2. Overview and Background

This is a course for computer system designers and builders, and for people who want to really understand how systems work, especially concurrent, distributed, and fault-tolerant systems.

The course teaches you
how to write precise specifications for any kind of computer system,
what it means for an implementation to satisfy a specification, and
how to prove that it does.
It also shows you how to use the same methods less formally, and gives you some suggestions for deciding how much formality is appropriate (less formality means less work, and often a more understandable spec, but also more chance to overlook an important detail).

The course also teaches you a lot about the topics in computer systems that we think are the most important: persistent storage, concurrency, naming, networks, distributed systems, transactions, fault tolerance, and caching. The emphasis is on
careful specifications of subtle and sometimes complicated things,
the important ideas behind good implementations, and
how to understand what makes them actually work.
We spend most of our time on specific topics, but we use the general techniques throughout. We emphasize the ideas that different kinds of computer system have in common, even when they have different names.

The course uses a formal language called Spec for writing specs and implementations; you can think of it as a very high level programming language. There is a good deal of written introductory material on Spec (explanations and finger exercises) as well as a reference manual and a formal semantics. We introduce Spec ideas in class as we use them, but we do not devote class time to teaching Spec per se; we expect you to learn it on your own from the handouts.

Because we write specs and do proofs, you need to know something about logic. Since many people don't, there is a concise treatment of the logic you will need at the end of this handout.

This is not a course in computer architecture, networks, operating systems, or databases. We will not talk in detail about how to implement pipelines, memory interconnects, multiprocessors, routers, data link protocols, network management, virtual memory, scheduling, resource allocation, SQL, relational integrity, or TP monitors, although we will deal with many of the ideas that underlie these mechanisms.

Topics

General

Specifications as state machines.
The Spec language for describing state machines (writing specs and implementations).
What it means to implement a spec.
Using abstraction functions and invariants to prove that a program implements a spec.

What it means to have a crash.
What every system builder needs to know about performance.

Specific

Disks and file systems.
Practical concurrency using mutexes (locks) and condition variables; deadlock.
Hard concurrency (without locking): models, specs, proofs, and examples.
Transactions: simple, cached, concurrent, distributed.
Naming: principles, specs, and examples.
Distributed systems: communication, fault-tolerance, and autonomy.
Networking: links, switches, reliable messages and connections.
Remote procedure call and network objects.
Fault-tolerance, availability, consensus and replication.
Caching and distributed shared memory.

Previous editions of the course have also covered security (authentication, authorization, encryption, trust) and system management, but this year we are omitting these topics in order to spend more time on concurrency and semantics and to leave room for project presentations.

Prerequisites

There are no formal prerequisites for the course. However, we assume some knowledge both of computer systems and of mathematics. If you have taken 6.033 and 6.042, you should be in good shape. If you are missing some of this knowledge you can pick it up as we go, but if you are missing a lot of it you can expect to have serious trouble. It's also important to have a certain amount of maturity: enough experience with systems and mathematics to feel comfortable with the basic notions and to have some reliable intuition.

If you know the meaning of the following words, you have the necessary background. If a lot of them are unfamiliar, this course is probably not for you.

Systems

Cache, virtual memory, page table, pipeline
Process, scheduler, address space, priority
Thread, mutual exclusion (locking), semaphore, producer-consumer, deadlock
Transaction, commit, availability, relational data base, query, join
File system, directory, path name, striping, RAID
LAN, switch, routing, connection, flow control, congestion
Capability, access control list, principal (subject)

If you have not already studied Lampson's paper on hints for system design, you should do so as background for this course. It is Butler Lampson, Hints for computer system design, *Proceedings of the Ninth ACM Symposium on Operating Systems Principles*, October 1983, pp 33-48. There is a pointer to it on the course Web page.

Programming

Invariant, precondition, weakest precondition, fixed point
Procedure, recursion, stack
Data type, sub-type, type-checking, abstraction, representation
Object, method, inheritance
Data structures: list, hash table, binary search, B-tree, graph

Mathematics

Function, relation, set, transitive closure
Logic: proof, induction, de Morgan's laws, implication, predicate, quantifier
Probability: independent events, sampling, Poisson distribution
State machine, context-free grammar
Computational complexity, unsolvable problem

If you haven't been exposed to formal logic, you should study the summary at the end of this handout.

References

These are places to look when you want more information about some topic covered or alluded to in the course, or when you want to follow current research. You might also wish to consult Prof. Saltzer's bibliography for 6.033, which you can find on the course web page.

Books

Some of these are fat books better suited for reference than for reading cover to cover, especially Cormen, Leiserson, and Rivest, Jain, Mullender, Hennessy and Patterson, and Gray and Reuter. But the last two are pretty easy to read in spite of their encyclopedic character.

Systems programming: Greg Nelson, ed., *Systems Programming with Modula-3*, Prentice-Hall, 1991. Describes the language, which has all the useful features of C++ but is much simpler and less error-prone, and also shows how to use it for concurrency (a version of chapter 4 is a handout in this course), an efficiently customizable I/O streams package, and a window system.

Performance: Jon Bentley, *Writing Efficient Programs*, Prentice-Hall, 1982. Short, concrete, and practical. Raj Jain, *The Art of Computer Systems Performance Analysis*, Wiley, 1991. Tells you much more than you need to know about this subject, but does have a lot of realistic examples.

Algorithms and data structures: Robert Sedgwick, *Algorithms*, Addison-Wesley, 1983. Short, and usually tells you what you need to know. Tom Cormen, Charles Leiserson, and Ron Rivest, *Introduction to Algorithms*, McGraw-Hill, 1989. Comprehensive, and sometimes valuable for that reason, but usually tells you a lot more than you need to know.

Distributed algorithms: Nancy Lynch, *Distributed Algorithms*, Morgan Kaufmann, 1996. The bible for distributed algorithms. Comprehensive, but a much more formal treatment than in this course. The topic is algorithms, not systems.

Computer architecture: John Hennessy and David Patterson, *Computer Architecture: A Quantitative Approach*, 2nd edition, Morgan Kaufmann, 1995. The bible for computer architecture. The second edition has lots of interesting new material, especially on multiprocessor memory systems and interconnection networks. There's also a good appendix on computer arithmetic; it's useful to know where to find this information, though it has nothing to do with this course.

Transactions, data bases, and fault-tolerance: Jim Gray and Andreas Reuter, *Transaction Processing: Concepts and Techniques*, Morgan Kaufmann, 1993. The bible for transaction processing, with much good material on data bases as well; it includes a lot of practical information that doesn't appear elsewhere in the literature.

Networks: Radia Perlman, *Interconnections: Bridges and Routers*, Addison-Wesley, 1992. Not exactly the bible for networking, but tells you nearly everything you might want to know about how packets are actually switched in computer networks.

Distributed systems: Sape Mullender, ed., *Distributed Systems*, 2nd ed., Addison-Wesley, 1993. A compendium by many authors that covers the field fairly well. Some chapters are much more theoretical than this course. Chapters 10 and 11 are handouts in this course. Chapters 1, 2, 8, and 12 are also recommended. Chapters 16 and 17 are the best you can do to learn about real-time computing; unfortunately, that is not saying much.

User interfaces: Alan Cooper, *About Face*, IDG Books, 1995. Principles, lots of examples, and opinionated advice, much of it good, from the original designer of Visual Basic.

Journals

You can find all of these in the LCS reading room. The cryptic strings in brackets are call numbers there. You can also find the last few years of the ACM publications in the ACM digital library at www.acm.org.

For the current literature, the best sources are the proceedings of the following conferences. 'Sig' is short for "Special Interest Group", a subdivision of the ACM that deals with one field of computing. The relevant ones for systems are SigArch for computer architecture, SigPlan for programming languages, SigOps for operating systems, SigComm for communications, SigMod for data bases, and SigMetrics for performance measurement and analysis.

> Symposium on Operating Systems Principles (SOSP; published as special issues of ACM SigOps *Operating Systems Review*; fall of odd-numbered years) [P4.35.06]
>
> Operating Systems Design and Implementation (OSDI; Usenix Association, now published as special issues of ACM *SigOps Review*; fall of even-numbered years, except spring 1999 instead of fall 1998) [P4.35.U71]
>
> Architectural Support for Programming Languages and Operating Systems (ASPLOS; published as special issues of ACM SigOps *Operating Systems Review*, SigArch *Computer Architecture News*, or *SigPlan Notices*; fall of even-numbered years) [P6.29.A7]

Applications, Technologies, Architecture, and Protocols for Computer Communication, (SigComm conference; published as special issues of ACM SigComm *Computer Communication Review*; annual) [P6.24.D31]

Principles of Distributed Computing (PODC; ACM; annual) [P4.32.D57]

Very Large Data Bases (VLDB; Morgan Kaufmann; annual) [P4.33.V4]

International Symposium on Computer Architecture (ISCA; published as special issues of ACM SigArch *Computer Architecture News*; annual) [P6.20.C6]

Less up to date, but more selective, are the journals. Often papers in these journals are revised versions of papers from the conferences listed above.

ACM Transactions on Computer Systems

ACM Transactions on Database Systems

ACM Transactions on Programming Languages and Systems

There are often good survey articles in the less technical IEEE journals:

IEEE *Computer*, *Networks*, *Communication*, *Software*

The Internet Requests for Comments (RFC's) can be reached from

`http://ds.internic.net/ds/rfc-index.html`

Rudiments of logic

Propositional logic

The basic type is `Bool`, which contains two elements `true` and `false`. Expressions in these operators (and the other ones introduced later) are called 'propositions'.

Basic operators. These are ∧ (and), ∨ (or), and ~ (not).[1] The meaning of these operators can be conveniently given by a 'truth table' which lists the value of `a op b` for each possible combination of values of `a` and `b` (the operators on the right are discussed later) along with some popular names for certain expressions and their operands.

		negation	conjunction	disjunction	equality		implication
		not	and	or			implies
a	b	~a	a ∧ b	a ∨ b	a = b	a ≠ b	a ⇒ b
T	T	F	T	T	T	F	T
T	F		F	T	F	T	F
F	T	T	F	T	F	T	T
F	F		F	F	T	F	T
name of a			conjunct	disjunct			antecedent
name of b			conjunct	disjunct			consequent

Note: In Spec we write ==> instead of the ⇒ that mathematicians use for implication. Logicians write ⊃ for implication, which looks different but is shaped like the > part of ⇒.

In case you have an expression that you can't simplify, you can always work out its truth value by exhaustively enumerating the cases in truth table style. Since the table has only four rows, there are only 16 Boolean operators, one for each possible arrangement of `T` and `F` in a column. Most of the ones not listed don't have common names, though 'not and' is called 'nand' and 'not or' is called 'nor' by logic designers.

