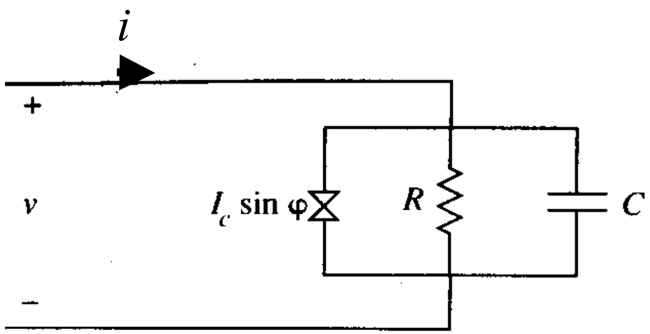
Josephson Circuits I.

Outline

1. RCSJ Model Review
2. Response to DC and AC Drives
   - Voltage standard
3. The DC SQUID
4. Tunable Josephson Junction

October 21, 2003

JJ RCSJ Model as Circuit Element

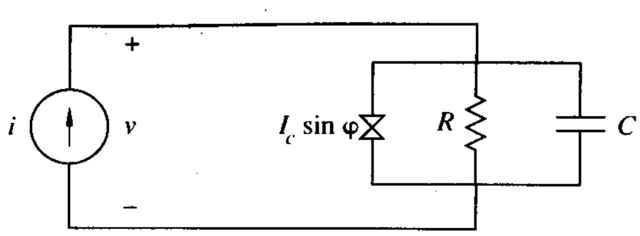


$$i = I_c \sin\varphi + \frac{v}{R} + C\frac{dv}{dt} \qquad \text{and} \qquad v = \frac{\Phi_o}{2\pi}\frac{d\varphi}{dt}$$

Therefore,

$$i = I_c \sin\varphi + \frac{\Phi_o}{2\pi R}\frac{d\varphi}{dt} + C\left(\frac{\Phi_o}{2\pi}\right)^2 \frac{d^2\varphi}{dt^2}$$

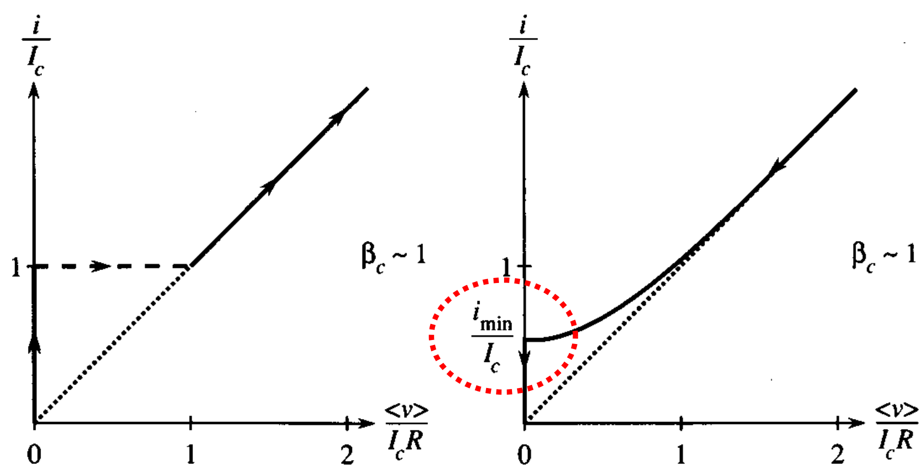
DC Current Drive



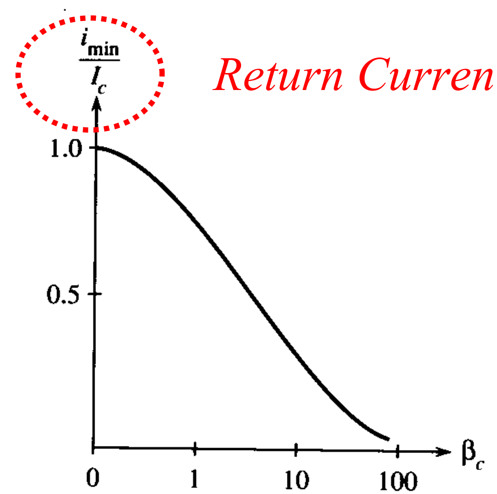
$$\beta_c = \frac{\tau_{RC}}{\tau_J} = \frac{R^2 C}{L_J} = \frac{2\pi I_c R^2 C}{\Phi_o}$$

A. Static Solution: $\varphi = \sin^{-1} \frac{i}{I_c}$ for $i \leq I_c$

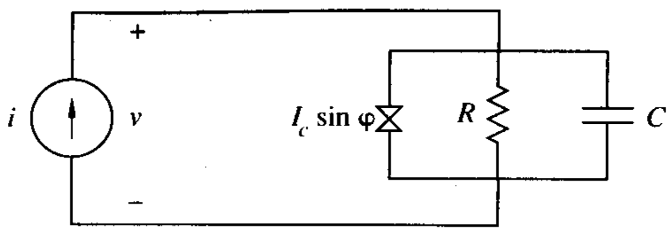
B. Dynamical Solution



Return Current



DC Voltage Drive



The voltage source is DC with v=V_0 , so that

$$\varphi(t) = \varphi(0) + \frac{2\pi}{\Phi_o} V_o t$$

The resulting current across the JJ is ac

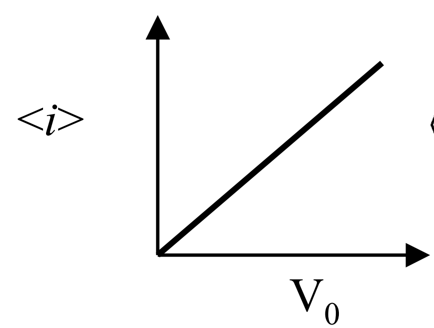
$$i_J = I_c \sin\left(\frac{2\pi}{\Phi_o} V_o t + \varphi(0)\right)$$

The current across the resistor is dc

$$i_R = \frac{V_o}{R}$$

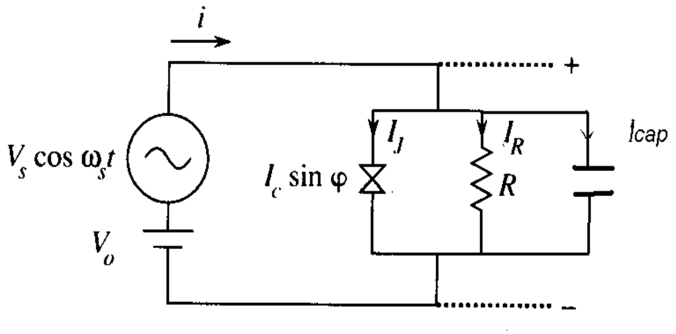
The total current is then

$$i = \frac{V_0}{R} + I_c \sin\left(\frac{2\pi}{\Phi_o} V_o t + \varphi(0)\right)$$



$$\langle i \rangle = \frac{V_o}{R}$$

AC Voltage Drive



The voltage source $v(t) = V_o + V_s \cos \omega_s t$

Then the guage-invariant phase is

$$\varphi(t) = \varphi(0) + \frac{2\pi}{\Phi_o} V_o t + \frac{2\pi V_s}{\Phi_o \omega_s} \sin \omega_s t$$

