Superconductivity¹

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117.1 Introduction

The fundamental ideal behind all of a superconductor's unique properties is that **superconductivity** is a quantum mechanical phenomenon on a macroscopic scale created when the motions of individual electrons are correlated. According to the theory developed by John Bardeen, Leon Cooper, and Robert Schrieffer (BCS theory), this correlation takes place when two electrons couple to form a Cooper pair. For our purposes, we may therefore consider the electrical charge carriers in a superconductor to be Cooper pairs (or more colloquially, superelectrons) with a mass m^* and charge q^* twice those of normal electrons. The average distance between the two electrons in a Cooper pair is known as the coherence length, ξ . Both the coherence length and the binding energy of two electrons in a Cooper pair, 2Δ , depend upon the particular superconducting material. Typically, the coherence length is many times larger than the interatomic spacing of a solid, and so we should not think of Cooper pairs as tightly bound electron molecules. Instead, there are many other electrons between those of a specific Cooper pair allowing for the paired electrons to change partners on a time scale of $h/(2\Delta)$, where h is Planck's constant.

If we prevent the Cooper pairs from forming by ensuring that all the electrons are at an energy greater than the binding energy, we can destroy the superconducting phenomenon. This can be accomplished, for example, with thermal energy. In fact, according to the BCS theory, the critical temperature, T_c , associated with this energy is

$$\frac{2\Delta}{k_B T_c} \approx 3.5 \tag{117.1}$$

where k_B is Boltzmann's constant. For low critical temperature (conventional) superconductors, 2 Δ is typically on the order of 1 meV, and we see that these materials must be kept below temperatures of about 10 K to exhibit their unique behavior. Superconductors with high critical temperature, in contrast, will superconduct up to temperatures of about 100 K, which is attractive from a practical view because the materials can be cooled cheaply using liquid nitrogen. A second way of increasing the energy of the electrons is electrically driving them. In other words, if the critical current density, J_c , of a superconductor is exceeded, the electrons have sufficient kinetic energy to prevent the formation of Cooper pairs. The necessary kinetic energy can also be generated through the induced currents created by an external magnetic field. As a result, if a superconductor is placed in a magnetic field larger than its critical field, H_c , it will return to its normal metallic state. To summarize, superconductors must be maintained under the appropriate temperature, electrical current density, and magnetic field conditions to exhibit its special properties. An example of this phase space is shown in Fig. 117.1.

¹ This chapter is modified from Delin, K. A. and Orlando, T. P. 1993. Superconductivity. In *The Electrical Engineering Handbook*, ed. R. C. Dorf, pp. 1114–1123. CRC Press, Boca Raton, FL.

117.2 General Electromagnetic Properties

The hallmark electromagnetic properties of a superconductor are its ability to carry a static current without any resistance and its ability to exclude a static magnetic flux from its interior. It is this second property, known as the Meissner effect that distinguishes a superconductor from merely being a perfect conductor (which conserves the magnetic flux in its interior). Although superconductivity is a manifestly quantum mechanical phenomenon, a useful classical model can be constructed around these two properties. In this section we will outline the rationale for this classical model, which is useful in engineering applications such as waveguides and high-field magnets.

The zeros DC resistance criterion implies that the superelectrons move unimpeded. The electromagnetic energy density, w, stored in a superconductor is therefore

$$w = \frac{1}{2}\varepsilon \mathbf{E}^2 + \frac{1}{2}\mu_o \mathbf{H}^2 + \frac{n^*}{2}m^* \mathbf{v}_s^2$$
(117.2)

where the first two terms are the familiar electric and magnetic energy densities, respectively. (Our electromagnetic notation is standard: ε is the permittivity, μ_o is the permeability, **E** is the electric field, and the magnetic flux density, **B**, is related to the magnetic field, **H**, via the constitutive law $\mathbf{B} = \mu_o \mathbf{H}$.) The last term represents the kinetic energy associated with the undamped superelectrons' motion (n^* and \mathbf{v}_s are the superelectrons' density and velocity, respectively). Because the supercurrent density, \mathbf{J}_s , is related to the superelectron velocity by $\mathbf{J}_s = n^* q^* \mathbf{v}_s$, the kinetic energy term can be rewritten

$$n^* \left(\frac{1}{2}m^* \mathbf{v}_s^2\right) = \frac{1}{2}\Lambda \mathbf{J}_s^2 \tag{117.3}$$

where Λ is defined as

$$\Lambda = \frac{m^*}{n^* (q^*)^2}$$
(117.4)

