Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.685 Electric Machines

Problem Set 3 Solutions October 2, 2005

Problem 1: Since the RMS value of internal voltage is:

$$E_{af} = \frac{\omega M I_f}{\sqrt{2}}$$

we know the mutual inductance:

$$M = \frac{\sqrt{2} E_{af}}{\omega I_{fnl}} = \frac{8000\sqrt{2}}{377 \times 1000} = .03\text{Hy} = 30\text{mHy}$$

Problem 2: The short circuit test will give:

$$I_a = \frac{E_{af}}{\omega L_s} = \frac{M I_{fsi}}{\sqrt{2} L_s}$$

which gives synchronous inductance:

$$L_s = \frac{M I_{fsi}}{\sqrt{2} I_a} = \frac{.03 \times 2000}{\sqrt{2} \times 20000} = 2.12\text{mHy}$$

Note that

$$X_s = \omega L_s = 0.8\Omega$$

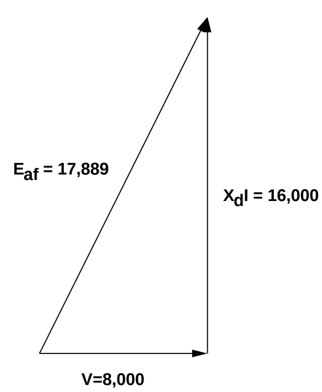

Figure 1: Phasor Diagram: Unity Power Factor, Rated Power

Problem 3: The phasor diagram for rated power, unity power factor operation is shown in Figure 1. Note voltage drop across the terminal inductance is $20,000A \times 0.8\Omega = 16,000V$. Internal Voltage is easily seen to be 17,889 V, so required field current is:

$$I_f = \frac{17,889 \times \sqrt{2}}{377 \times .03} = 2,237A$$

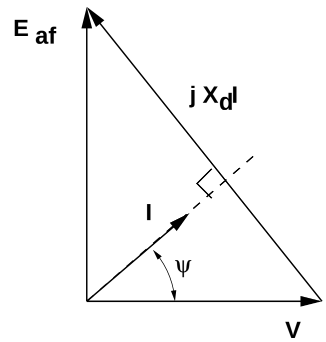

Figure 2: Phasor Diagram at underexcited stability limit

Problem 4: Now, to find the stability limit for underexcited operation, see that, when the torque angle is 90°, the phasor diagram must look like what is shown in Figure 2. It is pretty clear that, for power factor angle ψ,

$$X_d I_a \sin\psi = V$$

We may easily solve this for current: the limit for each power factor:

$$I_a = \frac{V}{X_d}\frac{1}{\sin\psi}$$

This evaluates to, for values of $\cos\psi$ of .4, .6 and .8 to be: 5,455 A, 6,250 A and 8,333 A, respectively. Since real power is

$$P = 2VI_a\cos\psi$$

we have real power of 52.4 MW, 90 MW and 160 MW for the three cases.

Problem 5: To get the required field current, start with the internal voltage:

$$E_f^2 = (V + X_s I_a \sin\psi)^2 + (X_s I_a \cos\psi)^2$$

and then

$$I_f = \frac{\sqrt{2}E_f}{\omega M} = I_{fnl}\frac{E_f}{V}$$

The results are shown in Figure 3. Using power factor as a parameter, the equation for E_f, above, is solved for a range of terminal current I_a ranging from zero to a limit which is either the armature rating (20,000 A) or the stability limit for underexcited operation. The resulting field current is plotted against real power.