The resulting current across the JJ is

$$i_J = I_c \sin\left(\varphi(0) + \frac{2\pi}{\Phi_o} V_o t + \frac{2\pi V_s}{\Phi_o \omega_s} \sin \omega_s t\right)$$

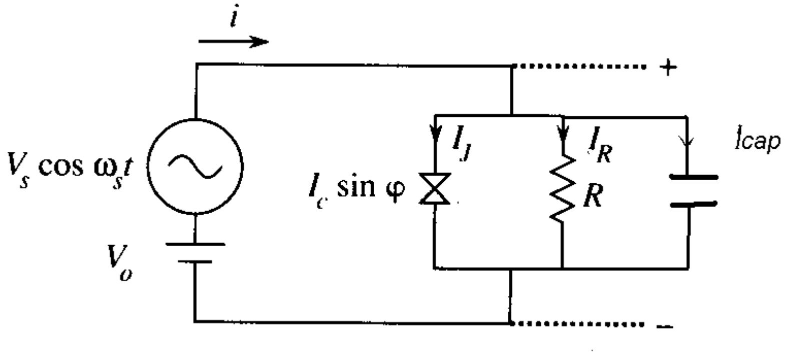
The current across the resistor is

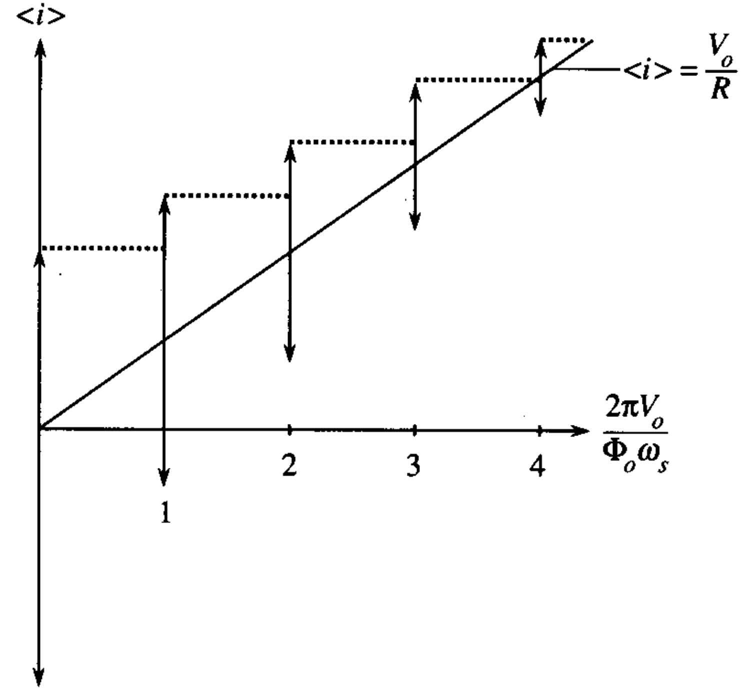
$$i_R(t) = \frac{V_o}{R} + \frac{V_s}{R} \cos \omega_s t$$

The current across the capacitor is $i_C(t) = -C\, V_s \omega_s \sin \omega_s t$

The total current is then $i(t) = i_R + i_J + i_C$

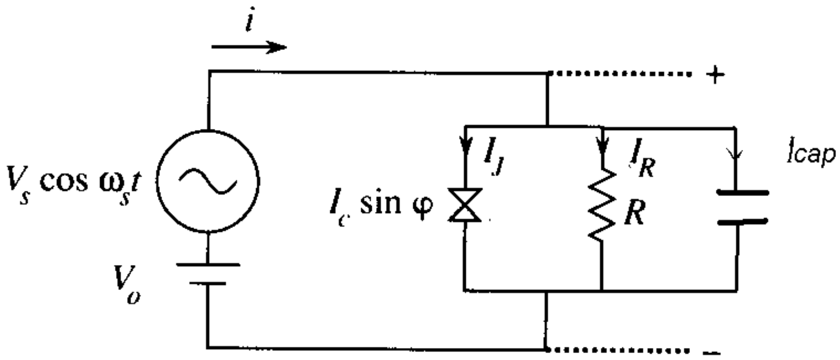
AC Voltage Drive

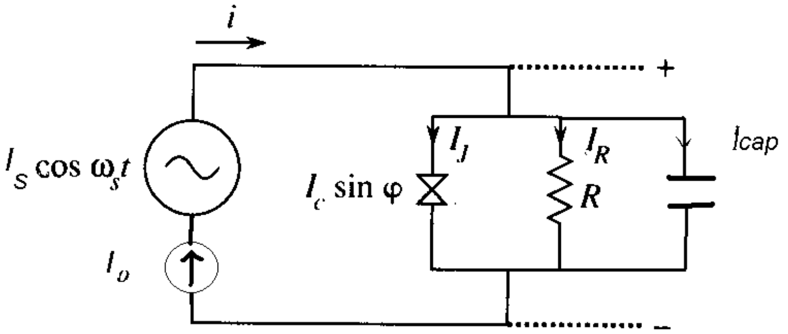


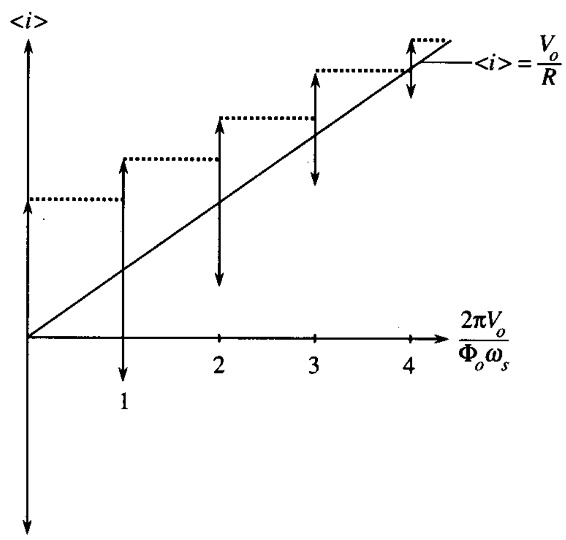


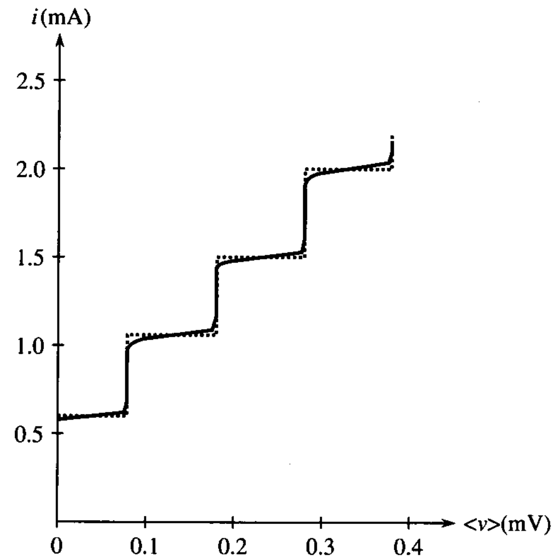
$$\langle i \rangle = \frac{V_o}{R} + I_c \sum_{n=-\infty}^{+\infty} (-1)^n J_n \left(\frac{2\pi V_s}{\Phi_o \omega_s} \right) \sin \varphi(0) \, \delta_{2\pi f_J, n\omega_s}$$

