

6.730 PHYSICS FOR SOLID STATE APPLICATIONS

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PROBLEM SET 6

Issued: 4-18-01

Due: 4-27-00, at the beginning of class.

Readings:

PSSA Chapter 9 and 10

Problem 6.1 2DEG from a Confining Potential (20 points)

This problem explores how a two dimensional electron gas (2DEG) in the conduction band can arise from a confining potential.

The conduction band for a three dimensional semiconductor has an energy $E(\mathbf{k})$ for energies greater than the conduction band minimum E_c given by

$$E(\mathbf{k}) = E_c + \frac{\hbar^2}{2} \left(\frac{k_x^2}{m_1} + \frac{k_y^2}{m_2} + \frac{k_z^2}{m_3} \right).$$

The electrons in the conduction band are now subjected to a confining potential which we model as a particle-in-a-box potential which restricts the electrons to a box in the z -direction of width L ,

$$V(\mathbf{r}) = \begin{cases} 0 & \text{if } |z| < L \\ \infty & \text{otherwise} \end{cases}$$

where L is much larger than the lattice constant. Assume the effects of this potential can be treated within the effective mass theorem.

(a) Use the Effective Mass Theorem to find the total energy of the electron wavepacket near the bottom of the conduction band.

(b) Find and sketch the areal density of states $g(E)$ for electrons in the conduction band when their energy is given as in part (a). Note that $g(E)$ has dimensions of number of electrons per unit area per unit energy.

(c) Write down both the total time-dependent envelope function and the total time-dependent wavefunction for the wavepacket for electrons with $k_z = \pi/2L$ but arbitrary k_x and k_y . Such electrons are said to be in the *first subband* and form a two dimensional electron gas.

(d) Assume that $\hbar^2 \pi^2 / 8L^2 \gg k_B T$. How many electrons per unit area can be added to the system before the occupancy of the second sub-band becomes significant.

Problem 6.2 Ring around the Work Function.

A ring of three metals of different work functions is formed as shown below.

- (a) Which metal has the highest electrostatic potential? Explain.
- (b) The ring is cut at A , and a gap of 1000 \AA is opened up. What is the magnitude and direction of the electric field in the gap?

Problem 6.3

An abrupt heterojunction is constructed from two semiconductors having the following data:

	Semiconductor 1	Semiconductor 2
Electron Affinity	2.0 eV	2.5 eV
Band gap	1.8 eV	1.1 eV
Doping	$N_A = 10^{17} \text{ cm}^{-3}$	$N_D = 10^{15} \text{ cm}^{-3}$
$N_V = N_C$	10^{19} cm^{-3}	10^{18} cm^{-3}

Make a sketch of the bands at equilibrium. Show important features of the band structure in the vicinity of the junction, and show the position of the Fermi level relative to the band structure in each semiconductor.