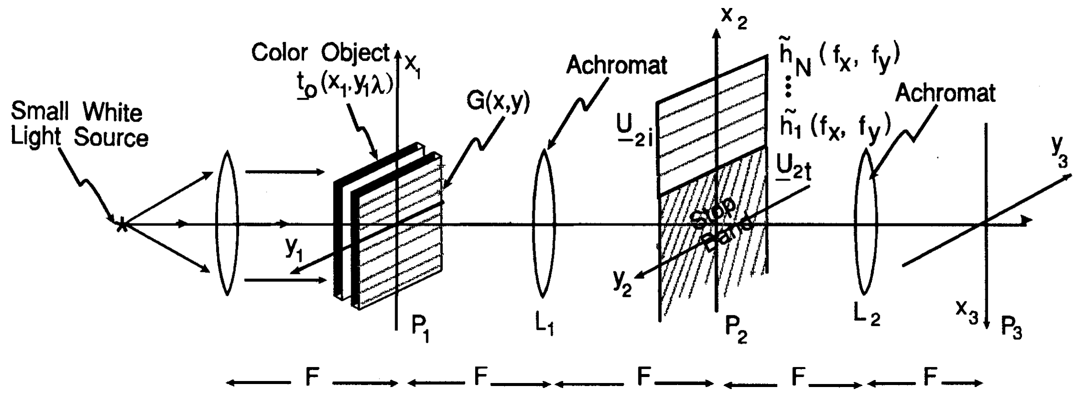
(d) Describe with words exactly what this matched filter is in real space, and write an expression for its transmission function, $\underline{h}(\overline{\rho})$.

(e) Draw a ray optics picture for the complete system to justify/support your wave-optics answers in (a), (b) and (c).

Problem 7.6 - 6.637 only

In the polychromatic optical processor shown below, S is a white-light point source, and L_0, L_1 and L_2 are achromatic lenses. Two three-color (each monochromatic) signal transparencies $\underline{g}_a(x, y, \lambda)$ and $\underline{g}_b(x, y, \lambda)$ are placed in the input plane P_1 at the points (0, d) and (0, -d) respectively and in contact with a high-efficiency diffraction grating $G(x, y)$. Assume the two input signal color transparencies, $\underline{g}_a(x, y, \lambda)$ and $\underline{g}_b(x, y, \lambda)$, each have a spatial frequency bandwidth limit of B_x x B_y. The transmission function of the in-plane grating is given by

$$t_G(x, y) = \frac{1}{2}[1 + cos(2\pi f_g x)]$$



(a) Write an expression for $U_{2i}(x, y, \lambda_n)$ where n is the color subscript [n =1(blue), 2(green), 3 (red)].

(b) Explain/interpret each term in the equation you have derived in (a). For n = 1, 2, 3 make a sketch of the pattern of the light falling on the P_2 plane, and label the position of the centroid of each of the components.

(c) Suppose the Fourier-plane composite filter in the P_2 plane is composed of a set of non-overlapping subfilters given by

$$\underline{t}_h(x_2, y_2) = \tilde{h}(f_x, f_y) = \Sigma_n(1/2)[1 + sin(2\pi d f_{yn})] \tag{1}$$

for the first-order spectra, but is opaque for the zero and -1 orders. Sketch the three subfilters, corresponding to the three values of n, in physical space.

(d) Write an expression for the output $\underline{U}_3(f_x, f_y, \lambda)$ in the P_3 plane for one of the colors using its corresponding Fourier-plane subfilter.

(e) To perform full color subtraction, we must synthesize the composite filter $\tilde{h}(f_x, f_y)$. Make a sketch of the desired composite filter. On your diagram specify and give expressions for: (1) its periodicity, Λ_{gy}, (2) the centroids and (3) widths of the filter segments (if any), and (4) the location of the desired subtraction image, $\underline{g}_a(x, y, \lambda)$ - $\underline{g}_b(x, y, \lambda)$, in the output plane.

(f) What are the conditions on λ_n, f_g, B_x, B_y and F so that the desired output image is clearly separated from other images in the output plane?

(g)) Comment on the practical limitations of this system.