AC Voltage vs Current Drives

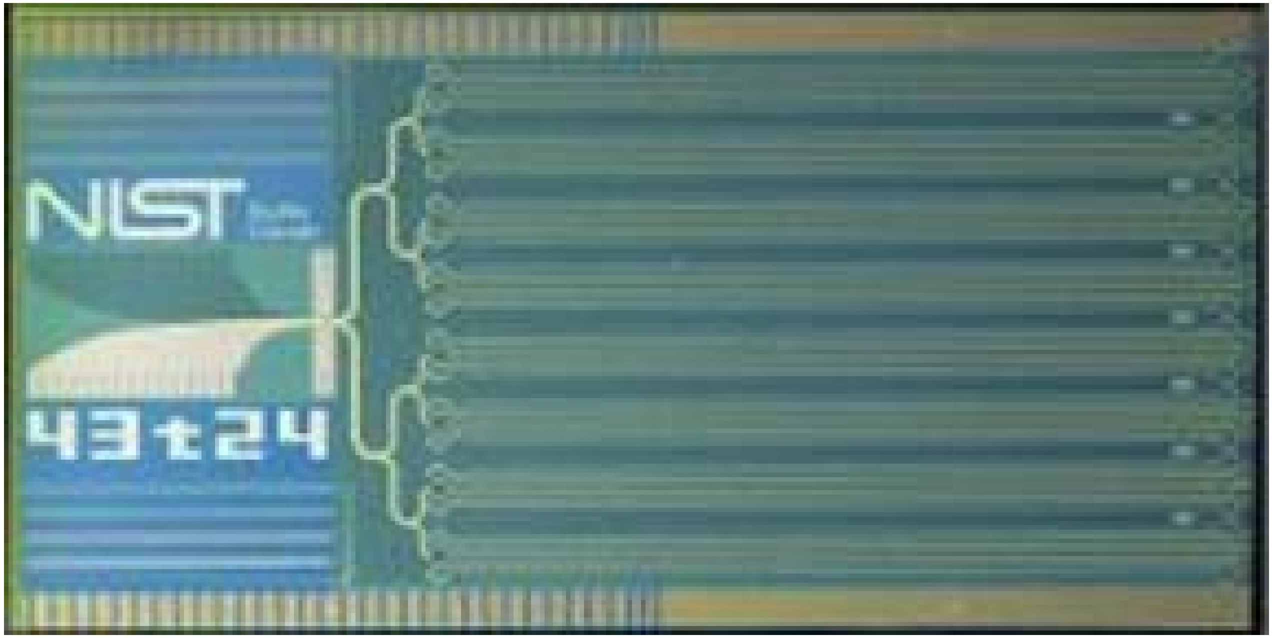








Voltage Standard



10 V conventional Josephson voltage standard chip. The chip is 1 cm x 2 cm and contains 20,208 series connected Nb-AlOx -Nb junctions.

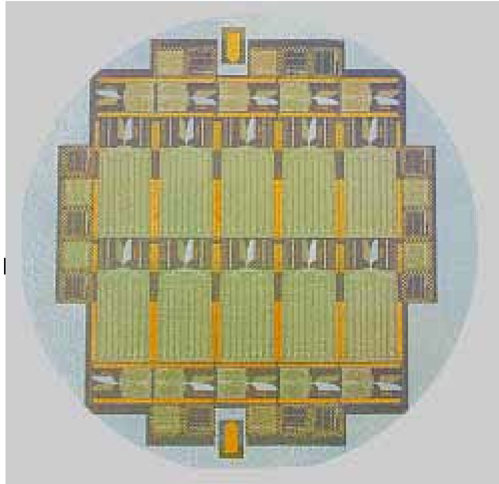
http://www.boulder.nist.gov/div814/div814/whatwedo/volt/dc/JVS.html

HYPRES: Voltage Standard Chip

HYPRES is the only commercial manufacturer of the superconducting integrated circuit used in Primary Voltage Standard Systems. HYPRES chips are used in the primary voltage standards in national laboratories around the world including Italy, France, United Kingdom, Australia, China, Malaysia, Japan, England, Canada, Norway, United States, Netherlands and Mexico.
The HYPRES Josephson Junction Array Voltage Standard circuits provide the ultimate accuracy for realizing and maintaining the SI Volt.

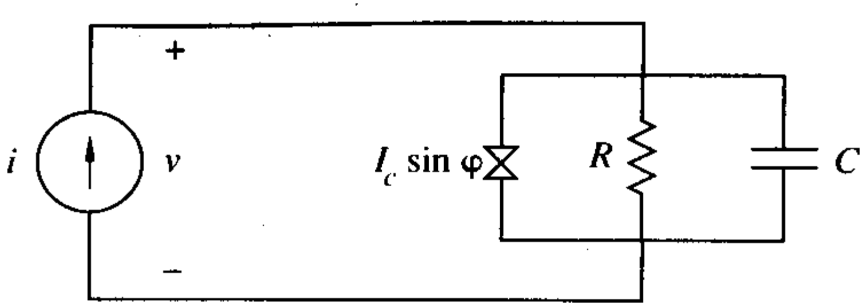
Features/Specifcations

- Niobium/Aluminum Oxide/Niobium, SiO_2 dielectric, Niobium wiring technology.
- All Niobium technology. Refractory. Impervious to moisture and thermal cycling.
- 20,208 Josephson junctions (10 V chip) 3,660 Josephson junctions (1 V chip)
- 18 x 38 micrometers junction area.
- Installed in a FR-4 epoxy glass mount.
- RF input WR-12 waveguide flange.
- RF Distribution - 16 way parallel x 1263 cells in series (10 V) - 4 way parallel x 915 cells i
- Designed for a frequency range of 72-78 GHz
- Operating temperature of 4.2 K
- Common DC terminal resistance is < 1 Ohm - typical
- Approximately 10 mW operating power at the input flange for 10 V chip (2 mW for 1 V)
- -11V to +11 V range for 10 V chip.
- -2.5 V to + 2.5 V range for 1 V chip.
- Stability time is typically 1 hour for the 10 V chip, 5 hours for the 1V chip
- 0.005 PPM accuracy at 10 V (10 V chip)
- 0.05 PPM accuracy at 1V (1V chip)
- Calibration certificate supplied with each chip.
- Two (2) year warranty



http://www.hypres.com/

Parametric Inductor



$$i(t) = I_c \sin\varphi(t) + \frac{v(t)}{R} + C\frac{d}{dt}v(t)$$

Take the time derivative of the currents, and for the Josephson term:

