6.728 Applied Quantum and Statistical Physics

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PROBLEM SET 1

Issued: 9-5-02 Due: 9-11-02, in-class

Problem 1.1 The goal of this course is to enable you to apply quantum and statistical physics to your everyday research. To this end, numerical analysis software can greatly enhance your ability to solve complex problems, especially those that have no exact solutions. Your first task, therefore, is to get familiar with a numerical analysis program such as `Matlab`. The class homepage has links to `Matlab` on Athena.

Problem 1.2 (You might want use a numerical analysis program for this problem.)

A 6.728 professor has decided to employ a probabilistic algorithm to determine the students' grades. The algorithm is simple: the professor rolls two dice (regular six-faced) for each student. The grade is determined by the sum of the two dice, and then multiplied by 100/N, where N is the maximum possible sum.

(a) Determine the probability *P(i)* that a student will achieve a score of i for the class.

(b) Make a bar plot of *P(i)* for $0 \le i \le 100$. You will find the `Matlab` functions `bar` ( ) or `stem` ( ) useful.

(c) Determine the mean $\langle i \rangle$ and the variance $\left\langle \left(i - \langle i \rangle\right)^2 \right\rangle$ for this distribution.

The professor now decides to use the sum of five dice for each student.

(d) Let the computer roll the dice for you (by using a random number generator such as `rand` ( )). After sufficiently many tries, plot a histogram of the so obtained scores, `hist` ( ). The histogram should resemble your results from part (b).

(e) Determine the mean $\langle i \rangle$ and the variance $\left\langle \left(i - \langle i \rangle\right)^2 \right\rangle$ for this distribution.

Problem 1.3 The Fourier transform and its inverse are given by

$$A(x) = \int_{-\infty}^{\infty} A(q) e^{iqx} \frac{dq}{2\pi}$$
$$A(q) = \int_{-\infty}^{\infty} A(x) e^{-iqx} dx$$

(a) Prove the following properties from the above definition:

(i) $A(bx)$ has the Fourier transform $\frac{1}{|b|} A(q/b)$

(ii) $A(x - x_0)$ has the Fourier Transform $e^{-iqx_0} A(q)$

(iii) $\frac{d}{dx} A(x)$ has the Fourier Transform $iqA(q)$.

(b) Find the Fourier Transform $A(q)$ of the following by using the Formula Sheet. Note that we use the physics convention, and that tables are typically published in terms of t and ω rather than in terms of x and q.

(i) $A(x) = xe^{-x^2}$

(ii) $A(x) = e^{ikx}$

(iii) $A(x) = \sin(kx)\, e^{-(x/b)^2}$

Problem 1.4

(a) What is the size of the smallest object which you will be able to see with a microscope. In order to answer this, you need to know that your eyes can see light of wavelengths ranging from l $\lambda = 380$ nm to $\lambda = 780$ nm.

(b) Researchers at a number of laboratories around the world have developed lasers which operate in the extreme ultraviolet and soft x-ray regimes. One such laser emits photons which have an energy of about 285 eV, which is interesting since at this energy water and protein "look" very different (water transmits and protein absorbs). Can this laser be used as a source for detailing protein structures. Explain.

(c) What level of resolution would be afforded by an <u>electron</u> microscope operating at 50 eV.