The ∧ and ∨ operators are
commutative and
associative and
distribute over each other.
That is, they are just like * (times) and + (plus) on integers, except that + doesn't distribute over *:

```
a + (b * c) ≠ (a + b) * (a + c)
```

but ∨ does distribute over ∧:

```
a ∨ (b ∧ c) = (a ∨ b) ∧ (a ∨ c)
```

An operator that distributes over ∧ is called 'conjunctive'; one that distributes over ∨ is called 'disjunctive'. So both ∧ and ∨ are both conjunctive and disjunctive. This takes some getting used to.

[1] It's possible to write all three in terms of the single operator 'nor' or 'nand', but our goal is clarity, not minimality.

The relation between these operators and ~ is given by DeMorgan's laws (sometimes called the "bubble rule" by logic designers), which say that you can push ~ inside ∧ or ∨ by flipping from one to the other:

```
~ (a ∧ b)  =  ~a ∨ ~b
~ (a ∨ b)  =  ~a ∧ ~b
```

Because `Bool` is the result type of relations like `=`, we can write expressions that mix up relations with other operators in ways that are impossible for any other type. Notably

```
(a = b)  =  ((a ∧ b) ∨ (~a ∧ ~b))
```

Some people feel that the outer = in this expression is somehow different from the inner one, and write it ≡. Experience suggests, however, that this is often a harmful distinction to make.

Implication. We can define an ordering on `Bool` with `false > true`, that is, `false` is greater than `true`. The non-strict version of this ordering is called 'implication' and written ⇒ (rather than ≥ or >= as we do with other types; logicians write it ⊃, which also looks like an ordering symbol). So `(true ⇒ false) = false` (read this as: "`true` is greater than or equal to `false`" is false) but all other combinations are `true`. The expression `a ⇒ b` is pronounced "`a` implies `b`", or "if `a` then `b`".[2]

There are lots of rules for manipulating expressions containing ⇒; the most useful ones are given below. If you remember that ⇒ is an ordering you'll find it easy to remember most of the rules, but if you forget the rules or get confused, you can turn the ⇒ into ∨ by the rule

```
(a ⇒ b)  =  ~a ∨ b
```

and then just use the simpler rules for ∧, ∨, and ~. So remember this even if you forget everything else.

The point of implication is that it tells you when one proposition is stronger than another, in the sense that if the first one is true, the second is also true (because if both `a` and `a ⇒ b` are `true`, then `b` must be `true` since it can't be `false`).[3] So we use implication all the time when reasoning from premises to conclusions. Two more ways to pronounce `a ⇒ b` are "`a` is stronger than `b`" and "`b` follows from `a`". The second pronunciation suggests that it's sometimes useful to write the operands in the other order, as `b ⇐ a`, which can also be pronounced "`b` is weaker than `a`" or "`b` only if `a`"; this should be no surprise, since we do it with other orderings.

Of course, implication has the properties we expect of an ordering:

Transitive: If `a ⇒ b` and `b ⇒ c` then `a ⇒ c`.[4]

[2] It sometimes seems odd that `false` implies `b` regardless of what `b` is, but the "if ... then" form makes it clearer what is going on: if `false` is `true` you can conclude anything, but of course it isn't. A proposition that implies `false` is called 'inconsistent' because it implies anything. Obviously it's bad to think that an inconsistent proposition is true. The most likely way to get into this hole is to think that each of a collection of innocent looking propositions is true when their conjunction turns out to be inconsistent.

[3] It may also seem odd that `false > true` rather than the other way around, since `true` seems better and so should be bigger. But in fact if we want to conclude lots of things, being close to `false` is better because if `false` is true we can conclude anything, but knowing that `true` is true doesn't help at all. Strong propositions are as close to `false` as possible; this is logical brinkmanship. For example, `a ∧ b` is closer to `false` than `a` (there are more values of the variables `a` and `b` that make it `false`), and clearly we can conclude more things from it than from `a` alone.

[4] We can also write this `((a ⇒ b) ∧ (b ⇒ c)) ⇒ (a ⇒ c)`.

Reflexive: `a ⇒ a`.

Anti-symmetric: If `a ⇒ b` and `b ⇒ a` then `a = b`.[5]

Furthermore, ~ reverses the sense of implication (this is called the 'contrapositive'):

```
(a ⇒ b)  =  (~b ⇒ ~a)
```

More generally, you can move a disjunct on the right to a conjunct on the left by negating it. Thus

```
(a ⇒ b ∨ c)  =  (a ∧ ~b ⇒ c)
```

As special cases in addition to the contrapositive we have

```
(a ⇒ b)  =  (a ∧ ~b ⇒ false )  =  ~ (a ∧ ~b) ∨ false = ~a ∨ b
(a ⇒ b)  =  (true   ⇒ ~a ∨ b)  =  false      ∨ ~a ∨ b = ~a ∨ b
```

since `false` and `true` are the identities for ∨ and ∧.

We say that an operator `op` is 'monotonic' in an operand if replacing that operand with a stronger (or weaker) one makes the result stronger (or weaker). Precisely, "`op` is monotonic in its first operand" means that if `a ⇒ b` then `(a op c) ⇒ (b op c)`. Both ∧ and ∨ are monotonic; in fact, any conjunctive operator is monotonic, because if `a ⇒ b` then `a = (a ∧ b)`, so `a op c = (a ∧ b) op c = a op c ∧ b op c ⇒ b op c`.

If you know what a lattice is, you will find it useful to know that the set of propositions forms a lattice with ⇒ as its ordering and (remember, think of ⇒ as "greater than or equal"):

top	= `false`		
bottom	= `true`		
meet	= ∧	least upper bound,	so `(a ∧ b) ⇒ a` and `(a ∧ b) ⇒ b`
join	= ∨	greatest lower bound,	so `a ⇒ (a ∨ b)` and `b ⇒ (a ∨ b)`

This suggests two more expressions that are equivalent to `a ⇒ b`:

`(a ⇒ b) = (a = (a ∧ b))`	'and'ing a weaker term makes no difference, because `a ⇒ b` iff `a =` least upper bound(`a`, `b`).
`(a ⇒ b) = (b = (a ∨ b))`	'or'ing a stronger term makes no difference, because `a ⇒ b` iff `b =` greatest lower bound(`a`, `b`).

Predicate logic

Propositions that have free variables, like `x < 3` or `x < 3 ⇒ x < 5`, demand a little more machinery. You can turn such a proposition into one without a free variable by substituting some value for the variable. Thus if `P(x)` is `x < 3` then `P(5)` is `5 < 3 = false`. To get rid of the free variable without substituting a value for it, you can take the 'and' or 'or' of the proposition for all the possible values of the free variable. These have special names and notation[6]:

`∀ x | P(x)  =  P(x1) ∧ P(x2) ∧ ...`	for all `x`, `P(x)`. In Spec, `(ALL x | P(x))` or `∧ : {x | P(x)}`

[5] Thus `(a = b) = (a ⇒ b ∧ b ⇒ a)`, which is why `a = b` is sometimes pronounced "`a` if and only if `b`" and written "`a` iff `b`".

[6] There is no agreement on what symbol should separate the ∀ `x` or ∃ `x` from the `P(x)`. We use '|' here as Spec does, but other people use '.' or ':' or just a space, or write `(∀ x)` and `(∃ x)`. Logicians traditionally write `(x)` and `(∃ x)`.

`∃ x | P(x) = P(x1) ∨ P(x2) ∨ ...` there exists an `x` such that `P(x)`. In Spec, `(EXISTS x | P(x))` or `∨ : {x | P(x)}`

Here the `xi` range over all the possible values of the free variables.[7] The first is called 'universal quantification'; as you can see, it corresponds to conjunction. The second is called 'existential quantification' and corresponds to disjunction. If you remember this you can easily figure out what the quantifiers do with respect to the other operators.

In particular, DeMorgan's laws generalize to quantifiers:

```
~ (∀ x | P(x)) = (∃ x | ~P(x))
~ (∃ x | P(x)) = (∀ x | ~P(x))
```

Also, because ∧ and ∨ are conjunctive and therefore monotonic, ∀ and ∃ are conjunctive and therefore monotonic.

It is not true that you can reverse the order of ∀ and ∃, but it's sometimes useful to know that having ∃ first is stronger:

```
∃ y | ∀ x | P(x, y) ⇒ ∀ x | ∃ y | P(x, y)
```

Intuitively this is clear: a `y` that works for every `x` can surely do the job for each particular `x`.

If we think of `P` as a relation, the consequent in this formula says that `P` is total (relates every `x` to some `y`). It doesn't tell us anything about how to find a `y` that is related to `x`. As computer scientists, we like to be able to compute things, so we prefer to have a function that computes `y`, or the set of `y`'s, from `x`. This is called a 'Skolem function'; in Spec you write `P.func` (or `P.setF` for the set). `P.func` is total if `P` is total. Or, to turn this around, if we have a total function `f` such that `∀ x | P(x, f(x))`, then certainly `∀ x | ∃ y | P(x, y)`; in fact, `y = f(x)` will do. Amazing.

[7] In general this might not be a countable set, so the conjunction and disjunction are written in a somewhat misleading way, but this complication won't make any difference to us.

Summary of logic

The ∧ and ∨ operators are commutative and associative and distribute over each other.

DeMorgan's laws:
```
~ (a ∧ b) = ~a ∨ ~b
~ (a ∨ b) = ~a ∧ ~b
```

Implication: `(a ⇒ b) = ~a ∨ b`

Implication is the ordering in a lattice (a partially ordered set in which every subset has a least upper and a greatest lower bound) with

top	= `false`	so `false ⇒ true`
bottom	= `true`	
meet	= ∧	least upper bound, so `(a ∧ b) ⇒ a`
join	= ∨	greatest lower bound, so `a ⇒ (a ∨ b)`

For all `x`, `P(x)`:

```
∀ x | P(x) = P(x1) ∧ P(x2) ∧ ...
```

There exists an `x` such that `P(x)`:

```
∃ x | P(x) = P(x1) ∨ P(x2) ∨ ...
```

Index for logic

3. Introduction to Spec

This handout explains what the Spec language is for, how to use it effectively, and how it differs from a programming language like C, Pascal, Clu, Java, or Scheme. Spec is very different from these languages, but it is also much simpler. Its meaning is clearer and Spec programs are more succinct and less burdened with trivial details. The handout also introduces the main constructs that are likely to be unfamiliar to a programmer. You will probably find it worthwhile to read it over more than once, until those constructs are familiar.