Assuming that all the charge carriers are superelectrons, there is no power dissipation inside the superconductor, and so Poynting's theorem over a volume V may be written

$$-\int_{V} \nabla \cdot (\mathbf{E} \times \mathbf{H}) \, dv = \int_{V} \frac{\partial w}{\partial t} \, dv \tag{117.5}$$

where the left side of the expression is the power flowing into the region. By taking the time derivative of the energy density and appealing to Faraday's and Ampère's laws to find the time derivatives of the field quantities, we find that the only way for Poynting's theorem to be satisfied is if

$$\mathbf{E} = \frac{\partial}{\partial t} (\Lambda \mathbf{J}_s) \tag{117.6}$$

This relation, known as the *first London equation* (after the London brothers, Heinz and Fritz), is thus necessary if the superelectrons have no resistance to their motion.

Equation (117.6) also reveals that the superelectrons' inertia creates a lag between their motion and that of the electric field. As a result, a superconductor can support a time-varying voltage drop across itself. The impedance associated with the supercurrent, therefore, is an inductor, and it will be useful to think of Λ as an inductance created by the correlated motion of the superelectrons.

If the first London equation is substituted into Faraday's law, $\nabla \times \mathbf{E} = -(\partial \mathbf{B}/\partial t)$, and integrated with respect to time, the *second London equation* results:

$$\nabla \times (\Lambda \mathbf{J}_{s}) = -\mathbf{B} \tag{117.7}$$

where the constant of integration has been defined to be zero. This choice is made so that the second London equation is consistent with the Meissner effect, as we now demonstrate. Taking the curl of the quasi-static form of Ampère's law, $\nabla \times \mathbf{H} = \mathbf{J}_s$, results in the expression $\nabla^2 \mathbf{B} = -\mu_o \nabla \times \mathbf{J}_s$, where a vector identity, $\nabla \times \nabla \times \mathbf{C} = \nabla$ ($\nabla \cdot C$) – $\nabla^2 C$; the constitutive relation, $\mathbf{B} = \mu_o \mathbf{H}$; and Gauss's law, $\nabla \cdot \mathbf{B} = 0$, have been used. By now appealing to the second London equation, we obtain the vector Helmholtz equation

$$\nabla^2 \mathbf{B} - \frac{1}{\lambda^2} \mathbf{B} = 0 \tag{117.8}$$

where the penetration depth is defined as

$$\lambda \equiv \sqrt{\frac{\Lambda}{\mu_o}} = \sqrt{\frac{m^*}{n^* (q^*)^2 \,\mu_o}} \tag{117.9}$$

From Eq. (117.8) we find that a flux density applied parallel to the surface of a semi-infinite superconductor will decay away exponentially from the surface on a spatial length scale of order λ . In other words, a bulk superconductor will exclude an applied flux as predicted by the Meissner effect.

The London equations reveal that there is a characteristic length λ over which electromagnetic fields can change inside a superconductor. This penetration depth is different from the more familiar skin depth of electromagnetic theory, the latter being a frequency-dependent quantity. Indeed, the penetration depth at zero temperature is a distinct material property of a particular superconductor.

Notice that λ is sensitive to the number of correlated electrons (the superelectrons) in the material. As previously discussed, this number is a function of temperature, and so only at T = 0 do *all* the electrons that usually conduct ohmically participate in the Cooper pairing. For intermediate temperatures, $0 < T < T_c$, there are actually two sets of interpenetrating electron fluids: the uncorrelated electrons providing ohmic conduction and the correlated ones creating supercurrents. This two-fluid model is a useful way to build temperature effects into the London relations.

