

6.730 PHYSICS FOR SOLID STATE APPLICATIONS

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PROBLEM SET 2

Issued: 2-15-02

Due: 2-22-02, at the beginning of class.

Readings:

PSSA Chapter 3

PSSA Chapter 4

Problem 2.1 *Classical Limit of Fermi-Dirac Statistics* PSSA Problem 3.1 parts a—d

This problem shows how if one assumes the Boltzmann factor (Eqn. 3.77), then the classical results that we quote many times in class follow.

Problem 2.2 *Two Dimensional Electron Gas* PSSA Problem 3.2

Problem 2.3 *Two-dimensional elastic continuum* PSSA Problem 4.5

This problem helps you review the arguments that were done in class and extend them to a different dimension.

Hint: $\mathbf{T} = \lambda e\mathbf{I} + 2\mu\mathbf{E}$.

Problem 2.5 *Conductivity of a Free Electron Gas in a Magnetic Field: The Hall Effect*

Consider an electron gas whose motion is confined to the $x - y$ plane, such as a thin film of a metal or the inversion layer in a MOSFET. Let a magnetic field $\mathbf{B} = B\hat{z}$ be applied along the z -axis as shown in the figure. Assume that the forces on the electron are the Lorentz force ($-e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$) and the drag force ($-m\mathbf{v}/\tau$), where τ is the scattering time.

1. Show that the electron satisfies the following equations of motion:

$$\frac{dv_x}{dt} + \frac{v_x}{\tau} = -\frac{e}{m}E_x - \omega_c v_y$$

$$\frac{dv_y}{dt} + \frac{v_y}{\tau} = -\frac{e}{m}E_y + \omega_c v_x$$

where $\omega_c = |e|B/m$. Notice that the effect of the magnetic field is to couple the x and y motion.

2. Show that

$$\begin{pmatrix} E_x(\omega) \\ E_y(\omega) \end{pmatrix} = \begin{pmatrix} \frac{1-i\omega\tau}{\sigma} & \frac{\omega_c\tau}{\sigma} \\ -\frac{\omega_c\tau}{\sigma} & \frac{1-i\omega\tau}{\sigma} \end{pmatrix} \begin{pmatrix} J_x(\omega) \\ J_y(\omega) \end{pmatrix}$$

where $\sigma = ne^2\tau/m$, n is the density of electrons, and the current density $\mathbf{J} = -en\mathbf{v}$. Note that Ohm's law is a tensor relationship in general.

3. In the Hall geometry, current is applied along the x – *direction* but no current flows along the y -direction ($J_y = 0$). (That is, there are leads for the current at the x -boundaries of the sample, but none at the y -boundary) In this case show that one has an electric field (and hence a voltage) in both the x and y directions, even though current flows only along the x direction; that is, show that

$$E_x(\omega) = [\sigma(\omega)]^{-1} J_x(\omega)$$

$$E_y(\omega) = [R_H B]^{-1} J_x(\omega)$$

Here $\sigma(\omega)$ is the usual Drude conductivity and $R_H = 1/ne$ is the Hall coefficient. This result implies that in a magnetic field a current in the x -direction gives rise not only to a voltage in the x -direction, but also to one in the y -direction.

4. Estimate the Hall coefficient R_H for copper.