Spec is a language for writing precise descriptions of digital systems, both sequential and concurrent. In Spec you can write something that differs from a practical implementation (for instance, one written in C) only in minor details of syntax. This sort of thing is usually called a program. Or you can write a very high level description of the behavior of a system, usually called a specification. A good specification is almost always quite different from a good program. You can use Spec to write either one, but not the same style of Spec. The flexibility of the language means that you need to know the purpose of your Spec in order to write it well.

Most people know a lot more about writing programs than about writing specs, so this introduction emphasizes how Spec differs from a programming language and how to use it to write good specs. It does not attempt to be either complete or precise, but other handouts fill these needs. The *Spec Reference Manual* (handout 4) describes the language completely; it gives the syntax of Spec precisely and the semantics informally. *Atomic Semantics of Spec* (handout 9) describes precisely the meaning of an atomic command; here ‘precisely’ means that you should be able to get an unambiguous answer to any question. The section “Non-Atomic Semantics of Spec” in handout 17 on formal concurrency describes the meaning of a non-atomic command.

Spec’s notation for commands, that is, for changing the state, is derived from Edsger Dijkstra’s guarded commands (E. Dijkstra, *A Discipline of Programming*, Prentice-Hall, 1976) as extended by Greg Nelson (G. Nelson, A generalization of Dijkstra’s calculus, *ACM TOPLAS* **11**, 4, Oct. 1989, pp 517-561). The notation for expressions is derived from mathematics.

This handout starts with a discussion of specifications and how to write them, with many small examples of Spec. Then there is an outline of the Spec language, followed by three extended examples of specs and implementations. At the end are two handy tear-out one-page summaries, one of the language and one of the official POCS strategy for writing specs and implementations.

What is a specification for?

The purpose of a specification is to communicate precisely all the essential facts about the behavior of a system. The important words in this sentence are:

communicate	The spec should tell both the client and the implementer what each needs to know.
precisely	We should be able to prove theorems or compile machine instructions based on the spec.
essential	Unnecessary requirements in the spec may confuse the client or make it more expensive to implement the system.
behavior	We need to know exactly what we mean by the behavior of the system.

Communication

Spec mediates communication between the client of the system and its implementer. One way to view the specification is as a contract between these parties:

> The client agrees to depend only on the system behavior expressed in the spec; in return it can count on the implementation to provide a system that actually does behave as the spec says it should.
>
> The implementer agrees to provide a system that behaves according to the spec; in return it is free to arrange the internals of the system however it likes, and it does not have to deliver anything not laid down in the spec.

Usually the implementer of a spec is a programmer, and the client is another programmer. Usually the implementer of a program is a compiler or a computer, and the client is a programmer.

Behavior

What do we mean by behavior? In real life a spec defines not only the functional behavior of the system, but also its performance, cost, reliability, availability, size, weight, etc. In this course we will deal with these matters informally if at all. The Spec language doesn’t help much with them.

Spec is concerned only with the possible state transitions of the system, on the theory that the possible state transitions tell the complete story of the functional behavior of a digital system. So we make the following definitions:

> A *state* is the values of a set of names (for instance, `x=3, color=red`).
>
> A *history* is a sequence of states such that each pair of adjacent states is a transition of the system (for instance, `x=1; x=2; x=5` is the history if the initial state is `x=1` and the transitions are “if `x = 1` then `x := x + 1`” and “if `x = 2` then `x := 2 * x + 1`”).
>
> A *behavior* is a set of histories (a non-deterministic system can have more than one history).

How can we specify a behavior?

One way to do this is to just write down all the histories in the behavior. For example, if the state just consists of a single integer, we might write

```
 1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 1  2  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
...
 1  2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1  2
....
 1  2  3  4  5  1  2  3  1  2  3  4  5  6  7  8  9 10
```

The example reveals two problems with this approach:

The sequences are long, and there are a lot of them, so it takes a lot of space to write them down. In fact, in most cases of interest the sequences are infinite, so we can't actually write them down.

It isn't too clear from looking at such a set of sequences what is really going on.

Another description of this set of sequences from which these examples are drawn is "18 integers, each one either `1` or one more than the preceding one." This is concise and understandable, but it is not formal enough either for mathematical reasoning or for directions to a computer.

Precise

In Spec the set of sequences can be described in many ways, for example, by the expression

```
{s: SEQ Int |    s.size = 18
            /\ (ALL i: Int | 0 <= i /\ i < s.size ==>
                             s(i) = 1 \/ (i > 0 /\ s(i) = s(i-1) + 1)) }
```

Here the expression in `{...}` is very close to the usual mathematical notation for defining a set. Read it as "The set of all `s` which are sequences of integers such that `s.size = 18` and ...". Spec sequences are indexed from `0`. The `(ALL ...)` is a universally quantified predicate, and `==>` stands for implication, since Spec uses the more familiar `=>` for 'then' in a guarded command. Throughout Spec the '`|`' symbol separates a declaration of some new names and their types from the scope in which they are meaningful.

Alternatively, here is a state machine that generates the sequences we want as the successive values of the variable `i`. We specify the transitions of the machine by starting with primitive *assignment commands* and putting them together with a few kinds of compound commands. Each command specifies a set of possible transitions.

```
VAR i, j |
    << i := 1; j := 1 >> ;
    DO << j < 18 => BEGIN i := 1 [] i := i + 1 END; j := j + 1 >> OD
```

Here there is a good deal of new notation, in addition to the familiar semicolons, assignments, and plus signs.

`VAR i, j |` introduces the local variables `i` and `j` with arbitrary values. Because `;` binds more tightly than `|`, the scope of the variables is the rest of the example.

The `<< ... >>` brackets delimit the atomic actions or transitions of the state machine. All the changes inside these brackets happen as one transition of the state machine.

`j < 18 => ...` is a transition that can only happen when `j < 18`. Read it as "if `j < 18` then `...`". The `j < 18` is called a *guard*. If the guard is false, we say that the entire command *fails*.

`i := 1 [] i := i + 1` is a *non-deterministic* transition which can either set `i` to `1` or increment it. Read `[]` as 'or'.

The `BEGIN ... END` brackets are just brackets for commands, like { ... } in C. They are there because `=>` binds more tightly than the `[]` operator inside the brackets; without them the meaning would be "either set `i` to `1` if `j < 18` or increment `i` and `j` unconditionally".

Finally, the `DO ... OD` brackets mean: repeat the `...` transition as long as possible. Eventually `j` becomes `18` and the guard becomes false, so the command inside the `DO ... OD` fails and can no longer happen.

The expression approach is better when it works naturally, as this example suggests, so Spec has lots of facilities for describing values: sequences, sets, and functions as well as integers and booleans. Usually, however, the sequences we want are too complicated to be conveniently described by an expression; a state machine can describe them much more easily.

State machines can be written in many different ways. When each transition involves only simple expressions and changes only a single integer or boolean state variable, we think of the state machine as a program, since we can easily make a computer exhibit this behavior. When there are transitions that change many variables, non-deterministic transitions, big values like sequences or functions, or expressions with quantifiers, we think of the state machine as a specification, since it may be much easier to understand and reason about it, but difficult to make a computer exhibit this behavior. In other words, large atomic actions, non-determinism, and expressions that compute sequences or functions are hard to implement. It may take a good deal of ingenuity to find an implementation that has the same behavior but uses only the small, deterministic atomic actions and simple expressions that are easy for the computer.

Essential

The hardest thing for most people to learn about writing specs is that *a spec is not a program*. A spec defines the behavior of a system, but unlike a program it need not, and usually should not, give any practical method for producing this behavior. Furthermore, it should pin down the behavior of the system only enough to meet the client's needs. Details in the spec that the client doesn't need can only make trouble for the implementer.

The example we just saw is too artificial to illustrate this point. To learn more about the difference between a spec and an implementation consider the following:

```
CONST eps := 10**-8

APROC SquareRoot0(x: Real) -> Real =
     << VAR y : Real | Abs(x - y*y) < eps => RET y >>
```

(Spec as described in the reference manual doesn't have a `Real` data type, but we'll add it for the purpose of this example.)

The combination of `VAR` and `=>` is a very common Spec idiom; read it as "choose a `y` such that `Abs(x - y*y) < eps` and do `RET y`". Why is this the meaning? The `VAR` makes a choice of any `Real` as the value of `y`, but the entire transition on the second line cannot occur unless the guard is true. The result is that the choice is restricted to a value that satisfies the guard.

What can we learn from this example? First, the result of `SquareRoot0(x)` is not determined by the value of `x`; any result whose square is within `eps` of `x` is possible. This is why `SquareRoot0` is written as a procedure rather than a function; the result of a function has to be determined by the arguments and the current state, so that the value of an expression like `f(x) = f(x)` will be `true`. In other words, `SquareRoot0` is non-deterministic.

Why did we write it that way? First of all, there might not be any `Real` (that is, any floating-point number of the kind used to represent `Real`) whose square exactly equals `x`.[1] Second, we may not want to pay for an implementation that gives the closest possible answer. Instead, we may settle for a less accurate answer in the hope of getting the answer faster.

You have to make sure you know what you are doing, though. This spec allows a negative result, which is perhaps not what we really wanted. We could have written (highlighting changes with boxes):

```
APROC SquareRoot1(x: Real) -> Real =
     << VAR y : Real | y >= 0 /\ Abs(x - y*y) < eps => RET y >>
```

to rule that out. Also, the spec produces no result if `x < 0`, which means that `SquareRoot1(-1)` will fail (see the section on commands for a discussion of failure). We might prefer a total function that raises an exception:

```
APROC SquareRoot2(x: Real) -> Real RAISES {undefined} =
     <<     x >= 0 => VAR y : Real | y >= 0 /\ Abs(x - y*y) < eps => RET y
       [*]  RAISE undefined >>
```

The `[*]` is 'else'; it does its second operand iff the first one fails. Exceptions in Spec are much like exceptions in CLU. An exception is contagious: once started by a `RAISE` it causes any containing expression or command to yield the same exception, until it runs into an exception handler (not shown here). The `RAISES` clause of a routine declaration must list all the exceptions that the procedure body can generate, either by `RAISES` or by invoking another routine.

An implementation of this spec would look quite different from the spec itself. Instead of the existential quantifier implied by the `VAR y`, it would have an algorithm for finding `y`, for

[1] We could accommodate this fact of life by specifying the closest floating-point number. This would still be non-deterministic in the case that two such numbers are equally close, so if we wanted a deterministic spec we would have to give a rule for choosing one of them, for instance, the smaller.

instance, Newton's method. In the algorithm you would only see operations that have obvious implementations in terms of the load, store, arithmetic, and test instructions of a computer. Probably the implementation would be deterministic.