Under the two-fluid model, the electrical current density, **J**, is carried by both the uncorrelated (normal) electrons and the superelectrons: $\mathbf{J} = \mathbf{J}_n + \mathbf{J}_s$, where \mathbf{J}_n is the normal current density. The two channels are modeled in a circuit, as shown in Fig. 117.2, by a parallel combination of a resistor (representing the ohmic channel) and an inductor (representing the superconducting channel). To a good approximation, the respective temperature dependences of the conductor and inductor are

$$\widetilde{\sigma}_{o}(T) = \sigma_{o}(T_{c}) \left(\frac{T}{T_{c}}\right)^{4} \quad \text{for } T \le T_{c}$$
(117.10)

$$\Lambda(T) = \Lambda(0) \left(\frac{1}{1 - (T/T_c)^4} \right) \quad \text{for } T \le T_c$$
(117.11)

where σ_o is the DC conductance of the normal channel. (Strictly speaking, the normal channel should also contain an inductance representing the inertia of the normal electrons, but typically such an inductor contributes negligibly to the overall electrical response.) Since the temperature-dependent penetration depth is defined as $\lambda(T) = \sqrt{\Lambda(T)/\mu_o}$, the effective conductance of a superconductor in the sinusoidal steady state is

$$\sigma = \widetilde{\sigma}_{o} + \frac{1}{j\omega\,\mu_{o}\lambda^{2}} \tag{117.12}$$

where the explicit temperature dependence notation has been suppressed.

Most of the important physics associated with the classical model is embedded in Eq. (117.12). As is clear from the lumped element model, the relative importance of the normal and superconducting channels is a function not

only of temperature but also of frequency. The familiar L/R time constant, here equal to $\Lambda \tilde{\sigma}_o$, delineates the frequency regimes where most of the total current is carried by \mathbf{J}_n (if $\omega \Lambda \tilde{\sigma}_o >> 1$) or \mathbf{J}_s (if $\omega \Lambda \tilde{\sigma}_o << 1$). This same result can also be obtained by comparing the skin depth associated with the normal channel, $\delta = \sqrt{2/(\omega \mu_o \tilde{\sigma}_o)}$, to the penetration depth to see which channel provides more field screening. In addition, it is straightforward to use Eq. (117.12) to rederive Poynting's theorem for systems that involve superconducting materials:

$$-\int_{\mathcal{V}} \nabla \cdot (\mathbf{E} \times \mathbf{H}) \, dv = \frac{d}{dt} \int_{\mathcal{V}} \left(\frac{1}{2} \, \varepsilon \mathbf{E}^2 + \frac{1}{2} \, \mu_o \mathbf{H}^2 + \frac{1}{2} \, \Lambda(T) \mathbf{J}_s^2 \right) dv$$

$$+ \int_{\mathcal{V}} \frac{1}{\widetilde{\sigma}_o(T)} \mathbf{J}_n^2 \, dv$$
(117.13)

Using this expression, it is possible to apply the usual electromagnetic analysis to find the inductance (L_o) , capacitance (C_o) , and resistance (R_o) per unit length along a parallel plate transmission line. The results of such analysis for typical cases are summarized in Table 117.1.

117.3 Superconducting Electronics

The macroscopic quantum nature of superconductivity can be usefully exploited to create a new type of electronic device. Because all the superelectrons exhibit correlated motion, the usual wave-particle duality normally associated with a single quantum particle can now be applied to the entire ensemble of superelectrons. Thus, there is a spatiotemporal phase associated with the ensemble that characterizes the supercurrent flowing in the material.

Naturally, if the overall electron correlation is broken, this phase is lost and the material is no longer a superconductor. There is a broad class of structures, however, known as *weak links*, where the correlation is merely perturbed locally in space rather than outright destroyed. Colloquially, we say that the phase "slips" across the weak link to acknowledge the perturbation.

