

Massachusetts Institute of Technology  
Department of Electrical Engineering and Computer Science  
6.685 Electric Machines

Problem Set 5

Issued October 2, 2005  
Due October 12, 2005

**Problem 1:** This problem concerns the same fanciful machine for which you plotted magnetic field in Problem Set 4, Problem 2. This is a three-phase, four-pole, round rotor machine with a rotor diameter of two meters and an active length of three meters. The air-gap is five centimeters. The stator winding consists of three phases, and we will consider only the case of a single, concentrated winding (one slot per pole per phase). The winding is driven by a balanced, three-phase current source of 10,000 A, peak, per phase, at 60 Hz. The rotor has a concentrated, full-pitch winding (we might call this a 'search coil' as it is not a real field winding). The rotor is turning with a rotational speed of 3,600 RPM (60 Hz).

Noting that the space fundamental waveform is synchronous with the rotor, we know that it will induce zero voltage. The lowest frequency component of voltage induced in the rotor will be sixth (time) harmonic, induced by some combination of the fifth and seventh space harmonics. For the purpose of this problem you may consider only the effects of fifth and seventh space harmonics.

1. Find the sixth harmonic voltage induced in that single rotor turn.
2. Note that the rotor, while turning at synchronous speed, may have a variable position (phase) with respect to the rotating flux wave of the stator. What are the maximum and minimum values of the induced voltage?
3. Plot the magnitude (peak amplitude) of the sixth harmonic voltage against the rotor phase angle. (that is, the position of the rotor with respect to the stator rotating flux wave).

**Problem 2:** Write a script to evaluate the torque-speed curve of an induction motor and use it to calculate and plot the motoring torque-speed curve of a ten horsepower motor with the following parameters:

Voltage	V	480	Line-line, RMS
Frequency	F	60	Hz
Number of Pole Pairs	p	2	
Stator Leakage Reactance	$X_1$	1.15	$\Omega$
Stator Resistance	$R_1$	0.5	$\Omega$
Rotor Leakage Reactance	$X_2$	3.4	$\Omega$
Rotor Resistance	$R_2$	0.3	$\Omega$
Magnetizing Reactance	$X_m$	116	$\Omega$

Plot torque over the range of speed from zero to 1,800 RPM.

Now: assume that through some mechanism the rotor resistance can be varied according to the formula:

$$R_2 = R_{2DC} \left( 1 + \sqrt{\frac{f_R}{f_0}} \right)$$

where  $f_R$  is frequency seen from the rotor and  $f_0$  is the base frequency (60 Hz).

Compute and plot the resulting torque/speed curve.

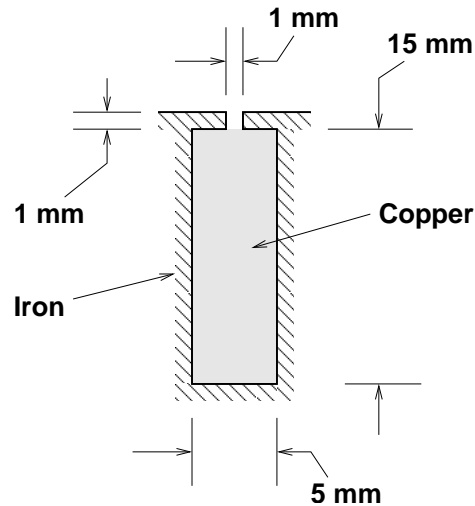


Figure 1: Phasor Diagram: Unity Power Factor, Rated Power

**Problem 3:** Figure 1 shows a highly simplified 'deep bar' as might be used in an induction motor in the fashion of the previous problem. The shaded region, 5 mm wide and 15 mm deep is a copper bar with conductivity  $5.8 \times 10^7$  S/m. It is in a slot in an iron surface which you should consider has infinite permeability. There is a slot 'depression' that is 1 mm square. The objective of this problem is to estimate the impedance, reactive and resistive, of this slot. That impedance (per unit length) is the axial electric field divided by total current in the slot.

Write a script which calculates the impedance of this bar (resistance and reactance per unit length) as a function of frequency. Plot a standard Bode plot of this impedance and a second plot of resistance per unit length as a function of frequency. (A Bode plot is a log-log plot of impedance magnitude vs. frequency and a separate plot of phase angle. The phase angle plot is linear in the phase angle against a logarithmic axis of frequency. Your resistance plot should be log-log. Run these plots over a frequency range of 0.1 Hz to 100 Hz.