Another way to write these specs is as functions that return the set of possible answers. Thus

```
FUNC SquareRoots1(x: Real) -> SET Real =
     RET {y : Real | y >= 0 /\ Abs(x - y*y) < eps}
```

Note that the form inside the `{...}` set constructor is the same as the guard on the `RET`. To get a single result you can use the set's `choose` method: `SquareRoots1(2).choose`.[2]

In the next section we give an outline of the Spec language. Following that are three extended examples of specs and implementations for fairly realistic systems. At the end is a one-page summary of the language.

[2] `r := SquareRoots1(x).choose` (using the function) is almost the same as `r := SquareRoot1(x)` (using the procedure). The difference is that because `choose` is a function it always returns the same element (even though we don't know in advance which one) when given the same set, and hence when `SquareRoots1` is given the same argument. The procedure, on the other hand, is non-deterministic and can return different values on successive calls.

An outline of the Spec language

The Spec language has two main parts:

- An *expression* describes how to compute a result (a value or an exception) as a function of other values: either literal constants or the current values of state variables.
- A *command* describes possible transitions of the state variables. Another way of saying this is that a command is a relation on states: it allows a transition from `s1` to `s2` iff it relates `s1` to `s2`.

Both are based on the *state*, which in Spec is a mapping from names to values. The names are called state variables or simply variables: in the sequence example above they are `i` and `j`. Actually a command relates states to *outcomes*; an outcome is either a state (a normal outcome) or a state together with an exception (an exceptional outcome).

There are two kinds of commands:

- An *atomic* command describes a set of possible transitions, or equivalently, a set of pairs of states. For instance, the command `<< i := i + 1 >>` describes the transitions `i=1`→`i=2`, `i=2`→`i=3`, etc. (Actually, many transitions are summarized by `i=1`→`i=2`, for instance, `(i=1, j=1)`→`(i=2, j=1)` and `(i=1, j=15)`→`(i=2, j=15)`). If a command allows more than one transition from a given state we say it is non-deterministic. For instance, on page 3 the command `BEGIN i := 1 [] i := i + 1 END` allows the transitions `i=2`→`i=1` and `i=2`→`i=3`.
- A *non-atomic* command describes a set of sequences of states (by contrast with the set of pairs for an atomic command). More on this below.

A sequential program, in which we are only interested in the initial and final states, can be described by an atomic command.

The meaning of an expression, which is a function from states to values (or exceptions), is much simpler than the meaning of an atomic command, which is a relation between states, for two reasons:

- The expression yields a single value rather than an entire state.
- The expression yields at most one value, whereas a non-deterministic command can yield many final states.

A atomic command is still simple, much simpler than a non-atomic command, because:

- Taken in isolation, the meaning of a non-atomic command is a relation between an initial state and a history. Again, many histories can stem from a single initial state.
- The meaning of the composition of two non-atomic commands is not any simple combination of their relations, such as the union, because the commands can interact if they share any variables that change.

These considerations lead us to describe the meaning of a non-atomic command by breaking it down into its atomic subcommands and connecting these up with a new state variable called a program counter. The details are somewhat complicated; they are sketched in the discussion of atomicity below, and described in handout 17 on formal concurrency.

The moral of all this is that you should use the simpler parts of the language as much as possible: expressions rather than atomic commands, and atomic commands rather than non-atomic ones. To encourage this style, Spec has a lot of syntax and built-in types and functions that make it easy to write expressions clearly and concisely. You can write many things in a single Spec expression that would require a number of C statements, or even a loop. Of course, an implementation with a lot of concurrency will necessarily have more non-atomic commands, but this complication should be put off as long as possible.

Organizing the program

In addition to the expressions and commands that are the core of the language, Spec has four other mechanisms that are useful for organizing your program and making it easier to understand.

- A *routine* is a named computation with parameters, in other words, an abstraction of the computation. Parameters are passed by value. There are four kinds of routine:

  A *function* (defined with `FUNC`) is an abstraction of an expression.

  An *atomic procedure* (defined with `APROC`) is an abstraction of an atomic command.

  A general procedure (defined with `PROC`) is an abstraction of a non-atomic command.

  A *thread* (defined with `THREAD`) is the way to introduce concurrency.

- A *type* is a highly stylized assertion about the set of values that a name or expression can assume. A type is also a convenient way to group and name a collection of routines, called its *methods*, that operate on values in that set.
- An *exception* is a way to report an unusual outcome.
- A *module* is a way to structure the name space into a two-level hierarchy. An identifier `i` declared in a module `m` has the name `m.i` throughout the program. A *class* is a module that can be instantiated many times to create many objects.

A Spec program is some global declarations of variables, routines, types, and exceptions, plus a set of modules each of which declares some variables, routines, types, and exceptions.

The next two sections describe things about Spec's expressions and commands that may be new to you. It doesn't answer every question about Spec; for those answers, read the reference manual and the handouts on Spec semantics. There is a one-page summary at the end of this handout.

Expressions, types, and functions

Expressions are for computing functions of the state. A Spec expression is a constant, a variable, or an invocation of a function on an argument that is some sub-expression. The values of these expressions are the constant, the current value of the variable, or the value of the function at the value of the argument. There are no side-effects; those are the province of commands. There is quite a bit of syntactic sugar for function invocations. An expression may be undefined in a state; if a simple command evaluates an undefined expression, the command fails (see below).

A Spec type defines two things:

A set of values; we say that a value *has* the type if it's in the set. The sets are not disjoint.

A set of functions called the *methods* of the type. There is convenient syntax `v.m` for invoking method `m` on a value `v` of the type.

Spec is strongly typed. This means that you are supposed to declare the types of your variables, just as you do in Pascal or CLU. In return the language defines a type for every expression[3] and ensures that the value of the expression always has that type. In particular, the value of a variable always has the declared type. You should think of a type declaration as a stylized comment that has a precise meaning and could be checked mechanically.

If `Foo` is a type, you can omit it in a declaration of the identifiers `foo, foo1, foo'` etc. Thus

```
VAR int1, bool2, char'' | ...
```

is short for

```
VAR int1: Int, bool2: Bool, char'': Char | ...
```

Spec has the usual types: `Int`, `Nat` (non-negative `Int`), `Bool`, functions, sets, records, tuples, and variable-length arrays called sequences. A sequence is a function whose domain is `{0, 1, ..., n-1}` for some `n`. In addition to the usual functions like `"+"` and `"\/"`, Spec also has some less usual operations on these types, which are valuable when you want to suppress implementation detail: constructors and combinations.

You can make a type with fewer values using `SUCHTHAT`. For example,

```
TYPE T = Int SUCHTHAT (\ i: Int | 0 <= i /\ i <= 4)
```

has the value set `{0, 1, 2, 3, 4}`. Here the `(\ ...)` is a lambda expression that defines a function from `Int` to `Bool`, and a value has type `T` if it's an `Int` and the function maps it to `true`.

Section 5 of the reference manual describes expressions and lists all the built-in operators. You should read the list, which also gives their precedence and has pointers to explanations of their meaning. Section 4 describes the types. Section 9 defines the built-in methods for sequences, sets, and functions; you should read it over so that you know the vocabulary.

[3] Note that a value may have many types, but a variable or an expression has exactly one type: for a variable, it's the declared type, and for a complex expression it's the result type of the top-level function in the expression.

Constructors

Constructors for functions, sets, and sequences make it easy to toss large values around. For instance, you can describe a database as a function `db` from names to data records with two fields:

```
TYPE DB = (String -> Entry)
TYPE Entry = [salary: Int, birthdate: Int]
VAR db := DB{}
```

Here `db` is initialized using a function constructor whose value is a function undefined everywhere. The type can be omitted in a variable declaration when the variable is initialized; it is taken to be the type of the initializing expression. The type can also be omitted when it is the upper case version of the variable name, `DB` in this example.

Now you can make an entry with

```
db := db{ "Smith" -> Entry{salary := 23000, birthdate := 1955} }
```

using another function constructor. The value of the constructor is a function that is the same as `db` except at the argument `"Smith"`, where it has the value `Entry{...}`, which is a record constructor. The assignment could also be written

```
db("Smith") := Entry{salary := 23000, birthdate := 1955}
```

which changes the value of the `db` function at `"Smith"` without changing it anywhere else. This is actually a shorthand for the previous assignment. You can omit the field names if you like, so that

```
db("Smith") := Entry{23000, 1955}
```

has the same meaning as the previous assignment. Obviously this shorthand is less readable and more error-prone, so use it with discretion. Another way to write this assignment is

```
db("Smith").salary := 23000; db("Smith").birthdate := 1955
```

The set of names in the database can be expressed by a set constructor. It is just

```
{n: String | db!n},
```

in other words, the set of all the strings for which the `db` function is defined ('`!`' is the 'is-defined' operator; that is, `f!x` is true iff `f` is defined at `x`). Read this "the set of strings `n` such that `db!n`". You can also write it as `db.dom`, the domain of `db`; section 9 of the reference manual defines lots of useful built in methods for functions, sets, and sequences. It's important to realize that you can freely use large (possibly infinite) values such as the `db` function. You are writing a specification, and you don't need to worry about whether the compiler is clever enough to turn an expensive-looking manipulation of a large object into a cheap incremental update. That's the implementer's problem (so you may have to worry about whether she is clever enough).

If we wanted the set of lengths of the names, we would write

```
{n: String | db!n | n.size}
```

This three part set constructor contains `i` if and only if there exists an `n` such that `db!n` and `i = n.size`. So `{n: String | db!n}` is short for `{n: String | db!n | n}`. You can introduce more than one name, in which case the third part defaults to the last name. For example, if we represent a directed graph by a function on pairs of nodes that returns `true` when there's an edge from the first to the second, then

```
{n1: Node, n2: Node | graph(n1, n2) | n2}
```

is the set of nodes that are the target of an edge, and the "`| n2`" could be omitted.