The unusual properties of this phase slippage were first investigated by Brian Josephson and constitute the central principles behind superconducting electronics. Josephson found that the phase slippage could be defined as the difference between the macroscopic phases on either side of the weak link. This phase difference, denoted as ϕ , determined the supercurrent, i_s , through and voltage, v, across the weak link according to the Josephson equations,

$$i_s = I_c \sin\phi \tag{117.14}$$

$$v = \frac{\Phi_o}{2\pi} \frac{\partial \phi}{\partial t} \tag{117.15}$$

where I_c is the critical (maximum) current of the junction and Φ_o is the quantum unit of flux. (The flux quantum has a precise definition in terms of Planck's constant, h, and the electron charge, $e: \Phi_o \equiv h/(2e) \approx 2.068 \times 10^{-15}$ Wb). As in the previous section, the correlated motion of the electrons, here represented by the superelectron phase, manifests itself through an inductance. This is straightforwardly demonstrated by taking the time derivative of Eq. (117.14) and combining this expression with Eq. (117.15). Although the resulting inductance is nonlinear (it depends on $\cos \phi$), its relative scale is determined by

$$L_{j} = \frac{\Phi_{o}}{2\pi l_{c}} \tag{117.16}$$

a useful quantity for making engineering estimates.

A common weak link, known as the Josephson tunnel junction, is made by separating two superconducting films with a very thin (typically 20 Å) insulating layer. Such a structure is conveniently analyzed using the resistively and capacitively shunted junction (RCSJ) model shown in Fig. 117.3. Under the RCSJ model an ideal lumped junction

[described by Eq. (117.14) and (117.15)] and a resistor R_j represent how the weak link structure influences the respective phases of the super and normal electrons, and a capacitor C_j represents the physical capacitance of the sandwich structure. If the ideal lumped junction portion of the circuit is treated as an inductor-like element, many Josephson tunnel junction properties can be calculated with the familiar circuit time constants associated with the model. For example, the quality factor Q of the RCSJ circuit can be expressed as

$$Q^{2} = \frac{R_{j}C_{j}}{L_{j}/R_{j}} = \frac{2\pi I_{c}R_{j}^{2}C_{j}}{\Phi_{o}} \equiv \beta$$
(117.17)

where β is known as the Stewart-McCumber parameter. Clearly, if $\beta \gg 1$, the ideal lumped junction element is underdamped in that the capacitor readily charges up, dominates the overall response of the circuit, and therefore creates a hysteretic *i*-*v* curve as shown in Fig. 117.4(a). In the case when the bias current is raised from zero, no time-averaged voltage is created until the critical current is exceeded. At this point the junction switches to the voltage $2\Delta/e$ with a time constant $\sqrt{L_jC_j}$. Once the junction has latched into the voltage state, however, the bias current must be lowered to zero before it can again be steered through the superconducting path. Conversely, $\beta \ll 1$ implies that the L_j/R_j time constant dominates the circuit response, so that the capacitor does not charge up and the *i*-*v* curve is not hysteretic [Fig. 117.4(b)].

Just as the correlated motion of the superelectrons creates the frequency-independent Meissner effect in a bulk superconductor through Faraday's law, so too the macroscopic quantum nature of superconductivity allows the possibility of a device whose output voltage is a function of a static magnetic field. If two weak links are connected in parallel, the lumped version of Faraday's law gives the voltage across the second weak link as $v_2 = v_1 + (d\Phi/dt)$, where Φ is the total flux threading the loop between the links. Substituting Eq. (117.15) and integrating with respect to time yields

$$\phi_2 - \phi_1 = (2\pi \Phi) / \Phi_o \tag{117.18}$$

showing that the spatial change in the phase of the macroscopic wavefunction is proportional to the local magnetic flux. The structure described is known as a *superconducting quantum interference device (SQUID)* and can be used as a highly sensitive magnetometer by biasing it with current and measuring the resulting voltage as a function of magnetic flux. Such SQUID structures have also been proposed for quantum bits in quantum computing. From this discussion, it is apparent that a duality exists in how fields interact with the macroscopic phase: electric fields are coupled to its rate of change in time and magnetic fields are coupled to its rate of change in space.

