6.730 PHYSICS FOR SOLID STATE APPLICATIONS

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PROBLEM SET 2

Issued: 2-13-04 Due: 2-20-04, at the beginning of class.

Readings:
PSSA Chapter 3
PSSA Chapter 4

Problem 2.1 *Conductivity of a Free Electron Gas in a Magnetic Field: The Hall Effect*

Consider an electron gas whose motion is confined to the $x-y$ plane, such as a thin film of a metal or the inversion layer in a MOSFET. Let a magnetic field $\mathbf{B} = B\hat{z}$ be applied along the z-axis. Assume that the forces on the electron are the Lorentz force $(-e(\mathbf{E} + \mathbf{v} \times \mathbf{B}))$ and the drag force $(-m\mathbf{v}/\tau)$, where τ is the scattering time.

1. Show that the electron satisfies the following equations of motion:

$$\frac{dv_x}{dt} + \frac{v_x}{\tau} = -\frac{e}{m}E_x - \omega_c v_y$$

$$\frac{dv_y}{dt} + \frac{v_y}{\tau} = -\frac{e}{m}E_y + \omega_c v_x$$

   where $\omega_c = |e|B/m$. Notice that the effect of the magnetic field is to couple the x and y motion.

2. Show that

$$\begin{pmatrix} E_x(\omega) \\ E_y(\omega) \end{pmatrix} = \begin{pmatrix} \frac{1-i\omega\tau}{\sigma} & \frac{\omega_c\tau}{\sigma} \\ \frac{-\omega_c\tau}{\sigma} & \frac{1-i\omega\tau}{\sigma} \end{pmatrix} \begin{pmatrix} J_x(\omega) \\ J_y(\omega) \end{pmatrix}$$

   where $\sigma = ne^2\tau/m$, n is the density of electrons, and the current density $\mathbf{J} = -en\mathbf{v}$. Note that Ohm's law is a tensor relationship in general.

3. In the Hall geometry, current is applied along the $x - direction$ h but no current flows along the y-direction ($J_y = 0$). (That is, there are leads for the current at the x-boundaries of the sample, but none at the y-boundary) In this case show that one has an electric field (and hence a voltage) in both the x and y directions, even though current flows only along the x direction; that is, show that

$$E_x(\omega) = [\sigma(\omega)]^{-1} J_x(\omega)$$

$$E_y(\omega) = [R_H B]^{-1} J_x(\omega)$$

   Here $\sigma(\omega)$ is the usual Drude conductivity and $R_H = 1/ne$ is the Hall coefficient. This result implies that in a magnetic field a current in the x-direction gives rise not only to a voltage in the x-direction, but also to one in the y-direction.

4. Estimate the Hall coefficient R_H for copper.

Problem 2.2 *Two-dimensional elastic continuum* **PSSA Problem 4.5**

This problem helps you review the arguments that were done in class and extend them to a different dimension.

Hint: $\mathbf{T} = \lambda e \mathbf{I} + 2\mu \mathbf{E}$.

Problem 2.3 *Lattice Mismatched Epitaxy* **PSSA Problem 4.8**

Also do the following additional part:

(f) The elastic energy is given by

$$W = \int dv \; (E_{xx}T_{xx} + E_{yy}T_{yy} + E_{zz}T_{zz} + E_{xy}T_{xy} + E_{zx}T_{zx} + E_{yz}T_{yz})$$

Show that

$$W = V \frac{\mu(3\lambda + 2\mu)}{\lambda + 2\mu} E_{\parallel}^2 = V \frac{E_Y}{1-\nu} E_{\parallel}^2$$

where V is the volume, and E_Y is Young's modulus.