6.730 PHYSICS FOR SOLID STATE APPLICATIONS

Department of Electrical Engineering and Computer Science
Massachusetts Institute of Technology

PROBLEM SET 9

Issued: 4-17-96 Due: 4-24-96, at the beginning of class.

Readings:
Semiclassical Equations of Motion: Chapter 9.4 in 6.730 notes and Chapter 12 in Ashcroft and Mermin.
Electrons and Holes.

Problem 9.1 This problem explores how a two dimensional electron gas (2DEG) in the conduction band can arise from a confining potential. (A former final problem.)

The conduction band for a three dimensional semiconductor has an energy $E(\mathbf{k})$ for energies greater than the conduction band minimum E_c given by

$$E(\mathbf{k}) = E_c + \frac{\hbar^2}{2}\left(\frac{k_x^2}{m_1} + \frac{k_y^2}{m_2} + \frac{k_z^2}{m_3}\right) .$$

The electrons in the conduction band are now subjected to a slowly varying potential

$$V(\mathbf{r}) = \frac{1}{2} m_3 \omega_o z^2$$

where ω_o is a positive constant of dimensions of inverse seconds.

(a) Use the Effective Mass Theorem to show that the total energy of the electron wavepacket near the bottom of the conduction band is given by

$$E = E_c + \frac{\hbar^2}{2}\left(\frac{k_x^2}{m_1} + \frac{k_y^2}{m_2}\right) + (\gamma + \frac{1}{2})\hbar\omega_o \, ,$$

where $\gamma = 0, 1, 2, 3, \ldots$.

(b) What is the areal density of states $g(E)$ for electrons in the conduction band when their energy is given as in part (a). Note that $g(E)$ has dimensions of number of electrons per unit area per unit energy.

(c)Write down the total time-dependent wavefunction for the wavepacket for electrons with $\gamma = 0$ but arbitrary k_x and k_y. Such electrons are said to be in the *first subband* and form a two dimensional electron gas.

(d) Assume there are n_{2D} electrons per unit area in the first subband. What is the maximum density such that there are no electrons in any other subband. You may assume that $\hbar\omega_o \gg k_B T$.

Problem 9.2 Do attached problem.

Final is Thursday, May 23, 1996 from 1:30 pm to 4:30 pm in room 16-310.