6.730 PHYSICS FOR SOLID STATE APPLICATIONS

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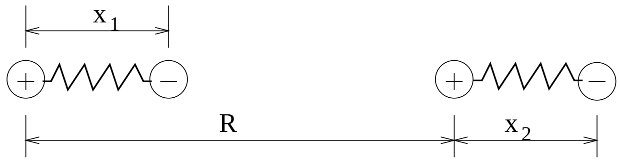
PROBLEM SET 2

Issued: 2-14-96 Due: 2-21-96, at the beginning of class.

Readings:
Aschroft and Mermin: Chapter 2.
6.728 Statistical Physics Notes: 1.3 and 3.2.
Senturia: 6.730 Course Notes, Chapter 3.

Problem 2.1 van der Waals Binding Energy.

Inert gases can bind to form molecular solids by inducing electrical dipole moments on each other. As a simplified model consider two atoms at a distance R apart. The electrons are modeled as bound by springs to their respective atoms; the natural frequency is ω_o. The electrons only move along the x-axis, see the Figure below.



Coordinates of the two oscillations

In the absence of coulomb interactions, the hamiltonian for the two uncoupled electrons with mass m is

$$\hat{\mathcal{H}}_o = \frac{p_1^2}{2m} + \frac{1}{2}m\omega_o^2 x_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}m\omega_o^2 x_2^2 \,.$$

Now adding the coulomb interactions gives an additional piece to the hamiltonian

$$\hat{\mathcal{H}}_1 = \frac{e^2}{4\pi\epsilon_o R} + \frac{e^2}{4\pi\epsilon_o(R + x_2 - x_1)} - \frac{e^2}{4\pi\epsilon_o(R + x_2)} - \frac{e^2}{4\pi\epsilon_o(R - x_1)}$$

(a) For $|x_1|$, $|x_2| \ll R$, show that

$$\hat{\mathcal{H}}_1 \approx -\frac{e^2 x_1 x_2}{2\pi\epsilon_o R^3} \,.$$

(b) This Hamiltonian ($\mathcal{H} = \mathcal{H}_o + \mathcal{H}_1$) can be solved exactly by making the change of variables to

$$y_1 = \frac{x_1 + x_2}{\sqrt{2}} \qquad y_2 = \frac{x_1 - x_2}{\sqrt{2}}$$

and similarly for the momentum.

$$P_1 = \frac{p_1 + p_2}{\sqrt{2}} \qquad P_2 = \frac{p_1 - p_2}{\sqrt{2}}$$

Then

$$\hat{\mathcal{H}} = \frac{P_1^2}{2m} + \frac{1}{2}m\omega_1^2 y_1^2 + \frac{P_2^2}{2m} + \frac{1}{2}m\omega_2^2 y_2^2 \,.$$

Find ω_1 and ω_2.

(c) The binding energy E_B due to the interaction can be approximated as the difference in the zero point energy of the two oscillations with and without the coulomb forces, that is,

$$E_{\mathrm{B}} = \frac{1}{2}\hbar(\omega_1 + \omega_2 - 2\omega_o) \,.$$

Assuming the correction is small, show that the van der Waals binding energy is $E_{\mathrm{B}} = -\frac{A}{R^6}$. Find A.

Problem 2.2 Heat Capacity for 2D electron gas. (statistical physics review problem).

(a) Given a 2D electron gas with density of n, show that its chemical potential is constant with temperature (ignore high order terms of T^4).

(b) Find the heat capacity of this 2D electron gas as a function of temperature.

Problem 2.3 Heat Capacity for 3D free electron gas (statistical physics review problem).

A particle in a box with a density of electrons n has a Fermi level at zero temperature of E_{F_0}.

(a) Using Sommerfeld expansion, show that for 3D electron gas, the chemical potential vs.T is

$$\mu(T) = E_{Fo}\left[1 - \frac{1}{3}\left(\frac{\pi k_B T}{2E_{Fo}}\right)^2\right] \,.$$

(b) What is the heat capacity of this 3D electron gas as a fucntion of temperature? How is it compared with the 2D case?

Problem 2.4 The Real World.

Look up data (in library if necessary) to answer the following questions (all at room temperatute). Cite your sources.

(a) Which is more compressible, water or polycrystalline aluminum?

(b) Which is "stronger" in terms of Young's modulus, aluminum or silicon?

(c) Which has the higher Young's modulus? aluminum or amorphous silicon dioxide? which has higher shear modulus? higher bulk modulus?

(d) If metals did not conduct electricity, would we be inclined to use them in devices?