117.4 Types of Superconductors

The macroscopic quantum nature of superconductivity also affects the general electromagnetic properties previously discussed. This is most clearly illustrated by the interplay of the characteristic lengths ξ , representing the scale of quantum correlations, and λ , representing the scale of electromagnetic screening. Consider the scenario where a magnetic field, H, is applied parallel to the surface of a semi-infinite superconductor. The correlations of the electrons in the superconductor must lower the overall energy of the system or else the material would not be superconducting in the first place. Because the critical magnetic field H_c destroys all the correlations, it is convenient to define the energy density gained by the system in the superconducting state as $(\frac{1}{2})\mu_o H_c^2$. The electrons in a Cooper pair are separated on a length scale of ξ , however, and so the correlations cannot be fully achieved until a distance roughly ξ from the boundary of the superconductor. There is thus an energy per unit area, $(\frac{1}{2})\mu_o H_c^2 \xi$, that is lost because of the presence of the boundary. Now consider the effects of the applied magnetic field on this system. It costs the superconductor energy to maintain the Meissner effect, $\mathbf{B} = 0$, in its bulk; in fact, the energy density required is $(\frac{1}{2})\mu_o H^2$. However, since the field can penetrate the superconductor a distance roughly λ , the system need not expend an some energy per unit area of $(\frac{1}{2})\mu_o H^2\lambda$ to screen over this volume. To summarize, more

than a distance ξ from the boundary, the energy of the material is lowered (because it is superconducting), and more than a distance λ from the boundary the energy of the material is raised (to shield the applied field).

Now, if $\lambda < \xi$, the region of superconducting material greater than λ from the boundary but less than ξ will be higher in energy than that in the bulk of the material. Thus, the surface energy of the boundary is positive and so costs the total system some energy. This class of superconductors is known as type I. Most elemental superconductors, such as aluminum, tin, and lead, are type I. In addition to having $\lambda < \xi$, type I superconductors are generally characterized by low critical temperatures (~ 5 K) and critical fields (~ 0.05 T). Typical type I superconductors and their properties are listed in Table 117.2.

Conversely, if $\lambda < \xi$, the surface energy associated with the boundary is negative and lowers the total system energy. It is therefore thermodynamically favorable for a normal–superconducting interface to form inside these type II materials. Consequently, this class of superconductors does not exhibit the simple Meissner effect as do type I materials. Instead, there are now two critical fields: for applied fields below the lower critical field, H_{c1} , a type II superconductor is in the Meissner state, and for applied fields greater than the upper critical field, H_{c2} , superconductivity is destroyed. The three critical fields are related to each other by $H_c \approx \sqrt{H_{c1}H_{c2}}$

In the range $H_{c1} < H < H_{c2}$, a type II superconductor is said to be in the vortex state because now the applied field can enter the bulk superconductor. Because flux exists in the material, however, the superconductivity is destroyed locally, creating normal regions. Recall that for type II materials the boundary between the normal and superconducting regions lowers the overall energy of the system. Therefore, the flux in the superconductor creates as many normal-superconductor in quantized bundles of magnitude Φ_o known as *vortices* or *fluxons* (the former name derives from the fact that current flows around each quantized bundle in the same manner as a fluid vortex circulates around a drain). The central portion of a vortex, known as the core, is a normal region with an approximate radius of ξ . If a defect-free superconductor is placed in a magnetic field, the individual vortices, whose cores essentially follow the local average field lines, form an ordered triangular array, or flux lattice. As the applied field is raised beyond H_{c1} (where the first vortex enters the superconductor), the distance between adjacent vortex cores decreases to maintain the appropriate flux density in the material. Finally, the upper critical field is reached when the normal cores overlap and the material is no longer superconducting. Indeed, a precise calculation of H_{c2} using the phenomenological theory developed by Vitaly Ginzburg and Lev Landau yields

$$H_{c^2} = \frac{\Phi_o}{2\pi\mu_o \xi^2}$$
(117.19)

which verifies out simple picture. The values of typical type II material parameters are listed in Tables 117.3 and 117.4.