Following standard mathematical notation, you can also write

```
{f :IN openFiles | f.modified}
```
to get the set of all open, modified files. This is equivalent to
```
{f: File | f IN openFiles /\ f.modified}
```
because if s is a `SET T`, then `IN s` is a type whose values are the `T`'s in `s`; in fact, it's the type `T SUCHTHAT (\ t | t IN s)`. This form also works for sequences, where the second operand of `:IN` provides the ordering. So if `s` is a sequence of integers, `{x :IN s | x > 0}` is the positive ones, `{x :IN s | x > 0 | x * x}` is the squares of the positive ones, and `{x :IN s | | x * x}` is the squares of all the integers, because an omitted predicate defaults to `true`.[4]

To get sequences that are more complicated you can use sequence generators with `BY` and `WHILE`.
```
{i := 1 BY i + 1 WHILE i <= 5 | true | i}
```
is `{1, 2, 3, 4, 5}`; the second and third parts could be omitted. This is just like the "for" construction in C. An omitted `WHILE` defaults to `true`, and an omitted `:=` defaults to an arbitrary choice for the initial value. If you write several generators, each variable gets a new value for each value produced, but the second and later variables are initialized first. So to get the sums of successive pairs of elements of `s`, write
```
{x := s BY x.tail WHILE x.size > 1 | | x(0) + x(1)}
```
To get the sequence of partial sums of `s`, write (eliding `| | sum` at the end)
```
{x :IN s, sum := 0 BY sum + x}
```
Taking `last` of this would give the sum of the elements of `s`. To get a sequence whose elements are reversed from those of `s`, write
```
{x :IN s, rev := {} BY {x} + rev}.last
```
To get the sequence `{f(e), f2(e), ..., fn(e)}`, write
```
{i :IN 1 .. n, iter := e BY f(iter)}
```
This uses the `..` operator; `i .. j` is the sequence `{i, i+1, ..., j-1, j}`.

Combinations

A combination is a way to combine the elements of a sequence or set into a single value using an infix operator, which must be associative, must have an identity, and must be commutative if it is applied to a set. You write "operator : sequence or set". Thus
```
+ : (SEQ String){"He", "l", "lo"} = "He" + "l" + "lo" = "Hello"
```
because + on sequences is concatenation, and
```
+ : {i :IN 1 .. 4 | | i**2} = 1 + 4 + 9 + 16 = 30
```

Existential and universal quantifiers make it easy to describe properties without explaining how to test for them in a practical way. For instance, a predicate that is `true` iff the sequence `s` is sorted is
```
(ALL i :IN 1 .. s.size-1   | s(i-1) <= s(i))
```
This is a common idiom; read it as
"for all `i` in `1 .. s.size-1`, `s(i-1) <= s(i)`".
This could also be written
```
(ALL i :IN (s.dom - {0}) | s(i-1) <= s(i))
```
since `s.dom` is the domain of the function `s`, which is the non-negative integers `< s.size`.

[4] In the sequence form, `IN s` is not a type but a special construct; treating it as a type would throw away the essential ordering information.

Because a universal quantification is just the conjunction of its predicate for all the values of the bound variables, it is simply a combination using `/\` as the operator:
```
(ALL    i | Predicate(i)) = /\ : {i | Predicate(i)}
```
Similarly, an existential quantification is just a similar disjunction, hence a combination using `\/` as the operator:
```
(EXISTS i | Predicate(i)) = \/ : {i | Predicate(i)}
```
Spec has the redundant `ALL` and `EXISTS` notations because they are familiar.

If you want to get your hands on a value that satisfies an existential quantifier, you can construct the set of such values and use the `choose` method to pick out one of them:
```
{i | Predicate(i)}.choose
```
This is deterministic: `choose` always returns the same value given the same set (a necessary property for it to be a function). It is undefined if the set is empty, which is the case in the example if no `i` satisfies `Predicate`.

The `VAR` command described in the next section on commands is another form of existential quantification that lets you get your hands on the value, but it is non-deterministic.

Functions

Like everything (except types), functions are ordinary values in Spec. Given a function, you can use a function constructor to make another one that is the same except at a particular argument, as in the `DB` example above. Another example is `f{x -> 0}`, which is the same as `f` except that it is 0 at `x`. If you have never seen a construction like this one, think about it for a minute. Suppose you had to implement it. If `f` is represented as a table of (argument, result) pairs, the implementation will be easy. If `f` is represented by code that computes the result, the implementation is less obvious, but you can make a new piece of code that says
```
(\ y: Int | ( (y = x) => 0 [*] f(y) ))
```
Here `'\'` is 'lambda', and the subexpression `( (y = x) => 0 [*] f(y) )` is a conditional, modeled on the conditional commands we saw in the first section; its value is `0` if `y = x` and `f(y)` otherwise, so we have changed `f` just at `0`, as desired. If the else clause `[*] f(y)` is omitted, the condition is undefined if `y # x`. Of course in a running program you probably wouldn't want to construct new functions very often, so a piece of Spec that is intended to be close to a practical implementation must use function constructors carefully.

Functions can return functions as results. Thus `T->U->V` is the type of a function that takes an `T` and returns a function of type `U->V`, which in turn takes a `U` and returns a `V`. If `f` has this type, then `f(t)` has type `U->V`, and `f(t)(u)` has type `V`. Compare this with `(T, U)->V`, the type of a function which takes an `T` and a `U` and returns a `V`. If `g` has this type, `g(t)` doesn't type-check, and `g(t, u)` has type `V`. Obviously `f` and `g` are closely related, but they are not the same.

You can define your own functions either by lambda expressions like the one above, or more generally by function declarations like this one
```
FUNC NewF(y: Int) -> Int = RET ( (y = x) => 0 [*] f(y) )
```
The value of this `NewF` is the same as the value of the lambda expression. To avoid some redundancy in the language, the meaning of the function is defined by a command in which `RET` sub-commands specify the value of the function. The command might be syntactically non-deterministic (for instance, it might contain `VAR` or `[]`), but it must specify at most one result

value for any argument value; if it specifies no result values for an argument or more than one value, the function is undefined there. If you need a full-blown command in a function constructor, you can write it with `LAMBDA` instead of `\`:

```
(LAMBDA (y: Int) -> Int = RET ( (y = x) => 0 [*] f(y) ))
```

You can compose two functions with the `*` operator, writing `f * g`. This means to apply `f` first and then `g`. It is often useful when `f` is a sequence (remember that a `SEQ T` is a function from `{0, 1, ..., size-1}` to `T`), since the result is a sequence with every element of `f` mapped by `g`. So:

```
(0 .. 4) * {\ i: Int | i*i} = (SEQ Int){0, 1, 4, 9, 16}
```

since `0 .. 4 = {0, 1, 2, 3, 4}` because `Int` has a method `..` with the obvious meaning: `i .. j = {i, i+1, ..., j-1, j}`. In the section on constructors we saw another way to write

```
(0 .. 4) * {\ i: Int | i*i},
```

as

```
{i :IN 0 .. 4 | | i*i}.
```

This is more convenient when the mapping function is defined by an expression, as it is here, but it's less convenient if the mapping function already has a name. Then it's shorter and clearer to write

```
(0 .. 4) * factorial
```

rather than

```
{i :IN 0 .. 4 | | factorial(i)}.
```

Methods

Methods are a convenient way of packaging up some functions with a type so that the functions can be applied to values of that type concisely and without mentioning the type itself. Look at the definitions in section 9 of the *Spec Reference Manual*, which give methods for the built-in types `SEQ T`, `SET T`, and `T->U`. If `s` is a `SEQ T`, `s.head` is `Sequence[T].Head(s)`, which is just `s(0)` (which is undefined if `s` is empty). You can see that it's shorter to write `s.head`.[5]

You can define your own methods by using `WITH`. For instance, consider

```
TYPE Complex = [re: Real, im: Real] WITH {"+":=Add, mag:=Mag}
```

`Add` and `Mag` are ordinary Spec functions that you must define, but you can now invoke them on a `c` which is `Complex` by writing `c + c'` and `c.mag`, which mean `Add(c, c')` and `Mag(c)`. You can use existing operator symbols or make up your own; see section 3 of the reference manual for lexical rules. You can also write `Complex."+"` and `Complex.mag` to denote the functions `Add` and `Mag`; this may be convenient if `Complex` was declared in a different module. Using `Add` as a method does not make it private, hidden, static, local, or anything funny like that.

When you nest `WITH` the methods pile up in the obvious way. Thus

```
TYPE MoreComplex = Complex WITH {"-":=Sub, mag:=Mag2}
```

has an additional method `"-"`, the same `"+"` as `Complex`, and a different `mag`. Many people call this 'inheritance' and 'overriding'.

[5] Of course, `s(0)` is shorter still, but that's an accident; there is no similar alternative for `s.tail`.

Commands

Commands are for changing the state. Spec has a few simple commands, and seven operators for combining commands into bigger ones. The main simple commands are assignment and routine invocation. There are also simple commands to raise an exception, to return a function result, and to `SKIP`, that is, do nothing. If a simple command evaluates an undefined expression, it fails (see below).

The operators on commands are:

- A conditional operator: `predicate => command`, read "if `predicate` then `command`". The predicate is called a *guard*.
- Choice operators: `c1 [] c2` and `c1 [*] c2`, read 'or' and 'else'.
- Sequencing operators: `c1 ; c2` and `c1 EXCEPT handler`. The `handler` is a special form of conditional command: `exception => command`.
- Variable introduction: `VAR id: T | command`, read "choose `id` of type `T` such that `command` doesn't fail".
- Loops: `DO command OD`.

Section 6 of the reference manual describes commands. *Atomic Semantics of Spec* gives a precise account of their semantics. It explains that the meaning of a command is a *relation* between a state and an outcome (a state plus an optional exception), that is, a set of possible state-to-outcome transitions.

Conditionals and choice

The figure below (copied from Nelson's paper) illustrates conditionals and choice with some very simple examples. Here is how they work:

The command

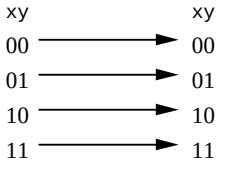
```
p => c
```

means to do `c` if `p` is true. If `p` is false this command fails; in other words, it has no outcome. More precisely, if `s` is a state in which `p` is false or undefined, this command does not relate `s` to any outcome.

