Problem Set #3 - Solutions

1) a) The important approximations are:

$$\int_0^\infty \varphi(E)\,\frac{dE}{1+\exp[\beta(E-\mu)]} = \int_0^\mu \varphi(E)\,dE + \frac{\pi^2}{6}(kT)^2\,\frac{d\varphi(E)}{dE}\Big|_\mu$$

$$\int_0^\infty \varphi(E)\left(-\frac{\partial f_0(E)}{\partial E}\right)dE = \varphi(\mu) + \frac{\pi^2}{6}(kT)^2\,\frac{d^2\varphi(E)}{d^2E}\Big|_\mu$$

b) (i) $g(E) = \dfrac{m^*}{\pi\hbar^2}$ $\left(\dfrac{\#\text{ state}}{\text{energy}\cdot\text{area}}\right)$ derived in recitation

(ii) $\displaystyle\int_0^{E_F} g(E)\,dE = n$ (at $T=0$)

$m^* E_F = n\pi\hbar^2$

$E_F(0K) = \dfrac{n\pi\hbar^2}{m^*}$

$$\int_0^\infty g(E)\,f(E-E_F)\,dE = \frac{m^*\mu}{\hbar^2\pi} + \frac{\pi^2}{6}(kT)^2\cdot\emptyset = n$$

$\mu(T) = \dfrac{n\pi\hbar^2}{m^*}$

(iii)
$$\sigma = e^2\int \tau\, v\cdot v\left(-\frac{\partial f_0}{\partial E}\right)\frac{d^2k}{2\pi^2}$$

$$= e^2\tau\int v\cdot v\left(-\frac{\partial f_0}{\partial E}\right)\frac{d|k|}{2\pi^2}\cdot 2\pi|k|$$

$$= e^2\tau\int \frac{|v^2|}{2}\left(-\frac{\partial f_0}{\partial E}\right)\frac{d|k|^2}{2\pi}$$

$$= e^2\tau\int \frac{E}{m^*}\left(-\frac{\partial f_0}{\partial E}\right)\cdot\frac{m^*}{\pi\hbar^2}\,dE$$

$$= \frac{e^2\tau}{m^*}\cdot\frac{m^*}{\pi\hbar^2}(\mu+0) = \frac{ne^2\tau}{m^*}$$

(iv) $$S = \frac{1}{eT}\,\frac{K_1}{K_0} = \frac{e}{T}\,\frac{K_1}{\sigma}$$

$$K_1 = \int \tau v v \left(-\frac{\partial f_0}{\partial E}\right)(E-\mu)\,\frac{d^2k}{2\pi^2}$$

$$= \tau \int \frac{|v^2|}{2}\left(-\frac{\partial f_0}{\partial E}\right)(E-\mu)\,\frac{2\pi|k|\,d|k|}{2\pi^2}$$

$$= \tau \int \frac{E}{m^*}\left(-\frac{\partial f_0}{\partial E}\right)(E-\mu)\,\frac{\pi\, d|k|^2}{2\pi^2}$$

$$= \frac{\tau}{m^*}\int E(E-\mu)\left(-\frac{\partial f_0}{\partial E}\right)\cdot\frac{m^*}{\pi\hbar^2}\,dE$$

$$= \frac{\tau}{m^*}\cdot\frac{m^*}{\pi\hbar^2}\left(\frac{2\pi^2}{6}(kT)^2\right)$$

$$S = \frac{e}{T}\cdot\frac{1}{e^2}\cdot\frac{\pi^2(kT)^2}{3\mu} = \frac{\pi^2 k}{3e}\cdot\frac{kT}{\mu}$$

2. Extension for $T \neq 0$.

We define small T as $kT \ll E_{overlap}$ so that

$E_c < E_F < E_v$

$$n_1 = \frac{1}{2\pi^2}\left(m_{et}^2 m_{el}\right)^{1/2}\left(\frac{2\sqrt{2}}{\hbar^3}\right)\left[\frac{2}{3}E_F^{e\,3/2} + \frac{\pi^2}{6}(kT)^2\frac{1}{2(E_F^e)^{1/2}}\right]$$

$$p = \underbrace{\frac{1}{2\pi^2}\left(m_{ht}^2 m_{hl}\right)^{1/2}\left(\frac{2\sqrt{2}}{\hbar^3}\right)}_{g_h}\left[\frac{2}{3}E_F^{h\,3/2} + \frac{\pi^2}{6}(kT)^2\frac{1}{2(E_F^h)^{1/2}}\right]$$