$$\frac{d}{dt}i_J(t) = \left[\frac{2\pi I_c}{\Phi_o}\cos\varphi(t)\right] v(t)$$

The parameteric (time-dependent) inductance can be defined as

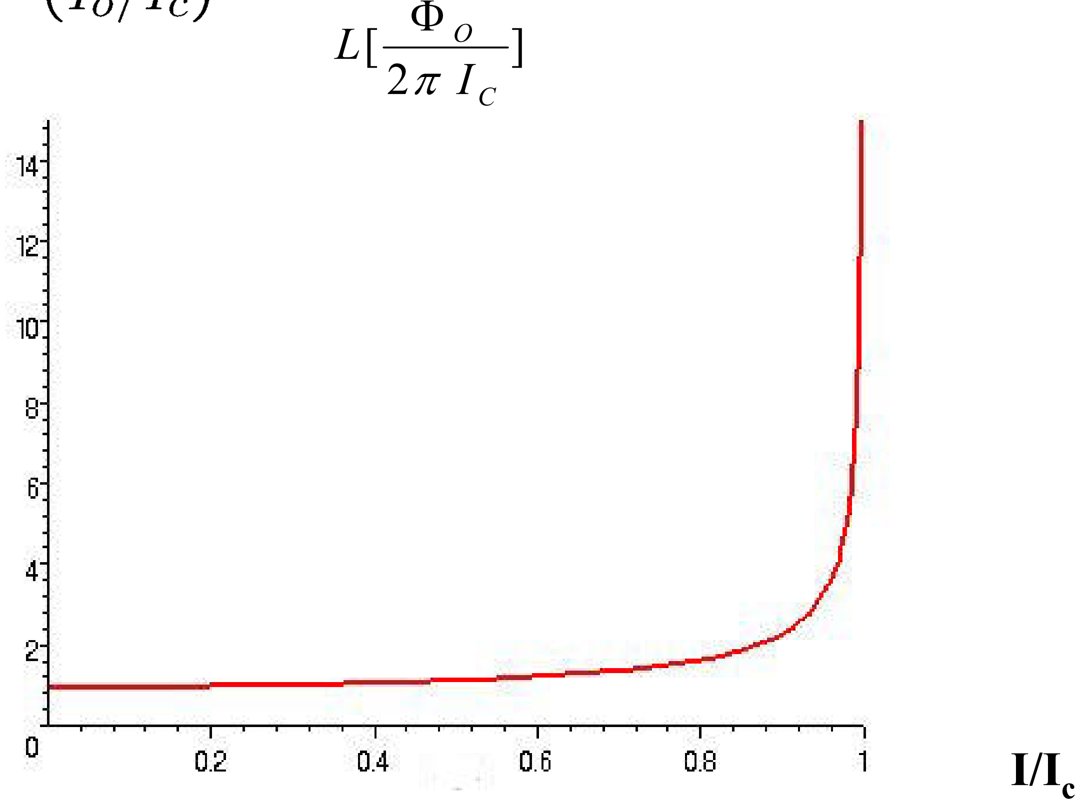
$$L_J = \frac{\Phi_o}{2\pi I_c \cos\varphi(t)}$$

*On the zero-voltage branch, for $I_s << I_0$ and $I_0 + I_s << I_c$, then $I_0 \sim I_c \sin\phi$, so that

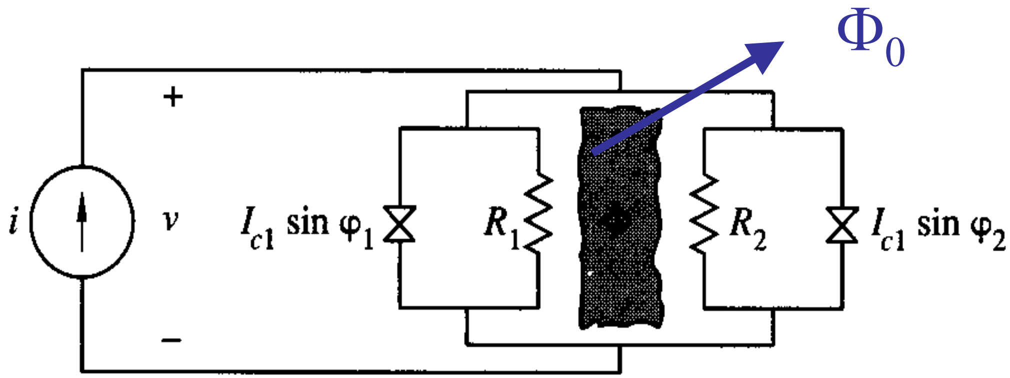
$$L_J \approx \frac{\Phi_o}{2\pi I_c\sqrt{1-(I_o/I_c)^2}}$$

Inductance along V=0 branch

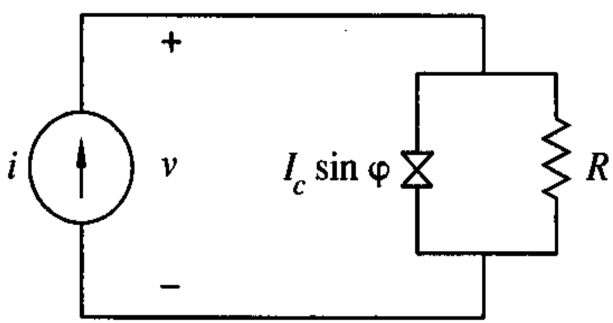
$$L_J \approx \frac{\Phi_o}{2\pi I_c \sqrt{1-(I_o/I_c)^2}}$$



The DC SQUID (damped)



(a)

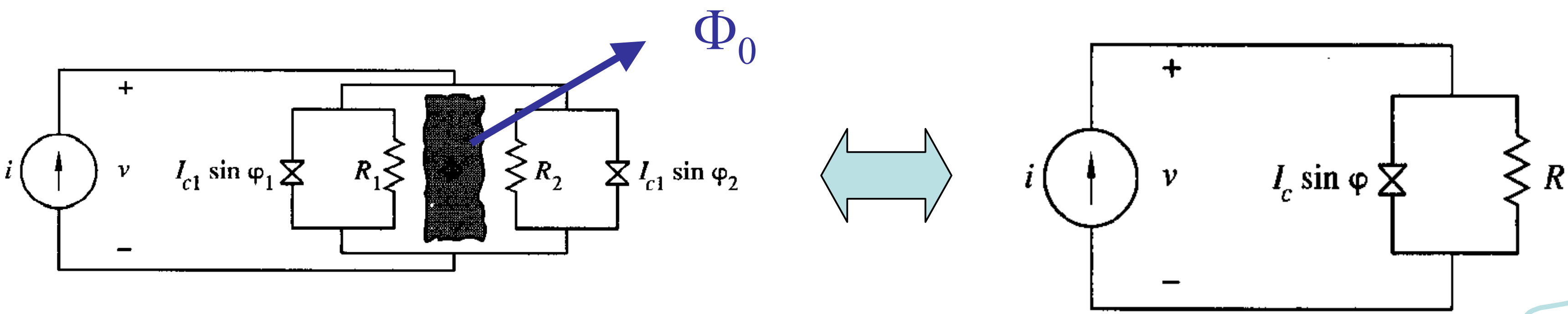


(b)

Our goal is to show that the DC SQUID circuit in (a) is equivalent to the circuit for a single junction with an effective I_c and an effective resistance. The inductance of the SQUID loop in (a) is considered negligible.

