# MASSACHUSETTS INSTITUTE of TECHNOLOGY <br> Department of Electrical Engineering and Computer Science 

6.161/6637 Practice Quiz 1

Issued 2:30pm 10/XX/2018
Fall Term, 2018
Due 4:00pm 10/XX/2018
Please utilize the space provided below each problem for your work. You may use extra paper if necessary. If you need additional clarification, be sure to ask. This quiz is open-book and open-notes. GOOD LUCK and DON'T STRESS!!! You may start as soon as you receive the quiz.

Please note that this sample quiz contains many more examples of the kind of problems you may see on an actual quiz. If you attempt all the questions, it should be roughly 3 times as long as the quiz you will take in class.

Please print your name here:
Please print your MIT ID number here: $\qquad$

## BASIC ELECTROMAGNETICS

## Problem 1

A diver shines his unpolarized collimated head-light laser beam toward the surface of the water at an angle of incidence, $\theta$. Complete the ray-optics pictures below to show where the light goes. Also show the polarization effects, if any, of the water on the beams you draw.


$$
\theta_{1}=\sin ^{-1}\left(\frac{1}{1.33}\right)
$$



$$
\theta_{2}=\tan ^{-1}\left(\frac{1}{1.33}\right)
$$

## Problem 2

The six $\bar{k}$-vectors given in the diagrams on the next page all lie in the $(x-z)$ plane. They correspond to six uniform plane waves each of amplitude $E_{0}$ traveling in the directions shown. The polarization vectors for the E and H fields are as shown.
(a) Draw the magnetic-field vector onto diagrams $a, b$ and $c$.
(b) Write expressions for $\bar{f}_{1}, \bar{f}_{2}$ and $\bar{f}_{3}$, the spatial frequency vectors corresponding to the waves in diagrams $a, b$ and $c$ respectively.
(c) Write complete expressions for the plane waves corresponding to diagrams $a, b$ and $c$.
(d) Which pairs of waves, if any, are phase-conjugate pairs?


## Problem 3

(a) Determine the critical angle for the 2-layer medium shown below, given that $n>n_{0}$.

(b) For the $m$-layered medium shown below, given that $n_{0}<n_{1}<n_{2}<\ldots<n_{m}<n$ determine the critical angle for the entire layered structure.


## Problem 4

(a) For the three waveplates described below, $\phi_{s}$ and $\phi_{f}$ are the phase shifts associated with the slow and fast axes of the plate of thickness, $d$. Fill in the table below.

| Plate | Desired value of $\phi_{\mathbf{s}}-\phi_{\mathbf{f}}$ | Formula for plate thickness |
| :---: | :--- | :--- |
| quarter-wave plate |  | $d=$ |
| half-wave plate |  | $d=$ |
| full-wave plate |  | $d=$ |
|  |  |  |

(b) You found a wave plate in laboratory and measured its thickness, which turns out to be $32 \frac{1}{2} \lambda$ microns, where $\lambda$ is the wavelength of laser light you happen to be using. The fast axis of the plate has a refractive index of 1.30 and a slow axis of refractive index 1.60. What kind of plate is this? (that is, quarter-, half-, or a full-wave plate?) No credit will be given for the correct answer without supporting calculation.

## Problem 5

Vertically polarized (along x ) laser light enters a halfwave plate whose fast and slow axes are oriented at $45^{0}$ to the x -axis as shown. The light next passes through a quarter-wave plate that has its fast axis along the x -axis. Thereafter it passes through a linear polarizer with its axis oriented at $45^{0}$ to the x -axis, before illuminating a polarization beam splitter. At each of the points A through E between the elements, write the polarization state and the intensity of the beam in the table below.


| Intensity | Polarization state of light | Your Reasoning here, please |
| :---: | :---: | :---: |
| $I_{A}=$ | $e_{A}=$ |  |
| $I_{B}=$ | $e_{B}=$ |  |
| $I_{C}=$ | $e_{C}=$ |  |
| $I_{D}=$ | $e_{D}=$ |  |
| $I_{E}=$ | $e_{E}=$ |  |

## GEOMETRIC OPTICS

## Problem 1

Complete the ray-optics picture for both diagrams shown below. Show all missing rays, their polarizations and angles.


## Problem 2

Consider an optical system consisting of two dielectric media with a planar interface. The medium on the left is free space and the medium on the right is isotropic with a refractive index, $n$. In the questions below, ignore all reflected beams and assume the paraxial approximation holds.

(a) Write the ABCD matrix for the dielectric boundary.
(b) Now consider a plane $P_{1}$ in the free-space medium that is a distance $d_{1}$ from the boundary, and a plane $P_{2}$ in the dielectric medium at a distance $d_{2}$ from the boundary. Find the ABCD matrix for the system bounded by the $P_{1}$ and $P_{2}$ planes, where $P_{1}$ is considered the input plane and $P_{2}$ the output plane.

(c) If an object is placed in the $P_{1}$ plane, what condition would have to hold such that $P_{2}$ is an image plane?

(d) Now, set $n=-1$, and determine the image distance, $d_{2}$, with respect to the object distance, $d_{1}$ (again, assume the boundary is matched, and that there is no reflection all rays incident from the first medium enter the second medium ).
(e) With $n=-1$, what will be the magnification of the image? Is it Real or Virtual at $d_{2}$ ? Assume the boundary is matched, and that there is no reflection - all rays incident from the first medium enter the second medium according to the paraxial approximation and snell's law.
(f) Draw the ray-optics picture on the diagram below for $n=-1$.


Note: Having a negative index of refraction does NOT violate Maxwell's equations. Researchers have several names for such media, such as 'negative media' or 'left-handed media.' Such media, made of metamaterials (mixtures of normal materials that exhibit peculiar electromagnetic properties) exhibit a bulk negative index of refraction for a small range of electromagnetic frequencies.