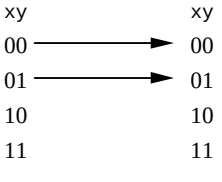
What good is such a command? One possibility is that `p` will be true some time in the future, and then the command will have an outcome and allow a transition. Of course this can only happen in a concurrent program, where there is something else going on that can make `p` true. Even if there's no concurrency, there might be an alternative to this command. For instance, it might appear in the larger command

```
      p  => c
[]    p' => c'
```

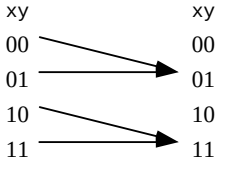
in which you read `[]` as 'or'. This fails only if each of `p` and `p'` is false or undefined. If both are true (as in the `00` state in the south-west corner of the figure), it means to do either `c` or `c'`; the choice is non-deterministic. If `p'` is `~ p` then they are never both false, and if `p` is defined this command is equivalent to



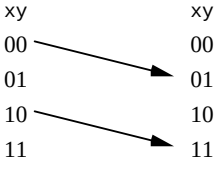
SKIP



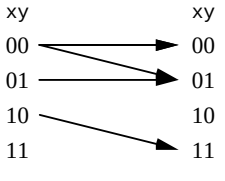
x = 0 => SKIP
(partial)



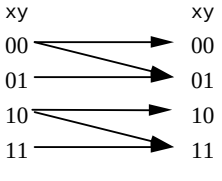
y := 1



y = 0 => y := 1
(partial)



x = 0 => SKIP
[] y = 0 => y := 1
(partial, non-deterministic)



SKIP
[] y = 0 => y := 1
(non-deterministic)

Combining commands

```
    p => c
[*] c'
```

in which you read `[*]` as ‘else’. On the other hand, if `p` is undefined the two commands differ, because the first one fails (since neither guard can be evaluated), while the second does `c'`.

Both `c1 [] c2` and `c1 [*] c2` fail only if *both* `c1` and `c2` fail. If you think of a Spec program operationally (that is, as executing one command after another), this means that if the execution makes some choice that leads to failure later on, it must ‘back-track’ and try the other alternatives until it finds a set of choices that succeed. For instance, no matter what `x` is, after

```
    y = 0 => x := x - 1; x < y => x := 1
[]  y > 0 => x := 3    ; x < y => x := 2
[*] SKIP
```


if `y = 0` initially, `x = 1` afterwards, if `y > 3` initially, `x = 2` afterwards, and otherwise `x` is unchanged. If you think of it relationally, `c1 [] c2` has all the transitions of `c1` (there are none if `c1` fails, several if it is non-deterministic) as well as all the transitions of `c2`. Both failure and non-determinism can arise from deep inside a complex command, not just from a top-level `[]` or `VAR`.

The precedence rules for commands are

`EXCEPT`	binds tightest
`;`	next
`=> |`	next (for the right operand; the left side is an expression or delimited by `VAR`)
`[] [*]`	bind least tightly.

These rules minimize the need for parentheses, which are written around commands in the ugly form `BEGIN ... END` or the slightly prettier form `IF ... FI`; the two forms have the same meaning, but as a matter of style, the latter should only be used around guarded commands. So, for example,

```
p => c1; c2
```

is the same as

```
p => BEGIN c1; c2 END
```

and means to do `c1` followed by `c2` if `p` is true. To guard only `c1` with `p` you must write

```
IF p => c1 [*] SKIP FI; c2
```

which means to do `c1` if `p` is true, and then to do `c2`. The `[*] SKIP` ensures that the command before the `";"` does not fail, which would prevent `c2` from getting done. Without the `[*] SKIP`, that is in

```
IF p => c1 FI; c2
```

if `p` is false the `IF ... FI` fails, so there is no possible outcome from which `c2` can be done and the whole thing fails. Thus `IF p => c1 FI; c2` has the same meaning as `p => BEGIN c1; c2 END`, which is a bit surprising.

Sequencing

A `c1 ; c2` command means just what you think it does: first `c1`, then `c2`. The command `c1 ; c2` gets you from state `s1` to state `s2` if there is an intermediate state `s` such that `c1` gets you from `s1` to `s` and `c2` gets you from `s` to `s2`. In other words, its relation is the composition of the relations for `c1` and `c2`; sometimes `';'` is called ‘sequential composition’. If `c1` produces an exception, the composite command ignores `c2` and produces that exception.

A `c1 EXCEPT ex => c2` command is just like `c1 ; c2` except that it treats the exception `ex` the other way around: if `c1` produces the exception `ex` then it goes on to `c2`, but if `c1` produces a normal outcome (or any other exception), the composite command ignores `c2` and produces that outcome.

Variable introduction

`VAR` gives you more dramatic non-determinism than `[]`. The most common use is in the idiom

```
VAR x: T | P(x) => c
```

which is read “choose some `x` of type `T` such that `P(x)`, and do `c`”. It fails if there is no `x` for which `P(x)` is true and `c` succeeds. If you just write

```
VAR x: T | c
```

then VAR acts like ordinary variable declaration, giving an arbitrary initial value to x.

Variable introduction is an alternative to existential quantification that lets you get your hands on the bound variable. For instance, you can write

```
IF  VAR n: Int, x: Int, y: Int, z: Int |
    (n > 2 /\ x**n + y**n = z**n) => out := n
[*] out := 0
FI
```

which is read: choose integers n, x, y, z such that n > 2 and $x^n + y^n = z^n$, and assign n to out; if there are no such integers, assign 0 to out.[6] The command before the [*] succeeds iff

```
(EXISTS n: Int, x: Int, y: Int, z: Int | n > 2 /\ x**n + y**n = z**n),
```

but if we wrote that in a guard there would be no way to set out to one of the n's that exist. We could also write

```
VAR s := { n: Int, x: Int, y: Int, z: Int
         | n > 2 /\ x**n + y**n = z**n
         | (n, x, y, z)}
```

to construct the set of all solutions to the equation. Then if s # {}, s.choose yields a tuple (n, x, y, z) with the desired property.

You can use VAR to describe all the transitions to a state that has an arbitrary relation R to the current state: VAR s' | R(s, s') => s := s' if there is only one state variable s.

The precedence of | is higher than [], which means that you can string together different VAR commands with [] or [*], but if you want several alternatives within a VAR you have to use BEGIN ... END or IF ... FI. Thus

```
        VAR x: T | P(x) => c1
    []  q => c2
```

is parsed the way it is indented and is the same as

```
        BEGIN VAR x: T | P(x) => c1 END
    []  BEGIN q => c2 END
```

but you must write the brackets in

```
VAR x: T |
    IF P(x) => c1
    [] Q(x) => c2
    FI
```

which might be formatted more concisely as

```
VAR x: T |
    IF  P(x) => c1
    []  R(x) => c2 FI
```

or even

```
VAR x: T | IF P(x) => c1 [] R(x) => c2 FI
```

You are supposed to indent your programs to make it clear how they are parsed.

[6] A correctness proof for an implementation of this spec defied the best efforts of mathematicians between Fermat's time and 1993.

Loops

You can always write a recursive routine, but often a loop is clearer . In Spec you use DO ... OD for this. These are brackets, and the command inside is repeated as long as it succeeds. When it fails, the repetition is over and the DO ... OD is complete. The most common form is

```
DO P => c OD
```

which is read "while P is true do c". After this command, P must be false. If the command inside the DO ... OD succeeds forever, the outcome is a looping exception that cannot be handled. Note that this is not the same as a failure, which simply means no outcome at all.

For example, you can zero all the elements of a sequence s with

```
VAR i := 0 | DO i < s.size => s(i) := 0; i - := 1 OD
```

or the simpler form (which also avoids fixing the order of the assignments)

```
DO VAR i | s(i) # 0 => s(i) := 0 OD
```

This is another common idiom: keep choosing an i as long as you can find one that satisfies some predicate. Since s is only defined for i between 0 and s.size-1, the guarded command fails for any other choice of i. The loop terminates, since the s(i) := 0 definitely reduces the number of i's for which the guard is true. But although this is a good example of a loop, it is bad style; you should have used a sequence method or function composition:

```
s := S.fill(0, s.size)
```

or

```
s := {x :IN s | | 0}
```

(a sequence just like s except that every element is mapped to 0), remembering that Spec makes it easy to throw around big things. Don't write a loop when a constructor will do, because the loop is more complicated to think about. Even if you are writing an implementation, you still shouldn't use a loop here, because it's quite clear how to write C code for the constructor.

To zero all the elements of s that satisfy some predicate P you can write

```
DO VAR i: Int | (s(i) # 0 /\ P(s(i))) => s(i) := 0 OD
```

Again, you can avoid the loop by using a sequence constructor and a conditional expression

```
s := {x :IN s | | (P(x) => 0 [*] x) }
```

Atomicity

Each <<...>> command is atomic. It defines a single transition, which includes moving the program counter (which is part of the state) from before to after the command. If a command is not inside <<...>>, it is atomic only if there's no reasonable way to split it up: SKIP, HAVOC, RET, RAISE. Here are the reasonable ways to split up the other commands:

- An assignment has one internal program counter value, between evaluating the right hand side expression and changing the left hand side variable.
- A guarded command likewise has one, between evaluating the predicate and the rest of the command.
- An invocation has one after evaluating the arguments and before the body of the routine, and another after the body of the routine and before the next transition of the invoking command.

Note that evaluating an expression is always atomic.

Modules and names

Spec's modules are very conventional. Mostly they are for organizing the name space of a large program into a two-level hierarchy: `module.id`. It's good practice to declare everything except a few names of global significance inside a module. You can also declare `CONST`'s, just like `VAR`'s.

```
MODULE foo EXPORT i, j, Fact =

CONST c := 1

VAR i := 0
    j := 1

FUNC Fact(n: Int) -> Int =
    IF   n <= 1 => RET 1
    [*] RET n * Fact(n - 1)
    FI

END foo
```

You can declare an identifier `id` outside of a module, in which case you can refer to it as `id` everywhere; this is short for `Global.id`, so `Global` behaves much like an extra module. If you declare `id` at the top level in module `m`, `id` is short for `m.id` inside of `m`. If you include it in `m`'s `EXPORT` clause, you can refer to it as `m.id` everywhere. All these names are in the *global* state and are shared among all the atomic actions of the program. By contrast, names introduced by a declaration inside a routine are in the *local* state and are accessible only within their scope.

The purpose of the `EXPORT` clause is to define the external interface of a module. This is important because module `T` implements module `S` iff `T`'s behavior at its external interface is a subset of `S`'s behavior at its external interface.

The other feature of modules is that they can be parameterized by types in the same style as CLU clusters. The memory systems modules in handout 5 are examples of this.

You can also declare a class, which is a module that can be instantiated many times. The `Obj` class produces a global `Obj` type that has as its methods the exported identifiers of the class plus a `new` procedure that returns a new, initialized instance of the class. It also produces a `ObjMod` module that contains the declaration of the `Obj` type, the code for the methods, and a state variable indexed by `Obj` that holds the state records of the objects. For example:

```
CLASS Stat EXPORT add, mean, variance, reset =

VAR  n          :  Int := 0
     sum        :  Int := 0
     sumsq      :  Int := 0

PROC add(i: Int) = n + := 1; sum + := i; sumsq + := i**2
FUNC mean() -> Int = RET sum/n
FUNC variance() -> Int = RET sumsq/n – self.mean**2
PROC reset() = n := 0; sum := 0; sumsq := 0

END Stat
```

Then you can write


```
VAR s: Stat | s := s.new(); s.add(x); s.add(y); print(s.variance)
```

In abstraction functions and invariants we also write `obj.n` for field `n` in `obj`'s state.