In fact, we will also show later that there is an equivalent statement even for the underdamped cass.

DC SQUID Equivalent Circuit



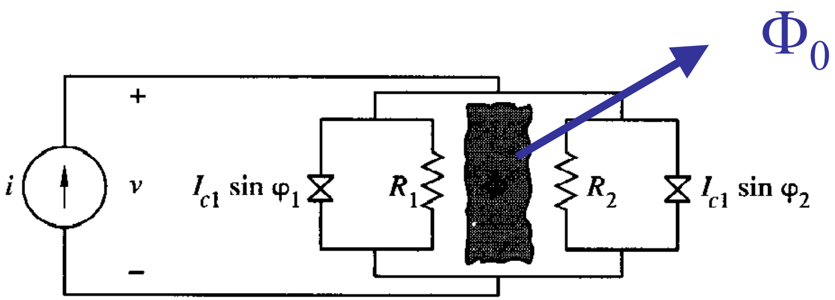
$$i = i_{J_1} + i_{J_2} + i_{R_1} + i_{R_2}$$
$$= I_{c1}\sin\varphi_1 + I_{c1}\sin\varphi_2 + \frac{v}{R_1} + \frac{v}{R_2}$$

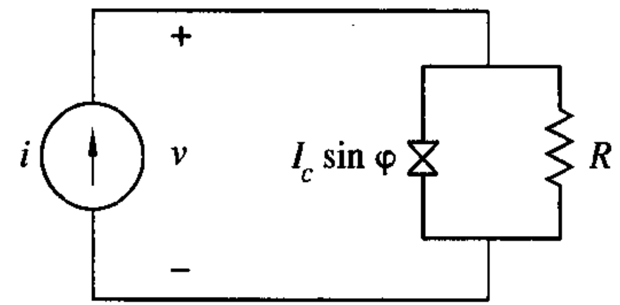
Use flux quantization, $\varphi_2 - \varphi_1 = \dfrac{2\pi\Phi}{\Phi_o}$ and the fact that the junctions are identical

$$i = 2I_{c1}\cos\left(\frac{\pi\Phi_{\text{ext}}}{\Phi_o}\right)\sin\left(\varphi_1 + \frac{\pi\Phi_{\text{ext}}}{\Phi_o}\right) + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v(t)$$

$$I = I_c \quad \sin \quad \varphi \quad + \quad 1/R \quad \frac{\Phi_0}{2\pi}\frac{d}{dt}\varphi$$

DC SQUID Equivalent Circuit



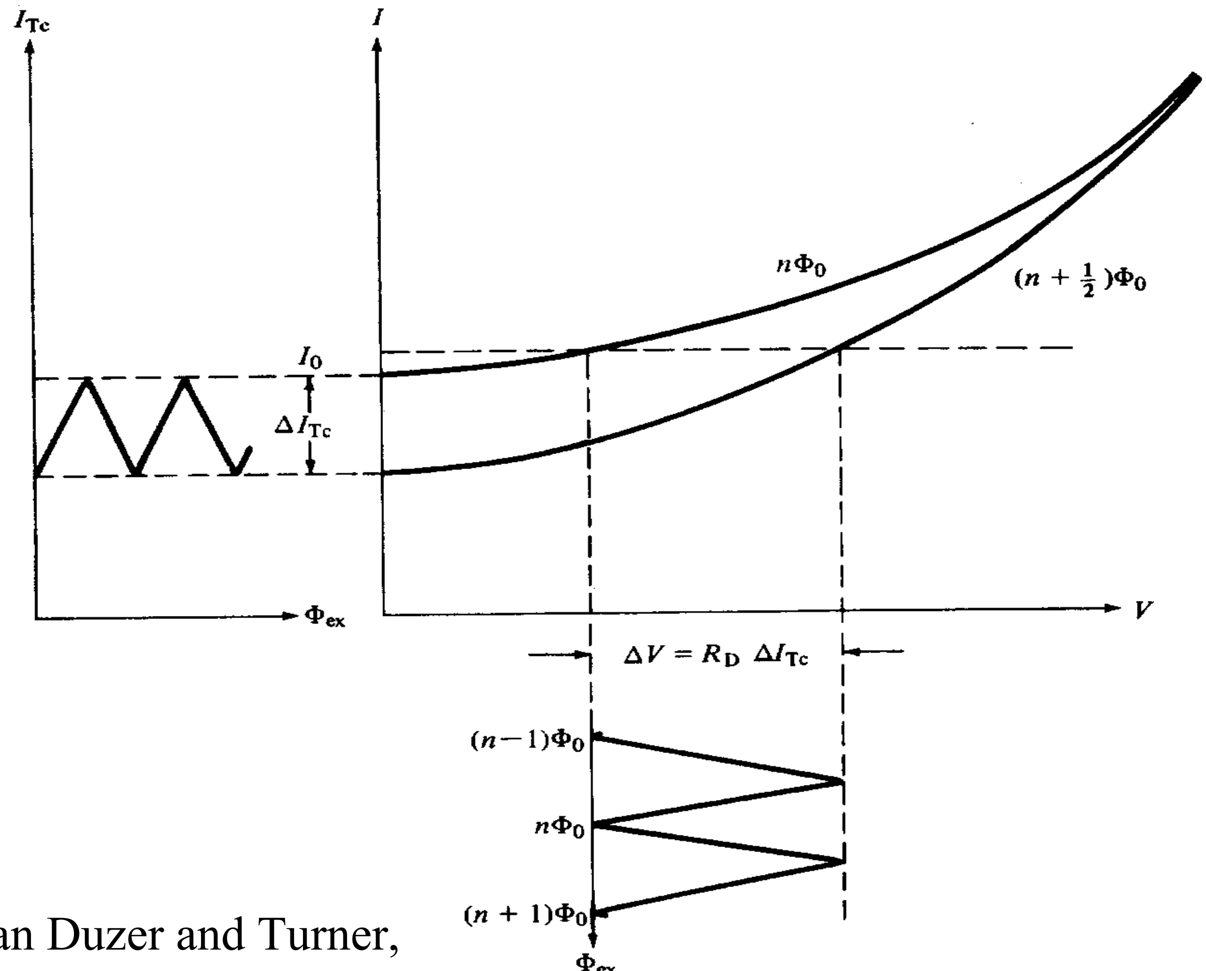


$$i = I_c \sin\varphi + \frac{1}{R}\frac{\Phi_o}{2\pi}\frac{d\varphi}{dt} \qquad \text{with} \qquad I_c = 2I_{c1}\cos\left(\frac{\pi\Phi_{\text{ext}}}{\Phi_o}\right)$$