$p = 3n_1$

$$g_h \cdot \frac{1}{\sqrt{E_F^h}}\left(\frac{2}{3}(E_F^h)^2 + \frac{\pi^2}{12}(kT)^2\right) = 3g_e\frac{1}{\sqrt{E_F^e}}\left(\frac{2}{3}(E_F^e)^2 + \frac{\pi^2}{12}(kT)^2\right)$$

Assume: $\sqrt{E_F^h} \sim \sqrt{E_F^h(T=0)}$ $\qquad E_F^h = E_{over} - E_F^e$

$$g_h \cdot \frac{1}{\sqrt{E_{over} - E_F^e(0)}}\left(\frac{2}{3}E_{over}^2 - \frac{4}{3}E_F^e E_{over} + \frac{2}{3}(E_F^e)^2 + \frac{\pi^2}{12}(kT)^2\right) -$$

$$-3g_e\frac{1}{\sqrt{E_F^e(0)}}\left(\frac{2}{3}(E_F^e)^2 + \frac{\pi^2}{12}(kT)^2\right) = 0$$

Solve quadratic equation $E_F^e(T)$

Find most significant term in $n(T)$: $E_F^{3/2}$ or $\dfrac{(kT)^2}{E_F^{1/2}}$

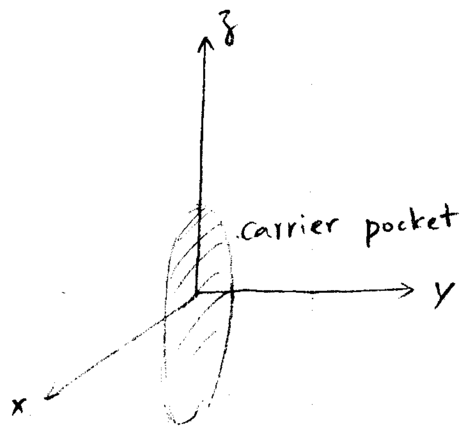
T-dependence of τ = T-dependence of n

2:

a) The carrier density in "one" electron pocket is

$$n_1 = \frac{1}{4\pi^3} \int f_0(E)\, d^3k .$$

We choose the axes of the carrier pocket as shown in the figure.



Let

$$k_x' = \sqrt{\frac{m_0}{m_{et}}}\, k_x$$

$$k_y' = \sqrt{\frac{m_0}{m_{et}}}\, k_y \qquad k_z' = \sqrt{\frac{m_0}{m_{el}}}\, k_y$$

Then

$$E(\vec{k}') = \frac{\hbar^2}{2m_0} k'^2$$

and

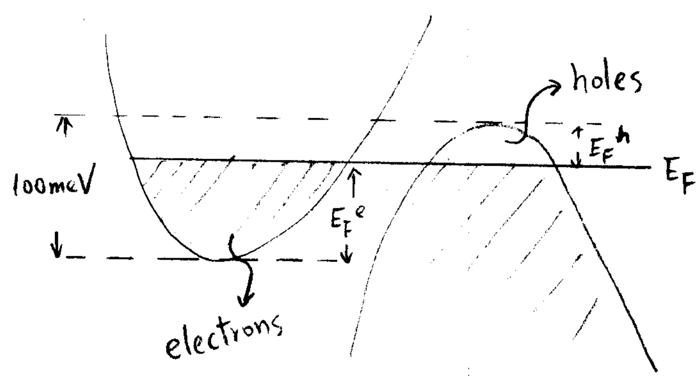
$$d^3k = \left(\frac{m_{et}^2 m_{el}}{m_0^3}\right)^{\frac{1}{2}} d^3k'$$

$$= \left(\frac{m_{et}^2 m_{el}}{m_0^3}\right)^{\frac{1}{2}} \cdot 2\pi \left(\frac{2m_0}{\hbar^2}\right)^{3/2} E^{1/2}\, dE$$

$$\therefore n_1 = \frac{1}{2\pi^2} \left(\frac{m_{et}^2 m_{el}}{m_0^3}\right)^{1/2} \left(\frac{2m_0}{\hbar^2}\right)^{3/2} \int_0^\infty E^{1/2} \cdot \frac{dE}{1+e^{(E-E_F^e)/kT}}$$

where $E=0$ is the bottom of conduction bands

If $|E_F^e| \gg kT$, we can assume that all states below E_F^e are essentially filled and all states above E_F^e are empty. Then we obtain



$$n_1 = \frac{1}{2\pi^2} \left(\frac{m_{et}^2 m_{el}}{m_0^3}\right)^{1/2} \left(\frac{2m_0}{\hbar^2}\right)^{3/2} \cdot \frac{2}{3} E_F^{e\ 3/2}$$