Extra Note that we have ignored the field current limit. (The problem solution did not ask about field limit). However, field windings synchronous machines are usually sized so that the machine can reach rated armature current only for overexcited power factors greater than some 'rating point'. For the purposes of this we assume a power factor at the rating point of 0.8. With that the per-unit field current limit is:

$$e_{afl}^2 = (v + x_d i_a \sin\psi)^2 + (x_d i_a \cos\psi)^2$$

which, for $v = 1$ and $i_a = 1$ and $\cos\psi = 0.8$ evaluates to about 2.7203. Using the law of cosines we have, for operation at some oprating point (actually this is the same equation):

$$e_{afl}^2 = v^2 + (x_d i_a)^2 + 2vx_d i_a \sin\psi$$

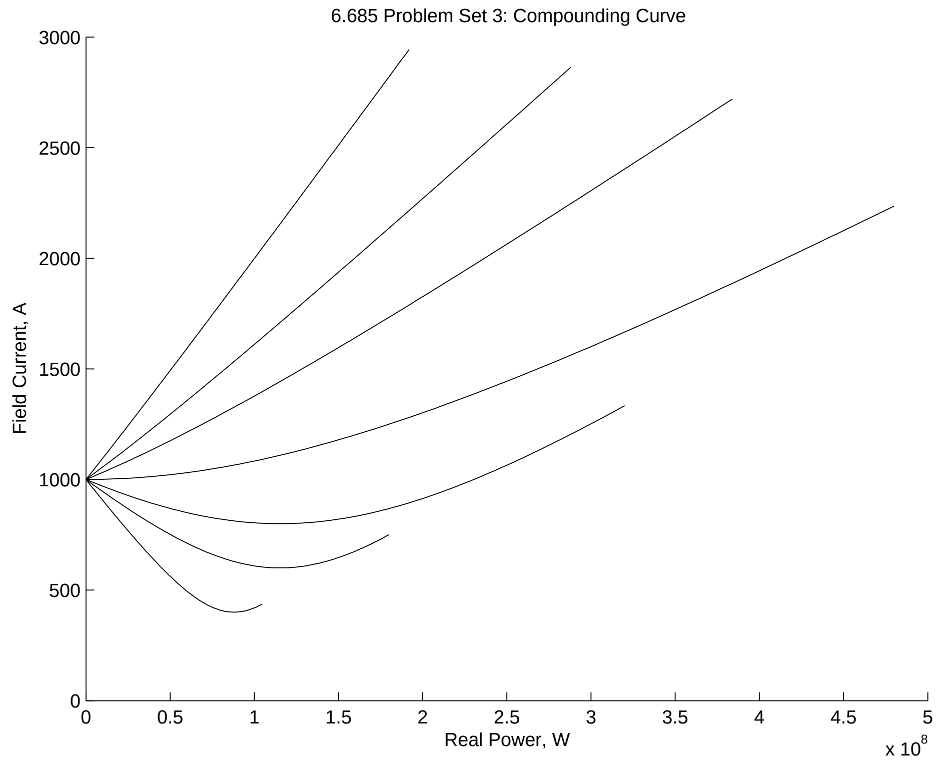

Figure 3: Compounding Curve, Ignoring Field Current Limit

which can be inverted to produce the limiting value of i_a:

$$i_{al} = \sqrt{(\frac{\sin\psi}{x_d})^2 + \frac{e_{afl}^2 - 1}{x_d^2}} - \frac{\sin\psi}{x_d}$$

This limit has been built into the script that generates the compounding curve, and the resulting curve is shown in Figure 4.