Therefore, for this overdamped equivalent circuit, for $i > I_C$

$$\langle v(t)\rangle = iR\sqrt{1-\left[\frac{2I_{c1}}{i}\cos\left(\frac{\pi\Phi_{\text{ext}}}{\Phi_o}\right)\right]^2} \qquad \langle v(t)\rangle = iR\sqrt{1-\left(\frac{i_{\max}}{i}\right)^2}$$

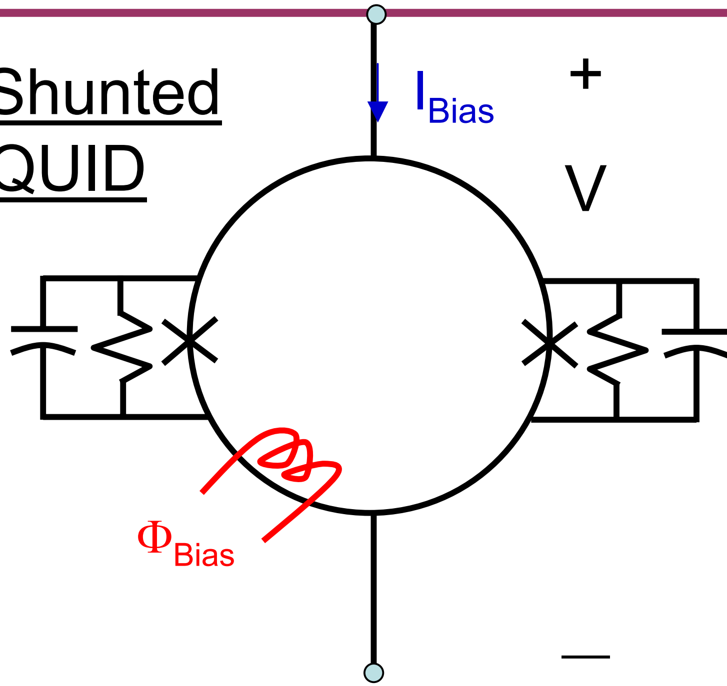
DC SQUID Voltage Modulation

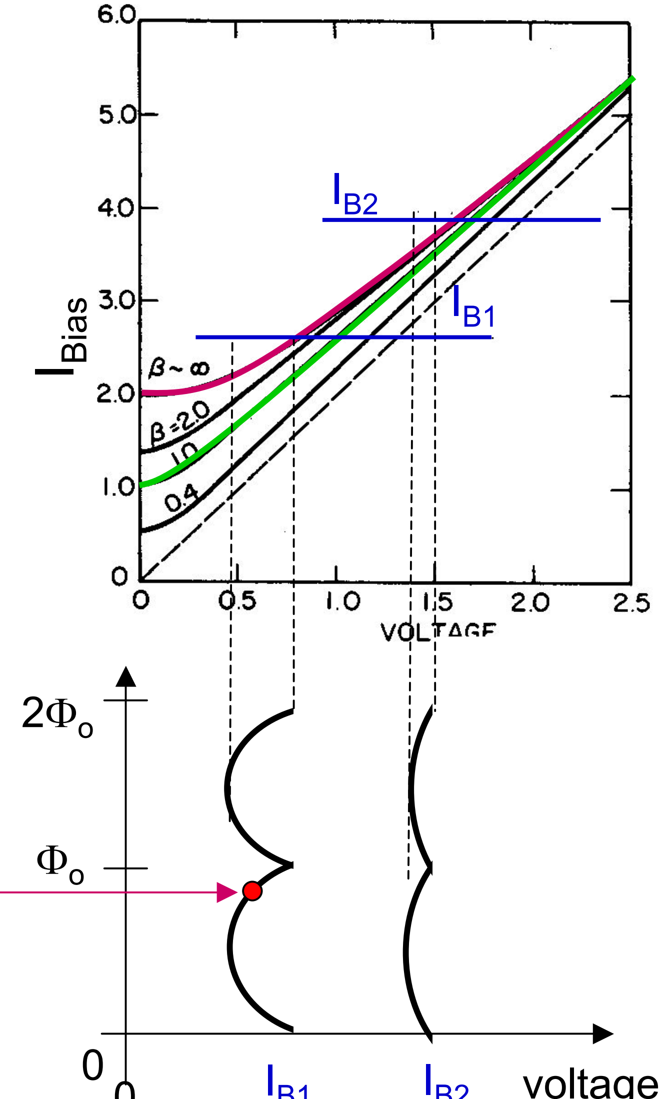


From Van Duzer and Turner, Figure 5.11a, page 272

DC SQUID Voltage Modulation

RC-Shunted SQUID



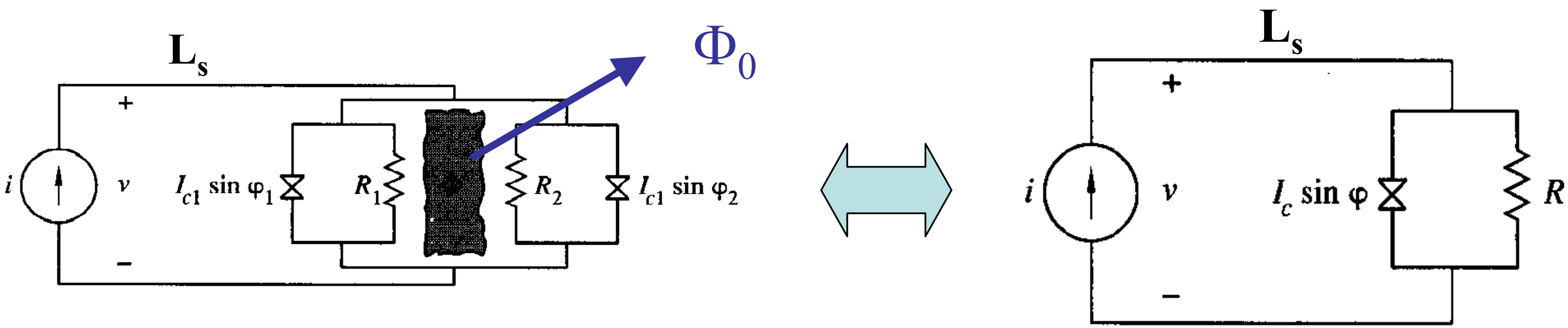


I_{bias} kept slightly above I_C

$\Phi_{bias} = 0.75\Phi_o$ for sensitivity

Goal is to maximize: $\dfrac{dV}{d\Phi} \approx \dfrac{R}{L}$

DC SQUID Sensitivity



Assume that $L_s > L_J = \Phi_0/2\pi I_c(\Phi_{ext})$, then the range of modulation of the current is about