Similarly, for hole density we have

$$P = \frac{1}{2\pi^2} \left(\frac{m_{ht}^2 m_{hl}}{m_0^3}\right)^{1/2} \left(\frac{2m_0}{\hbar^2}\right)^{3/2} \cdot \frac{2}{3} E_F^{h\ 3/2},$$

in which $E_F^e + E_F^h = E_{over} = 100\,\text{meV}$

The number of electrons in the conduction band should equal to that of holes in the valence band. However, there are three equivalent electron carrier pockets,

$\therefore \quad p = n = 3n_1$

$$\Rightarrow \quad (m_{ht}^2 m_{hl})^{1/2} \cdot (E_{over} - E_F^e)^{3/2} = 3(m_{et}^2 m_{el})^{1/2} \, E_F^{e\,3/2}$$

$$\Rightarrow \quad E_F^e = 69.05\ meV$$

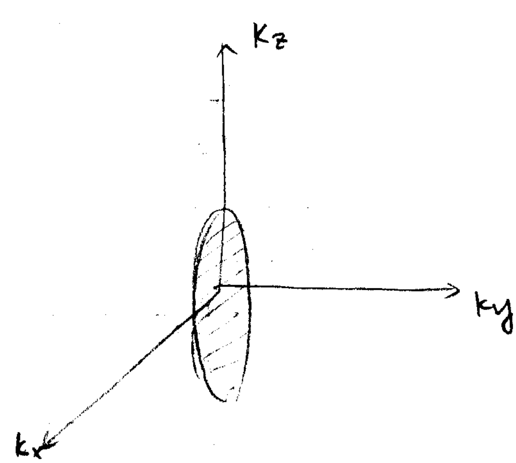
and $\quad E_F^h = 30.95\ meV$

Substitute E_F^e into n_1 and p, we have

$$n = 3n_1 = 2.5 \times 10^{24} \quad (1/m^3)$$
$$= 2.5 \times 10^{18} \quad (cm^{-3})$$
$$p = n = 2.5 \times 10^{18} \quad (cm^{-3}) \quad \#$$

b) We first look at the conductivity of a single electron carrier pocket. In the coordinate system of the pocket, (as shown below), the conductivity is

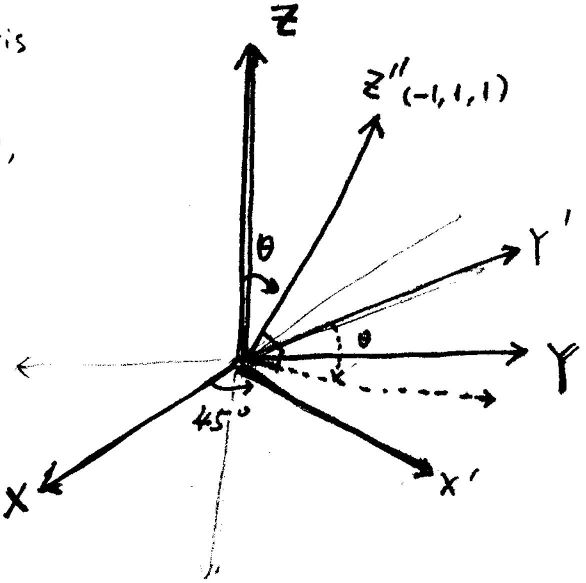
$$\overleftrightarrow{\sigma}_{pocket} = n_1 e^2 \tau_e \begin{pmatrix} \frac{1}{m_{et}} & 0 & 0 \\ 0 & \frac{1}{m_{et}} & 0 \\ 0 & 0 & \frac{1}{m_{el}} \end{pmatrix}$$



However, we have to transform the coordinate of the pocket axes to the Laboratory coordinate. And this transformation is different for each carrier pocket.

(i) For pocket along the $(1, \bar{1}, \bar{1})$ direction, we want to find a rotation R matrix so that R will rotate z-axis into $(-1, 1, 1)$ direction. (as shown next page).

To do this, we can rotate about z-axis 45° and have new x', y' axes, and then rotate about x' axis for angle θ, so that Z is rotated into Z'' axis.



As for θ, we have

$$\cos\theta = \frac{1}{\sqrt{3}}(-1,1,1)(0,0,1) = \frac{1}{\sqrt{3}}$$

The rotation matrix R is obtained by

$$R = \begin{pmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

and $$R^{-1} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\sqrt{\frac{2}{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

