6.728 Applied Quantum and Statistical Physics

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PROBLEM SET 10

Issued: 11-22-02 — Due: 12-04-02, in-class

Problem 10.1 Problem 23.3 in the notes.

Problem 10.2 Problem 25.2 in the notes.

Problem 10.3 Problem 26.1 in the notes.

Problem 10.4 Problem 26.2 in the notes. Use the temperature dependent approximation for $N_c(T)$ given in problem 26.1

Problem 10.5

A particle in a box with a density of electrons n has a Fermi level at zero temperature of $E_{F_0} = k_B T_0$.

1. Show that the chemical potential, μ, is a solution to

$$0 = 1 - \frac{3}{2}\left(\frac{T}{T_0}\right)^{3/2} \int_0^\infty \frac{x^{1/2}}{1 + e^{x - \mu/k_B T}}\, dx$$

2. Plot μ vs. T for a metal where $T_0 = 50{,}000\,\text{K}$ in the range of $0 \le T \le 10^5\,\text{K}$.
3. Plot the fermi-dirac distribution function, $f(E - \mu)$, for

$$T = \{500\ \text{K},\ 5000\,\text{K},\ 10^4\,\text{K},\ 5 \times 10^4\,\text{K},\ 10^5\,\text{K}\}$$

that is, plot $f(E - \mu)$ vs. E/k_B so that energy is expressed as a temperature.

4. Plot $-\frac{\partial f}{\partial E}$ for the same temperature range.
5. Plot the variance of $-\frac{\partial f}{\partial E}$ vs. temperature.
6. Plot $\langle E \rangle$ vs. temperature for $0 \le T \le 10^5\,\text{K}$.
7. What is $\langle E \rangle$ at zero temperature and at high temperatures. Explain.