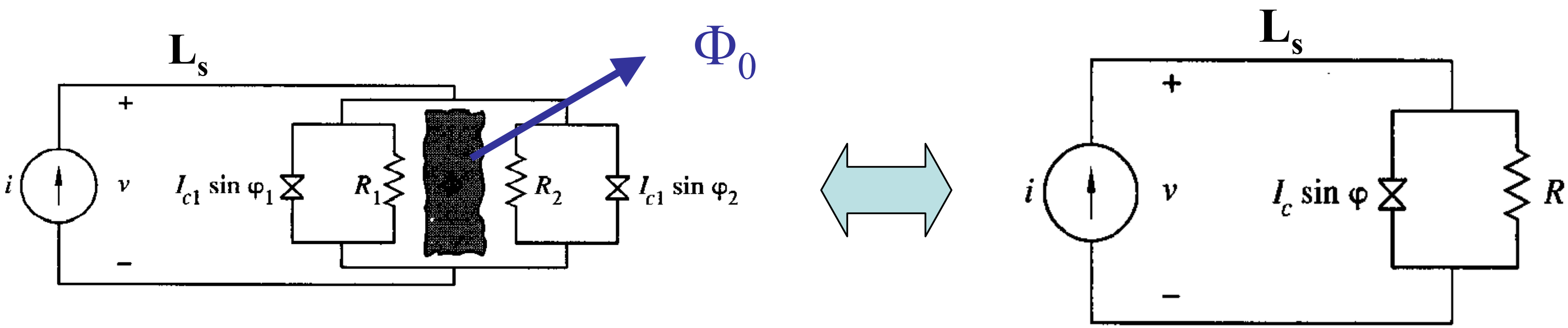
$$\delta i = i^{+}_{\text{max}} - i^{-}_{\text{max}} \approx \Phi_o/L$$

The sensitivity of the output voltage to the input flux is

$$\frac{\delta \langle v \rangle}{\delta \Phi_{\text{ext}}} \approx \frac{R_D \delta i}{\frac{1}{2}\Phi_o} = \frac{2R_D}{L}$$

With $R_D \sim R = 1$ Ohm and $L \sim 1$ nH, then the sensistivity is about one microvolt per flux quantum, so that small fractions of a flux quantum can be measured.

DC SQUID and Thermal Noise



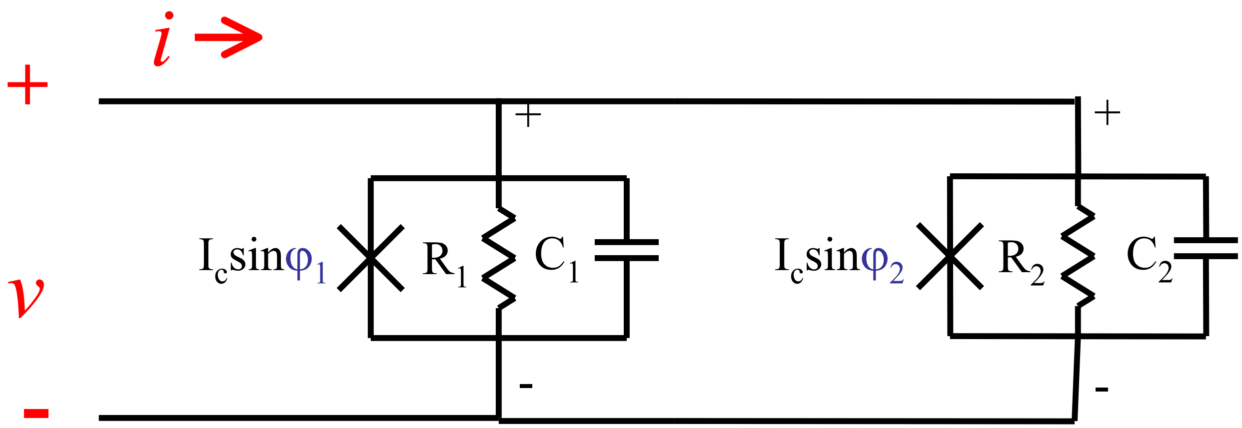
Assume again that $L_s > L_J = \Phi_0/2\pi I_c(\Phi_{ext})$, then to prevent thermal noise from effecting the SQUID's performance, one needs

Energy stored in the SQUID >> Thermal Energy

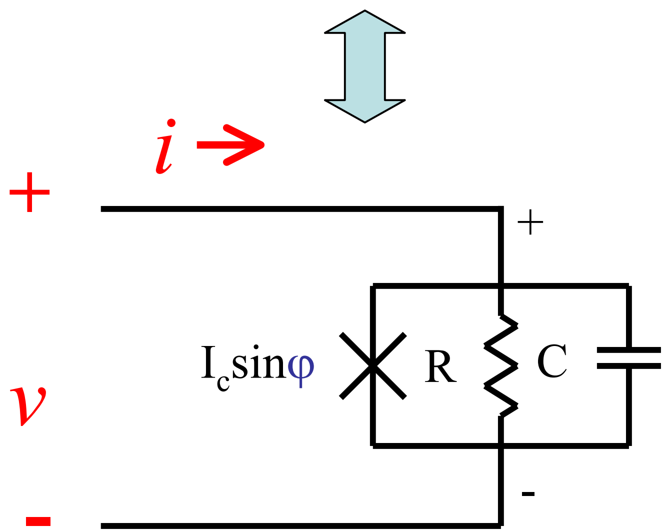
$$\frac{1}{2}\frac{\Phi_o^2}{L} \gg \frac{1}{2} k_B T$$

Therefore, L_s < 1 nH at 4 K, and 0.1 nH at 100 K.

Equivalent Tunable Junction



with $L_{loop} \ll L_J$



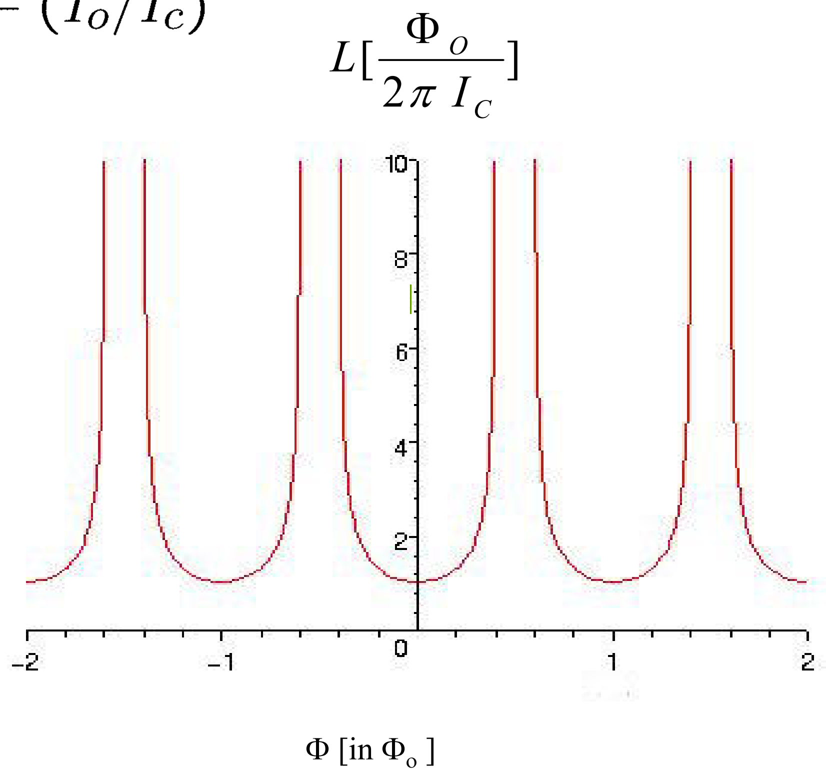
$$I_c = 2I_{c1} \cos\left(\frac{\pi \Phi_{ext}}{\Phi_o}\right)$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \qquad C = C_1 + C_2$$

