6.730 PHYSICS FOR SOLID STATE APPLICATIONS

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PROBLEM SET 6

Issued: 4-23-04 Due: 4-30-04, at the beginning of class.

Problem 6.1 Total Conductivity Tensor for for bulk Si

The 6 constant energy surfaces for energies just above the conduction band edge in silicon are shown below.

These surfaces are ellipsoidal and are described by two effective masses; m_t for the transverse mass and m_ℓ for the longitudinal mass. For example, for the ellipsoid along the z-axis, the surface of constant energy is

$$E(\mathbf{k}) = E_c + \frac{\hbar^2}{2}\left(\frac{(k_x - k_{x,0})^2}{m_t} + \frac{(k_y - k_{y,0})^2}{m_t} + \frac{(k_z - k_{z,0})^2}{m_\ell}\right).$$

(a) The acceleration $\mathbf{a}$ response to an external driving force $\mathbf{F}$ is given in terms of the the inverse of the effective mass tensor $\mathbf{M}_j^{-1}$ for each pocket j; namely, $\mathbf{a}_j = \mathbf{M}_j^{-1}\mathbf{F}$. Show that the total acceleration responds isotropically to the force; that is, $\mathbf{a}_{\text{total}} = M^{-1}\mathbf{F}$, where M is a scalar. Be sure to calculate the contributions from all six ellipsoids and find M explicitly.

(b) What is the numerical value of this effective mass ratio M/m_e for for Si.

(c) Refer to part (a) and write out the total effective mass tensor for the conductivity of electrons in Si explicitly.

(d) Do the same for the holes, assuming; that there are two hole bands with different effective masses. Also give a numerical value for the effective (conductivity) mass ratio for the holes in Si.

Problem 6.2 2DEG from a Confining Potential in Si

This problem explores how a two dimensional electron gas (2DEG) in the conduction band in Si can arise from a confining potential. The electrons in the conduction band of Si are subjected to a slowly varying potential caused by band bending. Model this potential as

$$V(\mathbf{r}) = \frac{1}{2} m_3 \omega_o^2 z^2 \qquad \text{for} \qquad z \leq 0$$

where ω_o is a positive constant of dimensions of inverse seconds. Assume the effects of this potential can be treated within the effective mass theorem.

(a) Use the Effective Mass Theorem to find the total energy of the electron wavepacket near the bottom of the conduction band for the ellipsoidal surface along the positive z-direction.

(b) Find and sketch the areal density of states $g(E)$ for these electrons in the conduction band when their energy is given as in part (a). Note that $g(E)$ has dimensions of number of electrons per unit area per unit energy.

(c) Write down both the total time-dependent envelope function and the total time-dependent wavefunction for the wavepacket for electrons with $k_z = \pi/2L$ but arbitrary k_x and k_y. Such electrons are said to be in the *first subband* and form a two dimensional electron gas.

(d) All 6 ellipsoidal energy pockets for electrons in the conduction band of Si are subjected to this confining potential. Let all the electrons be in the first subband so that they form a two dimensional electron gas (2DEG). If the electric field is confined in the xy-plane, find the corresponding conductivity tensor.

(e) What is the numerical value for this effective mass for Si. How does it with the full 3d conductivity effective mass and for the effective masses for 3D and 2D density of states?

Problem 6.3 Auger Recombination

A three-particle recombination process is described as follows

$$\text{electron } 1 + \text{electron } 2 + \text{hole} \Longrightarrow \text{one energetic electron}$$

In effect, the other or "Auger" electron carries off the necessary momentum and energy required for an electron and hole to recombine in an indirect gap material.

Assume that the recombination rate is of the form $R = An^2p$ where A is a constant.

Write the rate equation for the hole concentration, including both the generation and recombination terms, and show that for n-type material in low-level injection, the lifetime of excess carriers is $\tau_p = 1/An_0^2$, where n_0 is the equilibrium carrier concentration.