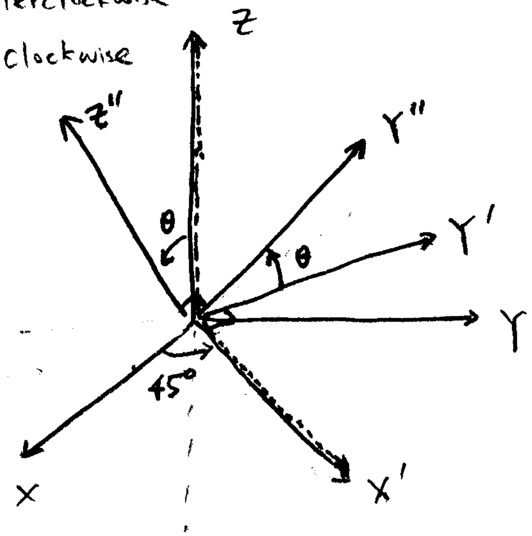
The electrical conductivity in the laboratary coordinate system is derived by

$$\overleftrightarrow{\sigma}_{1\bar{1}\bar{1}(Lab)} = R\,\overleftrightarrow{\sigma}_{pocket}\,R^{-1} = \frac{n_1 e^2 \tau_e}{3} \begin{pmatrix} \frac{2}{m_{et}} + \frac{1}{m_{el}} & \frac{1}{m_{et}} - \frac{1}{m_{el}} & \frac{1}{m_{et}} - \frac{1}{m_{el}} \\ \frac{1}{m_{et}} - \frac{1}{m_{el}} & \frac{2}{m_{et}} + \frac{1}{m_{el}} & \frac{-1}{m_{et}} + \frac{1}{m_{el}} \\ \frac{1}{m_{et}} - \frac{1}{m_{el}} & \frac{-1}{m_{et}} + \frac{1}{m_{el}} & \frac{2}{m_{et}} + \frac{1}{m_{el}} \end{pmatrix}$$

(ii) For electron pocket along $(\bar{1}, 1, \bar{1})$, we look for a rotation matrix that will tranform $(0,0,1)$ into $(1,-1,1)$ direction.

To do this, we first rotate about Z-axis counterclockwise for $45°$, and then rotate about X'-axis counterclockwise for angle θ.

The rotation matrix is then



$$R = \begin{pmatrix} \cos 45° & -\sin 45° & 0 \\ \sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ 0 & \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

$$R^{-1} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \sqrt{2/3} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 1/\sqrt{3} \end{pmatrix}$$

The conductivity of the electron pocket along $(\bar{1} 1 \bar{1})$ in the laboratory coordinate is

$$\overleftrightarrow{\sigma}_{\bar{1}1\bar{1}\,(\text{Lab})} = R \overleftrightarrow{\sigma}_{\text{pocket}} R^{-1}$$

$$= \frac{n_1 e^2 \tau_e}{3} \begin{pmatrix} \frac{2}{m_{et}} + \frac{1}{m_{el}} & \frac{1}{m_{et}} - \frac{1}{m_{el}} & \frac{-1}{m_{et}} + \frac{1}{m_{el}} \\ \frac{1}{m_{et}} - \frac{1}{m_{el}} & \frac{2}{m_{et}} + \frac{1}{m_{el}} & \frac{1}{m_{et}} - \frac{1}{m_{el}} \\ \frac{-1}{m_{et}} + \frac{1}{m_{el}} & \frac{1}{m_{et}} - \frac{1}{m_{el}} & \frac{2}{m_{et}} + \frac{1}{m_{el}} \end{pmatrix}$$

(iii) For election pocket along $(\bar{1}\bar{1}1)$ direction. the rotation matrix is chosen as:

$$R = \begin{pmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & -1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & -1/\sqrt{3} \\ 0 & \sqrt{2/3} & 1/\sqrt{3} \end{pmatrix}$$

and

$$\overleftrightarrow{\sigma}_{\bar{1}\bar{1}1} = R\overleftrightarrow{\sigma}_{pocket}R^{-1} = \frac{n_i e^2 \tau_e}{3} \begin{pmatrix} 2/m_{et} + 1/m_{el} & -1/m_{et} + 1/m_{el} & 1/m_{et} - 1/m_{el} \\ -1/m_{et} + 1/m_{el} & 2/m_{et} + 1/m_{el} & 1/m_{et} - 1/m_{el} \\ 1/m_{et} - 1/m_{el} & 1/m_{et} - 1/m_{el} & 2/m_{et} + 1/m_{el} \end{pmatrix}$$

(iv) For hole pocket along (111) direction, we can follow the same procedure and choose the rotation matrix R as

$$R = \begin{pmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -\sqrt{2/3} & 1/\sqrt{3} \end{pmatrix}$$

And

$$\overleftrightarrow{\sigma}_{111} = R\overleftrightarrow{\sigma}_{h\,pocket}R^{-1} = \frac{p e^2 \tau_h}{3} \begin{pmatrix} 2/m_{ht} + 1/m_{hl} & -1/m_{ht} + 1/m_{hl} & -1/m_{ht} + 1/m_{hl} \\ -1/m_{ht} + 1/m_{hl} & 2/m_{ht} + 1/m_{hl} & -1/m_{ht} + 1/m_{hl} \\ -1/m_{ht} + 1/m_{hl} & -1/m_{ht} + 1/m_{hl} & 2/m_{ht} + 1/m_{hl} \end{pmatrix}$$

