6.728 Applied Quantum and Statistical Physics

Department of Electrical Engineering and Computer Science
Massachusetts Institute of Technology

PROBLEM SET 8

Issued: 11-01-02 Due: 11-06-02, in-class

PROBLEMS

Problem 8.1 A Couple of SHO's.

Consider the system shown in Figure 1: a LC circuit and a mass on a spring where one end of the spring is connected to one of the capacitor plates. Assume that the mass and spring do not affect the capacitance of the capacitor.

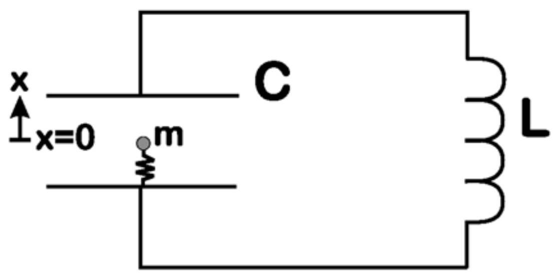

Figure 1: LC circuit with mass on a spring.

(a) For the case in which the mass is charge neutral, the Hamiltonian of the system is

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_0^2\hat{x}^2 + \frac{1}{2}L\hat{i}^2 + \frac{1}{2}C\hat{v}^2$$

What is the Hamiltonian written in terms of creation and annihilation operators?

(b) What is the ground state wave function of this system? What is the ground state energy?

(c) Consider now the case in which the mass has a charge $+e$. For small displacements of the mass, argue that the interaction term can be written as

$$\hat{V} = -\frac{e}{d}\hat{x}\hat{v}$$

where d is the gap between the capacitor plates.

What is this term in creation and annihilation operator notation?

(d) Consider a two-basis state approximation:

$$\Psi = c_1\psi_0\phi_1 + c_2\psi_1\phi_0$$

where ψ are the eigenstates of the LC circuit and ϕ are eigenstates of the mass on a spring. What are the elements of the Hamiltonian matrix below?

$$\begin{pmatrix} H_{11} & V \\ V & H_{22} \end{pmatrix}$$

(e) Consider the two-basis state approximation above. Under what conditions do you expect the wave function to oscillate completely from one basis state to the other?

Problem 8.2 Particle in a Two Dimensional Well.

A particle is confined in one dimension by an infinite square well of width W and in a second dimension by a harmonic potential. The specific Hamiltonian for this system is:

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \hat{V}(x) + \frac{\hat{p}_y^2}{2m} + \frac{1}{2}m\omega_0^2\hat{y}^2$$

where

$$\begin{aligned} \hat{V}(x) &= 0 \quad \text{for } 0 > x > W \\ &= \infty \quad \text{for } x \leq 0 \text{ and } x \geq W \end{aligned}$$

(a) What are the eigenfunctions and eigenenergies for this system?
(b) How many quantum states are there with total energy less than $\frac{3}{2}\hbar\omega_0$?
(c) A particle in this potential is described as a wavepacket at $t = 0$ with the following functional form:

$$\Psi(x, y, 0) = Ce^{-(x-W/2)^2/2L^2}e^{-(y-y_0)^2/2L^2}$$

where C is a normalization constant, $y_0 \neq 0$, $L = \sqrt{\hbar/2m\omega_0}$ and $W >> L$. Describe the behavior of this wavefunction for $t > 0$, both on a time scale on the order of $1/\omega_0$, and a time scale much longer than $1/\omega_0$.

Problem 8.3

Do problem 19.1 from page 412 of the notes.