6.730 PHYSICS FOR SOLID STATE APPLICATIONS

Department of Electrical Engineering and Computer Science
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PROBLEM SET 11

Issued: 5-1-96 Due: 5-8-96, at the beginning of class.

Problem 11.1 Total Conductivity Tensor for for bulk Si.
(a) Refer to Problem 10.1 and write out the effective mass tensor for electrons for Si explicitly.

(b) Do the same for the holes, assuming; that there are two hole bands with different effective masses. Also give a numerical value for the effective (conductivity) mass ratio for the holes in Si.

Problem 11.2 Conductivity Tensor for a 2DEG in Si.

The electrons in the conduction band of Si are subjected to a slowly varying potential caused by band bending. Model this potential as

$$V(\mathbf{r}) = \frac{1}{2} m_3 \omega_o^2 z^2 \qquad \text{for} \qquad z \leq 0$$

where ω_o is a positive constant of dimensions of inverse seconds.

(a) Use the Effective Mass Theorem to find the total energy of a wave packet near the bottom of the conduction band for each of the 6 ellipsoids.

(b) Let all the electrons be in the first subband so that they form a two dimensional electron gas (2DEG). If the electric field is confined in the xy-plane, find the corresponding conductivity tensor.

(c) What is the numerical value for this effective mass for Si. How does if compare with that in Problem 10.1 for the full 3d conductivity effective mass and for the effective masses for 3D and 2D density of states.

Problem 11.3 A Work Function Thickness Monitor? (former quiz problem).

Two thin slabs of aluminum, each having dimensions $10\,\mu\text{m} \times 10\,\mu\text{m}$ are positioned parallel to each other, separated by a distance of 1000 Å. They are electrically connected to each other through an ammeter using copper wires. A short pulse of gold atoms, enough to create a coating about 10 Å thick, is directed toward the gap between the plates at a grazing angle so that the gold atoms only strike the surface of the lower plate.

In principle, is it possible to detect the arrival of the pulse of gold atoms with the ammeter?

Explain your answer. If you decide that the answer is yes, then indicate which way the current must flow through the ammeter (lower-to-upper, or upper-to-lower), and estimate the total area under the current pulse (that is, estimate the total charge transfer that results from the arrival of the gold).

Problem 11.4 Diffusion Constant.
Using the results of solving the Boltzmann transport equation in the relaxation time approximation, calculate the room temperature electron diffusion constant for a semiconductor having a spherical parabolic band with $m^* = 1$, and assuming a uniform, constant relaxation time of 10^{-12} sec. Compare your result with the electron diffusion constant of lightly doped silicon.

Problem 11.5 Auger Recombination.

A three-particle recombination process is described as follows

$$\text{electron } 1 + \text{electron } 2 + \text{hole} \Longrightarrow \text{one energetic electron}$$

In effect, the other or "Auger" electron carries off the necessary momentum and energy required for an electron and hole to recombine in an indirect gap material.

Assume that the recombination rate is of the form $R = An^2p$ where A is a constant.

Write the rate equation for the hole concentration, including both the generation and recombination terms, and show that for n-type material in low-level injection, the lifetime of excess carriers is $\tau_p = 1/An_0^2$, where n_0 is the equilibrium carrier concentration.

Problem 11.6 The Hall Effect.

This problem generalizes the discussion of solving the Boltmann equation in Chapter 11 (*Non-equilibrium conditions: Transport*). Consider a uniform material and with no $(\partial f/\partial t)_{\text{trans}} = 0$ in an applied constant electric and magnetic fields so that an electron obeys the Lorentz force law,

$$\mathbf{F} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})\,.$$

(a) Show that to first order in the electric field $\mathbf{E}$, that the change in the distribution function $f_1 = f - f_0$ is given in the relaxation time approximation by

$$f_1 = \tau \left\{ -\frac{e}{k_B T} f_0(1-f_0)\mathbf{v}\cdot\mathbf{E} + \frac{1}{\hbar}(\mathbf{v}\times\mathbf{B})\cdot\nabla_{\mathbf{k}} f_1 \right\} .$$

(b) To find a solution to the above equation, let

$$f_1 = -\frac{e\tau}{k_B T} f_0(1-f_0)\mathbf{v}\cdot\mathbf{A}$$

where $\mathbf{A}$ to be determined. Show that $\mathbf{A}$ satisfies the equation

$$\mathbf{v}\cdot\mathbf{A} = \mathbf{E}\cdot\mathbf{A} - e\tau(\mathbf{v}\times\mathbf{B})\cdot\nabla_{\mathbf{k}}(\mathbf{v}\cdot\mathbf{A}) .$$

Now assume the effective mass approximation, where $E_n(\mathbf{k}) = E_o + (\hbar^2/2m^*)k^2$. Show that

$$\mathbf{A} = \frac{\mathbf{E} - \frac{e\tau}{m^*}\mathbf{B}\times\mathbf{E}}{1 + (\frac{e\tau B}{m^*})^2} .$$

(c) Argue that the current density is

$$\mathbf{J} = \sigma_0 \mathbf{A}$$

where σ_0 is the conductivity resulting from solving the same problem with the electric field only. Show that this results in

$$\mathbf{E} = \frac{1}{\sigma_0}\mathbf{J} + \frac{e\tau}{m^*\sigma_0}\mathbf{B}\times\mathbf{J} .$$

How would your answer change for holes?

(d) Let the magnetic field be applied perpendicular to the current. Find the resistivity along the direction of the current $\rho_{\|} = E_{\|}/J$, and the resistivity perpendicular to the current (the Hall resistivity) $\rho_{\mathrm{Hall}} = E_{\perp}/J$. Give your answer for electrons, "electrons", and "holes." Be careful of the signs.

Final is Thursday, May 23, 1996 from 1:30 pm to 4:30 pm in room 16-310.