Section 7 of the reference manual deals with modules. Section 8 summarizes all the uses of names and the scope rules. Section 9 gives several modules used to define abstract data types.

This completes the language summary; for more details and greater precision consult the reference manual. The rest of this handout consists of three extended examples of specifications and implementations written in Spec: topological sort, editor buffers, and a simple window system.

Example: Topological sort

Suppose we have a directed graph whose `n+1` vertexes are labeled by the integers `0 .. n`, represented in the standard way by a relation `g`; `g(v1, v2)` is true if `v2` is a successor of `v1`, that is, if there is an edge from `v1` to `v2`. We want a topological sort of the vertexes, that is, a sequence that is a permutation of `0 .. n` in which `v2` follows `v1` whenever `v2` is a successor of `v1`. Of course this possible only if the graph is acyclic.

```
MODULE TopologicalSort EXPORT V, G, Q, TopSort =

TYPE V = IN 0 .. n                                  % Vertex
     G = (V, V) -> Bool                             % Graph
     Q = SEQ V

PROC TopSort(g) -> Q RAISES {cyclic} =
     IF   VAR q | q IN (0 .. n).perms /\ IsTSorted(q, g)  => RET q
     [*] RAISE cyclic                               % g must be cyclic
     FI

FUNC IsTSorted(q, g) -> Bool =
% Not tsorted if v2 precedes v1 in q but is also a child
     RET ~ (EXISTS v1 :IN q.dom, v2 :IN q.dom | v2 < v1 /\ g(q(v1), q(v2))

END TopologicalSort
```

Note that this solution checks for a cyclic graph. It allows any topologically sorted result that is a permutation of the vertexes, because the `VAR q` in `TopSort` allows any `q` that satisfies the two conditions. The `perms` method on sets and sequences is defined in section 9 of the reference manual; the `dom` method gives the domain of a function. `TopSort` is a procedure, not a function, because its result is non-deterministic; we discussed this point earlier when studying `SquareRoot`. Like that one, this spec has no internal state, since the module has no `VAR`. It doesn't need one, because it does all its work on the input argument.

The following implementation is from Cormen, Leiserson, and Rivest. It adds vertexes to the front of the output sequence as depth-first search returns from visiting them. Thus, a child is added before its parents and therefore appears after them in the result. Unvisited vertexes are `white`, nodes being visited are `grey`, and fully visited nodes are `black`. Note that all the descendants of a `black` node must be `black`. The `grey` state is used to detect cycles: visiting a `grey` node means that there is a cycle containing that node.

This module has state, but you can see that it's just for convenience in programming, since it is reset each time `TopSort` is called.

```
MODULE TopSortImpl EXPORT V, G, Q, TopSort =        % implements TopSort

TYPE Color = ENUM[white, grey, black]               % plus the spec's types

VAR  out  : Q
     color: V -> Color                              % every vertex starts white

PROC TopSort(g) -> Q RAISES {cyclic} = VAR i := 0 |
     out := {}; color := {* -> white}
     DO VAR v | color(v) = white => Visit(v, g) OD;  % visit every unvisited vertex
     RET out

PROC Visit(v, g) RAISES {cyclic} =
     color(v) := grey;
     DO VAR v' | g(v, v') /\ color(v') # black =>   % pick an successor not done
          IF   color(v') = white => Visit(v', g)
          [*]  RAISE cyclic                         % grey — partly visited
          FI
     OD;
     color(v) := black; out := {v} + out            % add v to front of out
```

The implementation is as non-deterministic as the spec: depending on the order in which `TopSort` chooses `v` and `Visit` chooses `v'`, any topologically sorted sequence can result. We could get a deterministic implementation in many ways, for example by taking the smallest node in each case (the `min` method on sets is defined in section 9 of the reference manual):

```
     VAR v  := {v0 | color(v0) = white}.min              in TopSort
     VAR v' := {v0 | g(v, v0) /\ color(v') # black }.min  in Visit
```

An implementation in C would do something like this; the details would depend on the representation of `G`.

Example: Editor buffers

A text editor usually has a *buffer* abstraction. A buffer is a mutable sequence of `C`'s. To get started, suppose that `C = Char` and a buffer has two operations,

`Get(i)` to get character `i`

`Replace` to replace a subsequence of the buffer by a subsequence of an argument of type `SEQ C`, where the subsequences are defined by starting position and size.

We can make this specification precise as a Spec class.


```
CLASS Buffer EXPORT B, C, X, Get, Replace =

TYPE X = Nat                                        % indeX in buffer
     C = Char
     B = SEQ C                                      % Buffer contents

VAR  b : B := {}                                    % Note: initially empty

FUNC Get(x) -> C = RET b(x)                         % Note: defined iff 0<=x<b.size

PROC Replace(from: X, size: X, b': B, from': X, size': X) =
% Note: fails if it touches C's that aren't there.
     VAR b1, b2, b3 | b = b1 + b2 + b3 /\ b1.size = from /\ b2.size = size =>
       b := b1 + b'.seg(from', size') + b3

END Buffer
```

We can implement a buffer as a sorted array of *pieces* called a 'piece table'. Each piece contains a `SEQ C`, and the whole buffer is the concatenation of all the pieces We use binary search to find a piece, so the cost of `Get` is at most logarithmic in the number of pieces. `Replace` may require inserting a piece in the piece table, so its cost is at most linear in the number of pieces.[7] In particular, neither depends on the number of `C`'s. Also, each `Replace` increases the size of the array of pieces by at most two.

A piece is a `B` (in C it would be a pointer to a `B`) together with the sum of the length of all the previous pieces, that is, the index in `Buffer.b` of the first `C` that it represents; the index is there so that the binary search can work. There are internal routines `Locate(x)`, which uses binary search to find the piece containing `x`, and `Split(x)`, which returns the index of a piece that starts at `x`, if necessary creating it by splitting an existing piece. `Replace` calls `Split` twice to isolate the substring being removed, and then replaces it with a single piece. The time for `Replace` is linear in `pt.size` because on the average half of `pt` is moved when `Split` or `Replace` inserts a piece, and in half of `pt`, `p.x` is adjusted if `size' # size`.

[7] By using a tree of pieces rather than an array, we could make the cost of `Replace` logarithmic as well, but to keep things simple we won't do that. See `FSImpl` in handout 7 for more on this point.

```
CLASS BufImpl EXPORT B,C,X, Get, Replace =          % implements Buffer

TYPE                                                % Types as in Buffer, plus
     N   = X                                        % iNdex in piece table
     P   = [b, x]                                   % Piece: x is pos in Buffer.b
     PT  = SEQ P                                    % Piece Table

VAR  pt  := PT{}

ABSTRACTION FUNCTION buffer.b = + : {p :IN pt | | p.b}
% buffer.b is the concatenation of the contents of the pieces in pt

INVARIANT (ALL n :IN pt.dom |  pt(n).b # {}
                             /\pt(n).x = + :{i :IN 0 .. n-1 | | pt(i).b.size})
% no pieces are empty, and  x is the position of the piece in Buffer.b, as promised.

FUNC Get(x) -> C = VAR p := pt(Locate(x)) | RET p.b(x - p.x)

PROC Replace(from: X, size: X, b': B, from': X, size': X) =
     VAR n1 := Split(from); n2 := Split(from + size),
         new := P{b := b'.seg(from', size'), x := from} |
         pt :=  pt.sub(0, n1 - 1)
              + NonNull(new)
              + pt.sub(n2, pt.size - 1) * AdjustX(size' - size )

PROC Split(x) -> N =
% Make pt(n) start at x, so pt(Split(x)).x = x.  Fails if x > b.size.
% If  pt=abcd|efg|hi, then Split(4) is RET 1 and Split(5) is pt:=abcd|e|fg|hi; RET 2
     IF   pt = {} /\ x = 0 => RET 0
     [*] VAR n := Locate(x), p := pt(n), b1, b2 |
          p.b = b1 + b2 /\ p.x + b1.size = x =>
               VAR frag1 := p{b := b1}, frag2 := p{b := b2, x := x} |
                    pt :=  pt.sub(0, n - 1)
                         + NonNull(frag1) + NonNull(frag2)
                         + pt.sub(n + 1, pt.size - 1);
                    RET (b1 = {} => n [*] n + 1)
     FI

FUNC Locate(x) -> N = VAR n1 := 0, n2 := pt.size - 1 |
% Use binary search to find the piece containing x. Yields 0 if pt={},
% pt.size-1 if pt#{} /\ x>=b.size; never fails.  The loop invariant is
%  pt={} \/ n2 >= n1 /\ pt(n1).x <= x /\ ( x < pt(n2).x \/ x >= pt.last.x )
% The loop terminates because n2 - n1 > 1 ==> n1 < n < n2, so n2 – n1 decreases.
     DO n2 - n1 > 1 =>
          VAR n := (n1 + n2)/2 | IF pt(n).x <= x => n1 := n [*] n2 := n FI
     OD; RET (x < pt(n2).x => n1 [*] n2)

FUNC NonNull(p) -> PT = RET (p.b # {} => PT{p} [*] {})

FUNC AdjustX(dx: Int) -> (P -> P) = RET (\ p | p{x + := dx})

END BufImpl
```

If subsequences were represented by their starting and ending positions, there would be lots of extreme cases to worry about.

Suppose we now want each C in the buffer to have not only a character code but also some additional properties, for instance the font, size, underlining, etc. Get and Replace remain the same. In addition, we need a third exported methodApply that applies to each character in a subsequence of the buffer a map function C -> C. Such a function might make all the C's italic, for example, or increase the font size by 10%.

```
PROC Apply(map: C->C, from: X, size: X) =
     b :=   b.sub(0, from-1)
          +  b.seg(from, size) * map
          +  b.sub(from + size, b.size-1)
```

Here is an implementation for Apply that takes time linear in the number of pieces. It works by changing the representation to add a map function to each piece, and in Apply composing the map argument with the map of each affected piece. We need a new version of Get that applies the proper map function, to go with the new representation.

```
TYPE P    = [b, x, map: C->C]                       % x is pos in Buffer.b

ABSTRACTION FUNCTION buffer.b = + :{p :IN pt | | p.b * p.map}
% buffer.b is the concatenation of the pieces in p with their map's applied.
% This is the same AF we had before, except for the addition of * p.map.

FUNC Get(x) -> C = VAR p := pt(Locate(x)) | RET p.map(p.b(x - p.x))

PROC Apply(map: C->C, from: X, size: X) =
  VAR n1 := Split(from), n2 := Split(from + size) |
     pt :=  pt.sub(0 , n1 - 1)
          + pt.sub(n1, n2 - 1) * (\ p | p{map := p.map * map})
          + pt.sub(n2, pt.size - 1)
```

Note that we wrote Split so that it keeps the same map in both parts of a split piece. We also need to add map := (\ c | c) to the constructor for new in Replace.