Type II superconductors are of great technical importance because typical H_{c2} values are at least an order of magnitude greater than the typical H_c values of type I materials. It is therefore possible to use type II materials to make high-field magnet wire. Unfortunately, when current is applied to the wire, there is a Lorentz-like force on the vortices, causing them to move. Because the moving vortices carry flux, their motion creates a static voltage drop along the superconducting wire by Faraday's law. As a result, the wire no longer has a zero DC resistance, even though the material is still superconducting. To fix this problem, type II superconductors are usually fabricated with intentional defects, such as impurities or grain boundaries, in their crystalline structure to pin the vortices and prevent vortex motion. The pinning as created because the defect locally weakens the superconductivity in the material, and it is thus energetically favorable for the normal core of the vortex to overlap the nonsuperconducting region in the material. Critical current densities usually quoted for practical type II materials, therefore, really represent the depinning critical current density where the Lorentz-like force can overcome the pinning force. (The depinning critical current density should not be confused with the depairing critical current density, which represents the current when the Cooper pairs have enough kinetic energy to overcome their correlation. The depinning critical current density is typically an order of magnitude less than the depairing critical current density, the latter of which represents the theoretical maximum for J_c .)

By careful manufacturing, it is possible to make superconducting wire with tremendous amounts of currentcarrying capacity. For example, standard copper wire used in homes will carry about 10^7 A/m^2 , whereas a practical type II superconductor like niobium-titanium can carry current densities of 10^{10} A/m^2 or higher even in fields of several teslas. This property, more than a zero DC resistance, is what makes superconducting wire so desirable.

Defining Terms

Superconductivity: A state of matter whereby the correlation of conduction electrons allows a static current to pass without resistance and a static magnetic flux to be excluded from the bulk of the material.

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Further Information

Every two years an Applied Superconductivity Conference is held devoted to practical technological issues. The proceedings of these conferences have been published every other year from 1977 to 1991 in the *IEEE Transactions* on *Magnetics*.

In 1991 the *IEEE Transactions on Applied Superconductivity* began publication. This quarterly journal focuses on both the science and the technology of superconductors and their applications, including materials issues, analog and digital circuits, and power systems. The proceedings of the Applied Superconductivity Conference now appear in this journal.

Figure Captions

Figure 117.1 The phase space for the superconducting alloy niobium-titanium. The material is superconducting inside the volume of phase space indicated. [*Source:* Orlando, T. P. and Delin, K. A. 1991. *Foundations of Applied Superconductivity*. p. 10. Addison-Wesley, Reading, MA. With permission. (As adapted from Wilson, 1983.)]

Figure 117.2 A lumped element model of a superconductor.

Figure 117.3 A real Josephson tunnel junction can be modeled using ideal lumped circuit elements.

Figure 117.4 The *i*-*v* curves for a Josephson junction: (a) $\beta \gg 1$, (b) $\beta \ll 1$.

Transmission Line Geometry	L_0	C_0	R_0
Two identical, thin $\lambda \gg b$ superconducting plates	$\frac{\mu_{t}h}{d} + \frac{2\mu_{0}\lambda^{2}}{db}$	$\frac{\varepsilon_{\iota}d}{h}$	$\frac{8}{db\widetilde{\sigma}_{_{o}}} \left(\frac{\lambda}{\delta}\right)^{4}$
Two identical, thick $\lambda \ll b$ superconducting plates	$\frac{\mu_{t}h}{d} + \frac{2\mu_{0}\lambda}{d}$	$\frac{\varepsilon_{i}d}{h}$	$\frac{4}{d\delta\widetilde{\sigma}_{o}}\left(\frac{\lambda}{\delta}\right)^{3}$
One thick $\lambda \ll b$ superconducting plate and one thick $(\delta_n \ll b)$ ohmic plate	$\frac{\mu_i h}{d} + \frac{\mu_0 \lambda}{d} + \frac{\mu_n \delta_n}{2d}$	$\frac{\varepsilon_{,}d}{h}$	$\frac{1}{d\delta_{{}_{n}}\sigma_{{}_{o,n}}}$

 Table 117.1
 Lumped Circuit Element Parameters Per Unit Length for Typical

 Transverse Electromagnetic Parallel Plate Waveguides*

*The subscript *n* refers to parameters associated with a normal (ohmic) plate. Using these expressions, line input impedance, attenuation, and wave velocity can be calculated.