The conductivity is the sum of the conductivity of each carrier pocket,

$$\therefore \overleftrightarrow{\sigma} = \overleftrightarrow{\sigma}_{1\bar{1}\bar{1}} + \overleftrightarrow{\sigma}_{\bar{1}1\bar{1}} + \overleftrightarrow{\sigma}_{\bar{1}\bar{1}1} + \overleftrightarrow{\sigma}_{111}$$

$\therefore$ We get $\quad (3n_1 = n = p)$

$$\overleftrightarrow{\sigma} = \frac{ne^2\tau_e}{3}\begin{pmatrix} 2/m_{et} + 1/m_{el} & 1/3m_{et} - 1/3m_{el} & 1/3m_{et} - 1/3m_{el} \\ 1/3m_{et} - 1/3m_{el} & 2/m_{et} + 1/m_{el} & 1/3m_{et} - 1/3m_{el} \\ 1/3m_{et} - 1/3m_{el} & 1/3m_{et} - 1/3m_{el} & 2/m_{et} + 1/m_{el} \end{pmatrix}$$

$$+ \frac{pe^2\tau_h}{3}\begin{pmatrix} 2/m_{ht} + 1/m_{hl} & 1/m_{hl} - 1/m_{ht} & 1/m_{hl} - 1/m_{ht} \\ 1/m_{hl} - 1/m_{ht} & 2/m_{ht} + 1/m_{hl} & 1/m_{hl} - 1/m_{ht} \\ 1/m_{hl} - 1/m_{ht} & 1/m_{hl} - 1/m_{ht} & 2/m_{ht} + 1/m_{hl} \end{pmatrix}$$

$$= \frac{ne^2}{3m_0}\begin{pmatrix} 201\tau_e + 21\tau_h & 33\tau_e - 9\tau_h & 33\tau_e - 9\tau_h \\ 33\tau_e - 9\tau_h & 201\tau_e + 21\tau_h & 33\tau_e - 9\tau_h \\ 33\tau_e - 9\tau_h & 33\tau_e - 9\tau_h & 201\tau_e + 21\tau_h \end{pmatrix}$$

$$= \frac{ne^2}{m_0}\begin{pmatrix} 67\tau_e + 7\tau_h & 11\tau_e - 3\tau_h & 11\tau_e - 3\tau_h \\ 11\tau_e - 3\tau_h & 67\tau_e + 7\tau_h & 11\tau_e - 3\tau_h \\ 11\tau_e - 3\tau_h & 11\tau_e - 3\tau_h & 67\tau_e + 7\tau_h \end{pmatrix}$$

Let $\vec{E} = (1,1,1)\frac{E}{\sqrt{3}}$

$$\vec{J} = \overleftrightarrow{\sigma}\cdot\vec{E} = \frac{ne^2}{m_0}\frac{E}{\sqrt{3}}\begin{pmatrix} 89\tau_e + \tau_h \\ 89\tau_e + \tau_h \\ 89\tau_e + \tau_h \end{pmatrix} = \frac{ne^2}{m_0}(89\tau_e + \tau_h)\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}\frac{E}{\sqrt{3}}$$

which is parallel to $\vec{E}$!

$\therefore$ The conductivity along (111) is

$$\sigma_{111} = \frac{ne^2}{m_0}(89\tau_e + \tau_h) \quad \# \quad \checkmark$$

3.

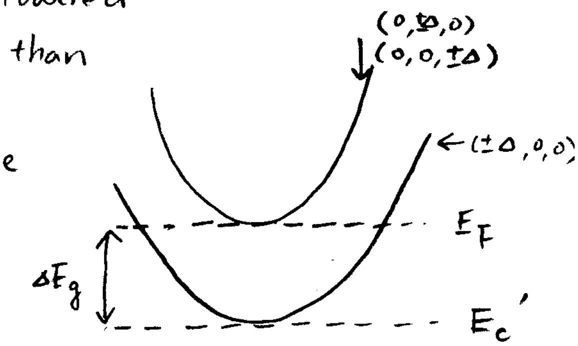
a) In the highly degenerate Si, the carrier density of "one" carrier pocket can be caculated just like a metal:

$$n_{(1)} = \int f_0(E)\, d^3 k$$

Using the technique as in Problem (2), we have

$$n_{(1)} = \frac{1}{2\pi^2}\left(\frac{m_t^2 m_\ell}{m_0^3}\right)^{1/2}\left(\frac{2m_0}{\hbar^2}\right)^{3/2}\int_0^\infty E^{1/2}\cdot\frac{dE}{1+e^{(E-E_F)/kT}}$$