## INTERFERENCE

## Problem 1

The light beams from two $\mathrm{He}-\mathrm{Ne}$ lasers that you found in lab are first expanded and collimated, and then combined on a screen as shown in the figure below. The wavelength of both lasers is 633 nm . Calculate the fringe separation and give the orientation of the fringes in the resulting interference pattern.


## Problem 2

A semiconductor laser diode with a wavelength of 500 nm and a coherence length of 10 cm is used as the light source in a Michelson interferometer. The fixed mirror is 10 cm from the beam splitter. A glass block ( $\mathrm{n}=1.5$ ) of thickness 5 cm is placed in the fixed arm as shown. The moveable mirror is permitted to move only through a range that causes the detector current to go through appreciable maxima and minima. What is this range? That is, what are the numerical values of $l_{\max }$ and $l_{\min }$, where $l$ is the distance of the movable mirror from the beam splitter?


## Problem 3

A thin soap film is stretched on a wire hoop as was demonstrated in the classroom and shown in the figure below. The index of refraction of the soap film is 1.33 (mostly water). A student views a particular spot $S$ on the soap film at a $30^{\circ}$ angle of reflection under the diffuse white light illumination present in the classroom. The film appears red at the spot $S$ with $\lambda=600 \mathrm{~nm}$.

(a) If $h$ is the thickness of the film and $n$ is its refractive index, what is the interference condition for a bright maximum in reflection for wavelength $\lambda$ incident at an angle $\theta$ ?
(b) Using the values given in the problem statement, what is one possible thickness of the soap film?

## DIFFRACTION

## Problem 1

In experiment No. 1, a thin long wire of diameter $a$ is illuminated at normal incidence by a very large cross-section plane wave of wavelength $\lambda$. In experiment No. 2 a narrow long slit of width $a$ is illuminated at normal incidence by a very large cross-section plane wave of the same wavelength $\lambda$. Assume both the wire and the slit are infinitely long in the $x$-direction and that they are centered around the $x$-axis on the $z=0$ plane of the co-ordinate system as shown.

(a) Write an equation for the aperture function $\underline{U}_{\text {wire }}(x, y)$ for the diffracting wire system
(b) Write an equation for the aperture function $\underline{U}_{\text {slit }}(x, y)$ for the diffracting slit system
(c) Describe (1) the differences and (2) the similarities of the two resulting far-field intensity diffraction patterns.

## Problem 2

The three objects ((a), (b), (c)) shown below are illuminated with coherent light of wavelength $\lambda$ : Object (a) is a transparent circular aperture of diameter $a$; Object (b) is a transparent annular aperture of inner diameter $b$ and outer diameter $(b+2 a)$ with $b>a$, and object (c) is a set of very long (length $l$ ) randomly oriented narrow opaque rods of width $a$. We note that $l \ggg a>\lambda$.

(a) What property does the far-field diffraction of these aperture have in common? Explain how you arrived at your answer.
(b) On the graphs below, sketch the far-field diffraction in the spatial frequency domain of the three objects labeling all the critical features of the diffration patterns. Use the same horizontal scale for all three graphs.


## DIFFRACTION

## Problem 1

Draw the far-field (Fraunhofer) diffracted intensity pattern for the objects pictured below; use your pencil or ink lines to indicate intensity maxima. These do not require any calculation; just draw a qualitative sketch, noting any special features (e.g., intensity peaks and troughs near the origin, as well as the overall diffraction patterns). (a) consists of randomly oriented, transmissive rectangular apertures (all of the same dimensions), (b) consists of an opaque square on a transparent substrate, (c) consists of a transmissive inner circle, and a transmissive outer annulus that imparts a phase change of $\pi$ radians relative to the inner circle. The inner circle and the outer annulus have the same area. The surrounding background is opaque.


4 (a) randomly oriented opaque apertures



4 (b) opaque square



4 (c)


## Problem 2

Below we have a set of nine transmissive apertures: A, B, S, L, Z, E, X, 2 and P. Match the appropriate aperture with its magnitude-squared Fourier transform (far-field intensity diffraction pattern) by inserting the letter or character in the box below its corresponding diffraction pattern. If you are not sure about an answer, explain your reasoning for partial credit. No credit will be given for illegible characters.

ABSLZE2XP


## Problem 3

Some of the following patterns were obtained in the laboratory by plotting the intensity as a function of space across a screen placed in the far-field of various diffracting apertures. In the spaces provided at the bottom of the next page, match the patterns to the apertures that produced them:

(1) multiple slit grating $\qquad$
(2) double slit grating $\qquad$
(3) single slit grating $\qquad$
(4) sinusoidal amplitude grating
(5) two partially-coherent point sources $\qquad$ $\sim \cap N A$ A
(i)
(6) phase-only grating $\qquad$
$\qquad$
(7) rectangular profile amplitude grating

## HOLOGRAPHY

## Problem 1

An ideal thick hologram is recorded, using the setup shown in Figure 1 below, with light of wavelength $\lambda$.

For the four read-out cases shown below,(a),(b),(c) and (d): (i) show the location of the output beams and (ii) label the beams (e.g., virtual image, real image, etc.). For (c) and (d) first draw in the readout beams before finishing these parts.


Figure 1. Recording geometry

(b) Reference wave readout

(c) Conjugate-of-object-wave readout


Show (i) readout wave, (ii) location of output beams and (iii) label output beams
(d) Optimal reference-beam readout configuration with a plane wave of wavelength $1.5 \lambda$

thick hologram

Show (i) readout angle $\alpha$, (ii) location of output beams and (iii) label output beams

THE END