Source: Orlando, T. P. and Delin, K. A. *Foundations of Applied Superconductivity*, p. 171. Addison-Wesley, Reading, MA. With permission.

Table 117.2 Material Parameters for Type I Superconductors*

Material	$T_c(\mathbf{K})$	λ_{o} (nm)	ξ_{o} (nm)	Δ_o (meV)	$\mu_o H_{co}(mT)$
Al	1.18	50	1600	0.18	10.5
In	3.41	65	360	0.54	23.0
Sn	3.72	50	230	0.59	30.5
Pb	7.20	40	90	1.35	80.0
Nb	9.25	85	40	1.50	198.0

*The penetration depth λ_o is given at zero temperature, as are the coherence length ξ_o , the thermodynamic critical field H_{co} , and the energy gap Δ_o .

Source: Donnelly, R. J. 1981. Cryogenics. In *Physics Vade Mecum*, ed. H. L. Anderson. American Institute of Physics, New York. With permission.

Table 117.3 Material Parameters for Conventional Type II Superconductors*

Material	$T_c(\mathbf{K})$	$\lambda_{\rm GL}$ (0)(nm)	$\xi_{\rm GL}$ (0)(nm)	$\Delta_o ({\rm meV})$	$\mu_{o} \operatorname{H}_{c2,o}(\mathrm{T})$
Pb-ln	7.0	150	30	1.2	0.2
Pb-Bi	8.3	200	20	1.7	0.5
Nb-Ti	9.5	300	4	1.5	13
Nb-N	16	200	5	2.4	15
PbMo ₆ S ₈	15	200	2	2.4	60
V ₃ Ga	15	90	2-3	2.3	23
V ₃ Si	16	60	3	2.3	20
Nb ₃ Sn	18	65	3	3.4	23
Nb ₃ Ge	23	90	3	3.7	38

*The values are only representative because the parameters for alloys and compounds depend on how the material is fabricated. The penetration depth $\lambda_{GL}(0)$ is given as the coefficient of the Ginzburg-Landau temperature dependence as $\lambda_{GL}(T) = \lambda_{GL}(0)(1 - T/T_c)^{-1/2}$; likewise for the coherence length where $\xi_{GL}(T) = \xi_{GL}(0)(1 - T/T_c)^{-1/2}$. The upper critical field $H_{c2,o}$ is given at zero temperature as well as the energy gap Δ_o .

Source: Donnelly, R. J. 1981. Cryogenics. IN Physics Vade Mecum, ed. H. L. Anderson. American Institute of Physics, New York. With permission.

Material	$T_c(\mathbf{K})$
BA _{1-x} K _x Bi O ₃	30
$Rb_{3}C_{60}$	33
M_gB_2	39
YBa ₂ Cu ₃ O ₇	95
Bi ₂ Sr ₂ CaCu ₂ O ₈	85
$Bi_2Sr_2Ca_2Cu_3O_{10}$	110
$TlBa_2Ca_2Cu_3O_{10}$	125
HgBa ₂ Ca ₂ Cu ₃ O ₈	131

 Table 117.4 Type II (Non-Conventional and High-Temperature Superconductors)

See the NIST WebHTS Database at http://www.ceramics.nist.gov/srd/hts/htsquery.htm for more information.

[Text for attached micrograph of superconducting quantum computer logic gate:]

Superconducting circuits have been proposed as one way of constructing logic gates for a quantum computer. The superconducting quantum bit (qubit) shown consists of a loop of superconductor interrupted by three submicron Josephson junctions. The two bits of information are stored persistent currents that flow clockwise or counterclockwise within the loop. Unlike classical information bits, the qubit can be in a quantum superposition of these two macroscopic quantum states, much like the fabled Schrödinger's cat. In the device pictured, the read-out of the qubit current is done with a sensitive superconducting SQUID magnetometer. One of the advantages of superconducting qubits is that high-speed superconducting digital electronics and microwave oscillators can be coupled to the qubit using standard fabrication technology. The device pictured was designed and studied by Prof. Terry Orlando's group at the Massachusetts Institute of Technology (MIT) and was fabricated in niobium at MIT Lincoln Laboratory in collaboration with Dr. Karl Berggren.