Again we assume that all states below E_F are completely filled and the states above E_F are empty. When we apply a stress Σ along $(1,0,0)$, the conduction band $(\Delta,0,0)$ and $(-\Delta,0,0)$ are lowered while the other four bands remain unchanged. Therefore, if all the carriers are in the lowered $(\pm\Delta,0,0)$ bands, E_F must be at least lower than the bottom of the four unchanged conduction band. From the figure we see that the change of band gap ΔE_g in $(\pm\Delta,0,0)$ satisfies



$$\Delta E_g \geq |E_F - E_c'|$$

The carrier density in the lowered conduction bands is

$$n = 2n_{(1)} = 2\,\frac{1}{2\pi^2}\left(\frac{m_t^2 m_\ell}{m_0^3}\right)^{1/2}\left(\frac{2m_0}{\hbar^2}\right)^{3/2}\cdot\frac{2}{3}E_F^{3/2}$$

$\therefore\ E_F \simeq 23.6\ \text{meV}$

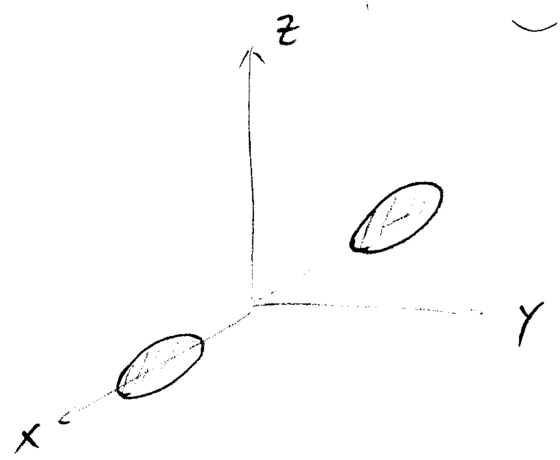
$\therefore\ \Delta E_g \gtrsim 23.6\ \text{meV}$

Since $\dfrac{\partial E_g}{\partial \Sigma} = \alpha\Sigma \qquad \therefore\ \Delta E_g = \dfrac{\alpha}{2}\Sigma^2$

$\therefore\ \Sigma \gtrsim \left(\dfrac{2E_F}{\alpha}\right)^{1/2}$ for all carriers to be $(\pm\Delta,0,0)$. Conduction bands. #

b) The electrical conductivity of "one" carrier pocket is

$$\overleftrightarrow{\sigma}_1 = n_{(1)} e^2 \tau \begin{pmatrix} \frac{1}{m_\ell} & 0 & 0 \\ 0 & \frac{1}{m_t} & 0 \\ 0 & 0 & \frac{1}{m_t} \end{pmatrix}$$



The two pockets have the same electrical conductivity tensor (see figure), therefore the total conductivity is

$$\overleftrightarrow{\sigma} = 2\overleftrightarrow{\sigma}_1 = 2n_{(1)} e^2 \tau \begin{pmatrix} \frac{1}{m_\ell} & 0 & 0 \\ 0 & \frac{1}{m_t} & 0 \\ 0 & 0 & \frac{1}{m_t} \end{pmatrix} = ne^2\tau \begin{pmatrix} \frac{1}{m_\ell} & 0 & 0 \\ 0 & \frac{1}{m_t} & 0 \\ 0 & 0 & \frac{1}{m_t} \end{pmatrix}$$

Let $\vec{E} = \frac{E}{\sqrt{2}}(0, 1, 1)$

$$\vec{j} = \overleftrightarrow{\sigma} \cdot \vec{E} = ne^2\tau \cdot \frac{E}{\sqrt{2}} \begin{pmatrix} 0 \\ \frac{1}{m_t} \\ \frac{1}{m_t} \end{pmatrix} = \frac{ne^2\tau}{m_t} \cdot \frac{E}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \frac{ne^2\tau}{m_t} \vec{E}$$

which is parallel to $\vec{E}$!!

$\therefore$ The conductivity along (011) is

$$\sigma_{011} = \frac{ne^2\tau}{m_t} \qquad \left[\text{if we assume } \tau \simeq 10^{-14} \text{ (sec)}\right]$$

$$= 8.88 \times 10^3 \quad (\Omega\text{-m})^{-1}$$

c) To calculate the Seeback Coefficient, we first have to calculate:

$$\overleftrightarrow{K}_0 = \frac{\overleftrightarrow{\sigma}}{e^2}$$

and

$$\overleftrightarrow{K}_1 = \frac{1}{4\pi^3} \int \tau \vec{v}\vec{v} (E - E_F) \left(-\frac{\partial f_0}{\partial E}\right) d^3k$$

For a ellipsoid constant energy surface, we use again the same technique in (a) (or problem 2).