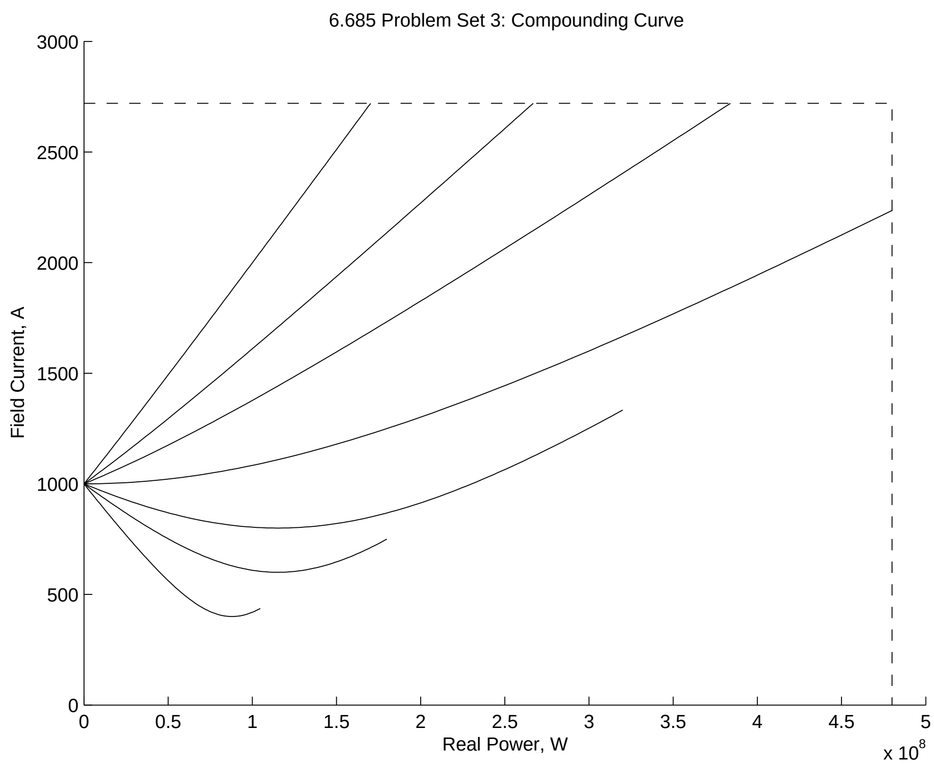

Figure 4: Compounding Curve, Field Current Limit Included

```
% Compounding Curve for a round rotor machine

% parameters
Ifnl = 1000;
V0 = 8000;
Xd = .8;
Ial = 20000;

% set up power factor angles:
Psi = [ acos(.4) acos(.6) acos(.8) 0 -acos(.8) -acos(.6) -acos(.4)];

figure(1)
hold on

for i =1:length(Psi)
    psi = Psi(i);
    if psi<0
        Ias = (V0/Xd)/sin(-psi);
    else
        Ias = Ial;
    end
    Il = min(Ias, Ial);
    I = 0:Il/100:Il;

    Ef = sqrt((V0 + Xd*sin(psi) .* I) .^2 + (Xd*cos(psi) .* I) .^2);
    If = (Ifnl/V0) .* Ef;
    P = 3*V0*cos(psi) .* I;
    plot(P, If)
end
title('6.685 Problem Set 3: Compounding Curve')
ylabel('Field Current, A');
xlabel('Real Power, W')
```

```
% Compounding Curve for a round rotor machine
% field current limit is included in this script
% parameters
Ifnl = 1000;
V0 = 8000;
Xd = .8;
Ial = 20000;
pfrat = .8;

% find limiting eaf:
xd = Xd*Ial/V0;                    % per-unit
psil = acos(pfrat);                % power factor angle at limit
eafl = sqrt((1+xd*sin(psil))^2 + (xd*cos(psil))^2);

% set up power factor angles:
Psi = [ acos(.4) acos(.6) acos(.8) 0 -acos(.8) -acos(.6) -acos(.4)];

figure(1)
clf
hold on

for i =1:length(Psi)
    psi = Psi(i);
    if psi<0
        Ias = (V0/Xd)/sin(-psi);
    else
        ias = sqrt((sin(psi)/xd)^2 + (eafl^2-1)/xd^2)- sin(psi)/xd
        Ias = ias*Ial;
    end
    Il = min(Ias, Ial);
    I = 0:Il/100:Il;

    Ef = sqrt((V0 + Xd*sin(psi) .* I) .^2 + (Xd*cos(psi) .* I) .^2);
    If = (Ifnl/V0) .* Ef;
    P = 3*V0*cos(psi) .* I;
    plot(P, If)
end
Ifl = Ifnl*eafl;
Pal = 3*Ial*V0
plot([0 Pal Pal], [Ifl Ifl 0], '--')
title('6.685 Problem Set 3: Compounding Curve')
ylabel('Field Current, A');
xlabel('Real Power, W')
```