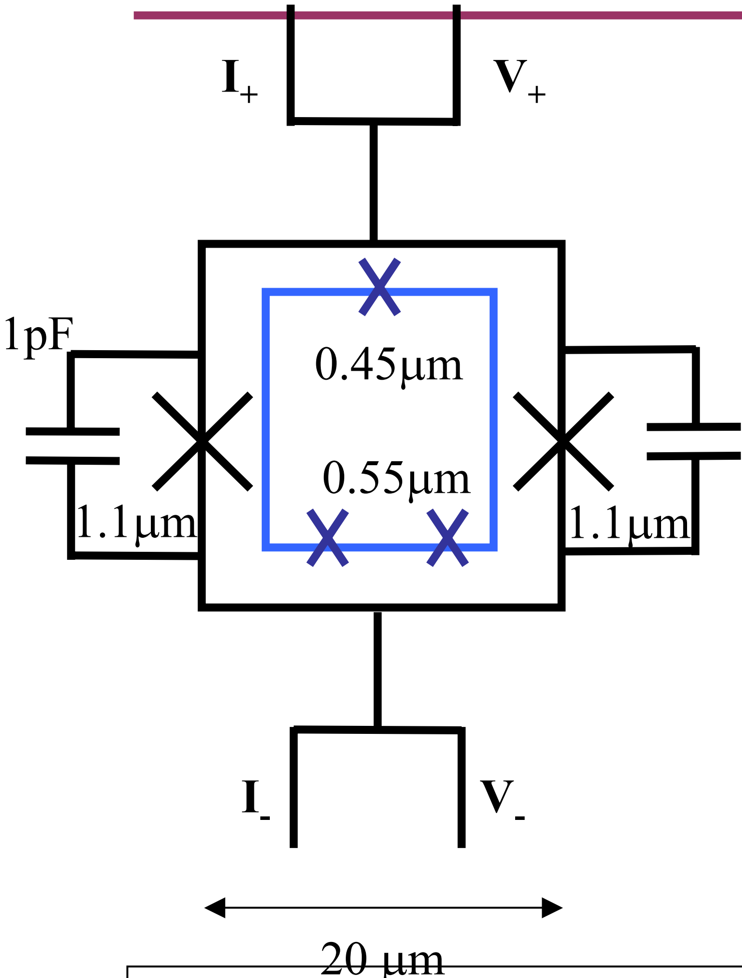
$$\beta_c = \frac{\tau_{RC}}{\tau_J} = \frac{R^2 C}{L_J} = \frac{2\pi I_c R^2 C}{\Phi_o}$$

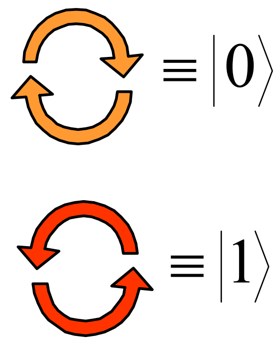
Tunable Inductance along V=0 branch

$$L_J \approx \frac{\Phi_o}{2\pi I_c\sqrt{1-(I_o/I_c)^2}}$$



Three-Junction Loop Measurements





Three-junction Loop

Jct. Size ~ 0.45µm, 0.55µm

Loop size ~16x16µm²

$L_{3\text{-junction}}$ ~ 30pH

I_c~1 & 2µA

E_J/E_c ~ 350 & 550

DC SQUID

Shunt capacitors ~ 1pF

Jct. Size ~ 1.1µm

Loop size ~20x20µm²

L_{SQUID} ~ 50pH

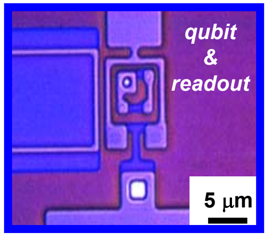
I_c~10 & 20µA

M~35pH

J_c~350 & 730A/cm²

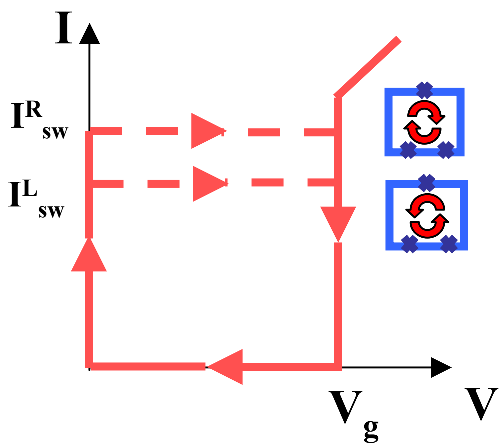
- **Measure switching current of DC SQUID**
- **Vary external flux, temperature and SQUID ramp rate**

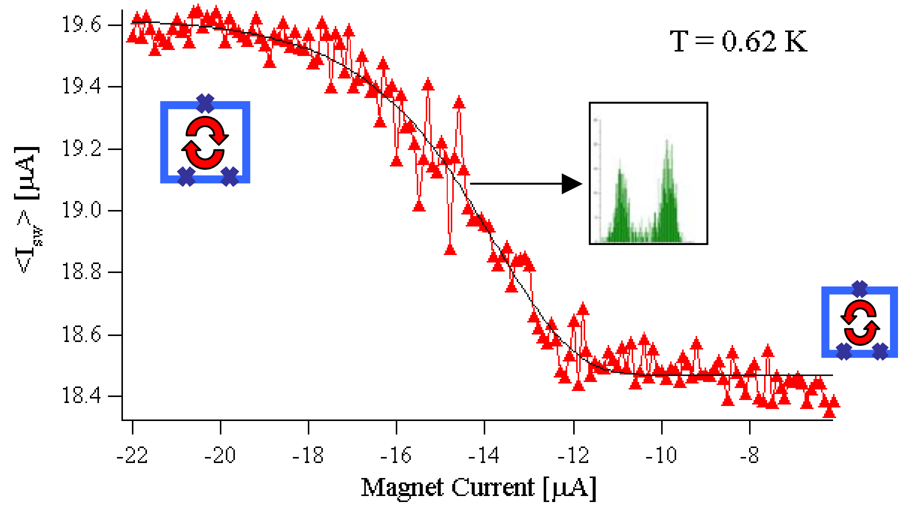
Thermal Activation of Nb Persistent Current Qubit



SFQ on-chip oscillator and qubit

SQUID Detector





	J_c	E_J	E_J/E_c	Jct. width	α	Q
Device A:	365 A/cm²	2400 μeV	380	0.563 μm	0.613	1.0×10^6
Device B:	730 A/cm²	4000 μeV	560	0.529 μm	0.589	1.2×10^6

Highest Q of any submicron Nb junction

Coupling between qubits