Let $k_x' = \sqrt{\frac{m_0}{m_l}}\, k_x$, $k_y' = \sqrt{\frac{m_0}{m_t}}\, k_y$, $k_z' = \sqrt{m_0/m_t}\, k_z$

$$\vec{v} = \frac{1}{\hbar}\frac{\partial E}{\partial \vec{k}} = \frac{\hbar k_x}{m_l}\hat{x} + \frac{\hbar k_y}{m_t}\hat{y} + \frac{\hbar k_z}{m_t}\hat{z}$$

$$\therefore v_x v_x = \frac{\hbar^2 k_x^2}{m_l^2} = \frac{1}{m_l}\cdot\frac{\hbar^2 k_x'^2}{m_0}$$

Similarly $v_y v_y = \frac{1}{m_t}\cdot\frac{\hbar^2 k_y'^2}{m_0}$, $v_z v_z = \frac{1}{m_t}\frac{\hbar^2 k_z'^2}{m_0}$

$$\therefore (K_1)_{xx} = \frac{m_0}{m_l}\cdot\frac{1}{4\pi^3}\int \tau\left(\frac{\hbar^2 k_x'^2}{m_0^2}\right)(E-E_F)\left(-\frac{\partial f_0}{\partial E}\right)d^3k$$

$$= \frac{1}{m_l}\cdot\frac{1}{2\pi^2}\left(\frac{m_t^2 m_l}{m_0^3}\right)^{1/2}\left(\frac{2m_0}{\hbar^2}\right)^{3/2}\int_0^\infty \tau\cdot\left(\frac{2}{3}E\right)\cdot(E-E_F)\left(-\frac{\partial f_0}{\partial E}\right)E^{1/2}\,dE$$

($\because$ the integrate of $k_x'^2$ over a sphere of radius k' is $\frac{1}{3}k'^2$)

$$\int_0^\infty \tau\cdot\left(\frac{2}{3}E\right)(E-E_F)\left(-\frac{\partial f_0}{\partial E}\right)E^{1/2}\,dE$$ (assume τ is a constant)

$$= \frac{2\tau}{3}\int_0^\infty E^{3/2}(E-E_F)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$= \frac{2\tau}{3}\cdot\frac{\pi^2}{6}(kT)^2\cdot\left(2\cdot\frac{3}{2}E_F^{1/2}\right) = \tau\cdot\frac{\pi^2}{6}(kT)^2 E_F^{1/2}$$

(refer to lecture notes on page. 92 for the evaluation of $\int G(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$)

$$\therefore (K_1)_{xx} = \frac{1}{m_l}\cdot\frac{1}{2\pi^2}\left(\frac{m_t^2 m_l}{m_0^3}\right)^{1/2}\left(\frac{2m_0}{\hbar^2}\right)^{3/2}\tau\cdot\frac{\pi^2}{3}(kT^2)E_F^{1/2}$$

$$= \frac{1}{m_l}\cdot\frac{n_{(1)}}{2}\frac{(\pi kT)^2}{E_F}\tau$$

For $(K_1)_{yy}$, $(K_1)_{zz}$, we have similar results:

$$(K_1)_{yy} = \frac{1}{m_t}\cdot\frac{n_{(1)}}{2}\frac{(\pi kT)^2}{E_F}\tau \qquad (K_1)_{zz} = \frac{1}{m_t}\frac{n_{(1)}}{2}\cdot\frac{(\pi kT)^2}{E_F}\tau$$

However, since the integration of $k_x'k_y'$ over a sphere vanishes, we have

$$\overleftrightarrow{K}_{1(1)} = \frac{n_{(1)}}{2}\frac{(\pi kT)^2}{E_F}\tau\begin{pmatrix}\frac{1}{m_l} & 0 & 0\\ 0 & \frac{1}{m_t} & 0\\ 0 & 0 & \frac{1}{m_t}\end{pmatrix}$$

Since there are two identical carrier pockets, we have

$$\overleftrightarrow{K}_1 = 2\overleftrightarrow{K}_{1(1)} = \frac{n}{2}\frac{(\pi kT)^2}{E_F}\tau\begin{pmatrix}\frac{1}{m_l} & 0 & 0\\ 0 & \frac{1}{m_t} & 0\\ 0 & 0 & \frac{1}{m_t}\end{pmatrix}$$

$$\overleftrightarrow{S} = \frac{1}{eT}\,\overleftrightarrow{K_0}^{-1}\cdot\overleftrightarrow{K_1}$$

$$= \frac{1}{eT}\quad\frac{1}{n\tau}\begin{pmatrix} m_\ell & 0 & 0 \\ 0 & m_t & 0 \\ 0 & 0 & m_t \end{pmatrix}\cdot\frac{n}{2}\,\frac{(\pi k T)^2}{E_F}\begin{pmatrix} \frac{1}{m_\ell} & 0 & 0 \\ 0 & \frac{1}{m_t} & 0 \\ 0 & 0 & \frac{1}{m_t} \end{pmatrix}$$

$$= \frac{\pi^2 k^2 T}{2e\,E_F}\,\overleftrightarrow{\mathbb{1}} \quad \#$$

4(a) $E_d = 13.6 \frac{m^*}{m_0 \varepsilon^2} = 13.6 \frac{\sqrt[3]{m_t^2 m_l}}{m_0 \varepsilon^2} \sim 10^{-5}\, eV \sim k_B$

For all relevant temperatures, the donor impurities will be ionized.

(b) Since the conduction and valence bands mirror each other $E_F(\text{intrinsic}) = \frac{1}{2} E_g = 0.155\, eV$ independent of T.

$$n_i = 4 \int_{E_c}^{\infty} g(E) f_0(E - E_F)\, dE$$

$$= 4 \frac{\sqrt{2}}{\hbar^3 \pi^2} \sqrt{m_t^2 m_l} \int_0^{\infty} \sqrt{E}\, f(E - E_F)\, dE$$

$$= \frac{4\sqrt{2}}{\pi^2 \hbar^3} \sqrt{m_t^2 m_l} \cdot \left[\int_0^{\infty} \sqrt{E}\, e^{-\frac{1}{kT}(E - E_F)}\, dE \right]$$

$$= 6.244 \cdot 10^{54}\, J^{-3/2} m^{-3} \cdot \left[e^{E_F/kT} \cdot (kT)^{3/2} \cdot \int_0^{\infty} \sqrt{\epsilon}\, e^{-\epsilon}\, d\epsilon \right]$$

$$= 6.244 \cdot 10^{54} \cdot 2.6 \cdot 10^{-3} \cdot 2.7 \cdot 10^{-31} \cdot 0.89\ m^{-3}$$

$$= 3.8 \cdot 10^{21}\ m^{-3} = 3.8 \cdot 10^{15}\ cm^{-3}$$

(c) $n_e = 4 \int_0^{E_F} g(E)\, dE = 6.244 \cdot 10^{54} \int_0^{E_F} \sqrt{E}\, dE$

$$= 6.244 \cdot 10^{54} \cdot \frac{2}{3} (E_F)^{3/2} = 10^{16} \cdot 10^{6}$$

$E_F = 1.8 \cdot 10^{-22}\, J \cong 1\, meV$ @ 0K

At 300°K n_e and n_i are of about the same order of magnitude. We can predict E_F is in the bandgap and the Boltzman distribution is approximately correct.

$$n_e - n_h = 6.244 \cdot 10^{54} \cdot \left(e^{E_F/kT} - e^{-\frac{E_g + E_F}{kT}} \right) \cdot 2.7 \cdot 10^{-31} \cdot 0.89 = 10^{16} \cdot 10^{6}\ m^{-3}$$

$$e^{E_F/kT} - e^{-\frac{E_g+E_F}{kT}} = 6.66\cdot 10^{-3}$$

$$E_F = -0.130 \text{ eV}$$

$$n_e \sim 10^{16} \text{ cm}^{-3}$$

(d) $n_e = 6.244\cdot 10^{54}\cdot \frac{2}{3}(E_F)^{3/2} = 10^{25} \text{ m}^{-3}$ at $T = 0°K$

$$E_F = 1.8\cdot 10^{-20} \text{ J} = 0.112 \text{ eV}$$

Now we must use F.D. distribution:

$$n_e \sim 6.244\cdot 10^{54}\cdot \left[\frac{2}{3} E_F^{3/2} + \frac{\pi^2}{6}(kT)^2 \frac{1}{2\sqrt{E_F}} \right] = 10^{25}$$

Assume $E_F(300°K) \sim E_F(0°K)$:

$$\frac{\pi^2}{6}(kT)^2 \frac{1}{2\sqrt{E_F}} = 1.06\cdot 10^{-31} \text{ J}^{3/2}$$

$$E_F^{3/2} = 2.24\cdot 10^{-30} \Rightarrow E_F = 1.7\cdot 10^{-20} \text{ J} = 0.107 \text{ eV}$$

$$n_e \sim 10^{19} \text{ cm}^{-3}.$$