This implementation was used in the Bravo editor for the Alto, the first what-you-see-is-what-you-get editor. It is still used in Microsoft Word.

Example: Windows

A window (the kind on your computer screen, not the kind in your house) is a map from points to colors. There can be lots of windows on the screen; they are ordered, and closer ones block the view of more distant ones. Each window has its own coordinate system; when they are arranged on the screen, an offset says where each window's origin falls in screen coordinates.

```
MODULE Window EXPORT Get, Paint =

TYPE I         = Int
     Coord     = Nat
     Intensity = IN 0 .. 255
     P         = [x: Coord, y: Coord] WITH {"-":=PSub}       % Point
     C         = [r: Intensity, g: Intensity, b: Intensity]  % Color
     W         = P -> C                                      % Window

FUNC PSub(p1, p2) -> P = RET P{x := p1.x - p2.x, y := p1.y - p2.y}
```

The shape of the window is determined by the points where it is defined; obviously it need not be rectangular in this very general system. We have given a point a "-" method that computes the vector distance between two points.

A 'window system' consists of a sequence of `[w, offset: P]` pairs; we call a pair a `V`. The sequence defines the ordering of the windows (closer windows come first in the sequence); it is indexed by 'window number' `WN`. The `offset` gives the screen coordinate of the window's `(0, 0)` point, which we think of as its upper left corner. There are two main operations: `Paint(wn, p, c)` to set the value of `P` in window `wn`, and `Get(p)` to read the value of `p` in the topmost window where it is defined (that is, the first one in the sequence). The idea is that what you see (the result of `Get`) is the result of painting the windows from last to first, offsetting each one by its `offset` component and using the color that is painted later to completely overwrite one painted earlier. Of course real window systems have other operations to change the shape of windows, add, delete, and move them, change their order, and so forth, as well as ways for the window system to suggest that newly exposed parts of windows be repainted, but we won't consider any of these complications.

First we give the spec for a window system initialized with `n` empty windows. It is customary to call the coordinate system used by `Get` the screen coordinates. The `v.offset` field gives the screen coordinate that corresponds to `{0, 0}` in `v.w`. The `v.c(p)` method below gives the value of `v`'s window at the point corresponding to `p` after adjusting by `v`'s offset. The state `ws` is just the sequence of `V`'s. For simplicity we initialize them all with the same offset `{10, 5}`, which is not too realistic.

`Get` finds the smallest `WN` that is defined at `p` and uses that window's color at `p`. This corresponds to painting the windows from last (biggest `WN`) to first with opaque paint, which is what we wanted. `Paint` uses window rather than screen coordinates.

The state (the `VAR`) is a single sequence of windows.

```
TYPE WN          =  IN 0 .. n-1                              % Window Number
     V           =  [w, offset: P]                           % window on the screen
                    WITH {c:=(\ v, p | v.w(p - v.offset))}   % C of a screen point p

VAR  ws          := {i :IN 0..n-1 | | V{{}, P{10,5}}}        % the Window System

FUNC Get(p) -> C = VAR wn := {wn' | V.c!(ws(wn'), p)}.min | RET ws(wn).c(p)

PROC Paint(wn, p, c) = ws(wn).w(p) := c

END Window
```

Now we give an implementation that only keeps track of the visible color of each point (that is, it just keeps the pixels on the screen, not all the pixels in windows that are covered up by other windows). We only keep enough state to handle `Get` and `Paint`.

The state is one `W` that represents the screen, plus an `exposed` variable that keeps track of which window is exposed at each point, and the offsets of the windows. This is sufficient to implement `Get` and `Paint`; to deal with erasing points from windows we would need to keep more information about what other windows are defined at each point, so that `exposed` would have a type `P -> SET WN`. Alternatively, we could keep track for each window of where it is defined.

Real window systems usually do this, and represent `exposed` as a set of visible regions of the various windows. They also usually have a 'background' window that covers the whole screen, so that every point on the screen has some color defined; we have omitted this detail from the spec and the implementation.

We need a history variable `wH` that contains the `w` part of all the windows. The abstraction function just combines `wH` and `offset` to make `ws`. The important properties of the implementation are contained in the invariant, from which it's clear that `Get` returns the answer specified by `Window.Get`. Another way to do it is to have a history variable `wsH` that is equal to `ws`. This makes the abstraction function very simple, but then we need an invariant that says `offset(wn) = wsH(n).offset`. This is perfectly correct, but it's usually better to put as little stuff in history variables as possible.

`MODULE WinImpl` `EXPORT Get, Paint =`

```
VAR  w       :=  W{}                                  % no points defined
     exposed :   P -> WN := {}                        % which wn shows at p
     offset  :=  {i :IN 0..n-1 | | P(5, 10)}         %
     wH      :=  {i :IN 0..n-1 | | W{}}               % history variable

ABSTRACTION FUNCTION ws = (\ wn | V{w := wH(wn), offset := offset(wn)})

INVARIANT
     (ALL p |    w!p = exposed!p
             /\ (w!p ==>    {wn | V.c!(ws(wn), p)}.min = exposed(p)
                         /\ w(p) = ws(exposed(p)).c(p) ) )
```

The invariant says that each visible point comes from some window, `exposed` tells the topmost window that defines it, and its color is the color of the point in that window. Note that for convenience the invariant uses the abstraction function; of course we could have avoided this by expanding it in line, but there is no reason to do so, since the abstraction function is a perfectly good function.

```
FUNC Get(p) -> C = RET w(p)

PROC Paint(wn, p, c) =
     VAR p0 | p = p0 - offset(wn) =>                  % the screen coordinate
       IF wn <= exposed(p0) => w(p0) := c; exposed(p0) := wn [*] SKIP FI;
       wH(wn)(p) := c                                 % update the history var

END WinImpl
```

Index

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Operators (§ 5, § 9)

Op	*Pr*	*Type*	x op y *is*
.	9	Any	x's y field/method
IS	8	Any	does x have type y?
AS	8	Any	x with type y
**	8	Int	x^y
*	7	Int	x × y
		func	composition
		relation	composition
/	7	Int	x/y rounded to 0
//	7	Int	mod: x – (x/y)*y
+	6	Int	x + y
		func	overlay
		seq	concatenation
-	6	Int	x – y
		set	set difference
		seq	multiset diff
!	6	func	x defined at y
!!	6	func	x!y ∧ x(y) not ex
..	5	Int	seq {x, x+1,..., y}
=	4	Any	x = y
#	4	Any	x ≠ y
==	4	seq	x = y as multisets
<=	4	Int	x ≤ y
		set	x ⊆ y (subset)
		seq	x a prefix of y
<<=	4	seq	x a sub-seq of y
IN	4	set/seq	x ∈ y (member)
~	3	Bool	not x (unary)
/\	2	Bool	x ∧ y (and)
		set	x ∩ y (intersection)
\/	1	Bool	x ∨ y (or)
		set	x ∪ y (union)
==>	0	Bool	x implies y

Operators associate to the left.

Methods (§ 9)

set	*Ops:* /\ \/ - <= IN, op:	
	size	number of members
	choose	some member of s
	seq	s as some sequence
	pred	s.pred(x) = (x ∈ s)
	fmax/min	some max/min by f_1
	max/min	some max/min by <=
set/seq	perms	set of all perms of sq
	fsort	sq sorted (q stably) by f_1
	sort	sq sorted (q stably) by <=
func	*Ops:* * + ! !!	
	dom, rng	domain, range
	inv	inverse
	restrict	domain to set s_1
	rel	r(x,y) = (f(x)=y)
predicate	set	s = {x | pred(x)}
relation	*Ops:* * and func +	
	dom, rng	domain, range
	inv	inverse
	setF	f(x) = {y | r(x,y)}
	func	f(x) = setF(x).choose
graph	isPath	is q_1 a path in g?
	closure	transitive closure of g
seq (also see set/seq and func above)	*Ops:* + - .. <= <<= IN, op:, func * !	
	size	number of elements
	head	q(0)
	tail	{q(1),...,q(q.size-1)}
	remh	remove head = tail
	last	q(q.size-1)
	reml	{q(0),...,q(q.size-2)}
	sub	{q(i_1),...,q(i_2)}
	seg	{q(i_1),...}, i_2 elements
	fill	i_2 copies of x_1
	lexLE	q lexically <= q_1 by f_2?
	fsorter	perm sorts q stably by f_1
	count	number of x_1's in q
	set	q as a set, = q.rng
	tuple	tuple with q's values
tuple	seq	seq with tu's values

Types (§ 4)

Any, Null, Bool, Int, Nat, Char, String	basic
SET T, IN s	set
T_1 -> T_2	function
APROC T_1 -> T_2	procedures
PROC T_1 -> T_2	
SEQ T	sequence
(T_1,...,T_n)	tuple
[f_1: T_1,...,f_n: T_n]	record
(T_1 + ... + T_n)	union
T WITH {m_1:=f_1,...}	add methods
T SUCHTHAT pred	limit values

Commands (§ 6)

	Pr	
SKIP, HAVOC, RET e, RAISE ex		simple
p(e)		invocation
x := e, x := p(e), (x_1, ...) := e		assignment
c_1 EXCEPT ex=>c_2	3	handle ex
c_1 ; c_2	2	sequential
VAR n: T | c	1	new var n
pred => c	1	if (guarded cmd)
c_1 [] c_2	0	or (ND choice)
c_1 [*] c_2	0	else
<< c >>		atomic c
BEGIN c END		brackets
IF c FI		
DO c OD		loop until fail

Command operators associate to the left, but EXCEPT associates to the right.

Modules (§ 7)

```
MODULE/CLASS M
  [T1 WITH {m1: T11->T12,...},...]
  EXPORT n1, ... =

TYPE T1 = SET T2
     T3 = ENUM[n1,...]
CONST n: T := e
VAR   n: T := e
EXCEPTION ex = {ex11,...} + ex2 + ...
FUNC f(n1: T1,...) -> T = c
APROC, PROC, THREAD similarly
END M
```

Expression forms (§ 5)

f(e)	func	function invocation
op : sq	set/seq	sq(0) op sq(1) ...
(ALL x | pred)	Bool	pred(x_1) ∧ ... ∧ pred(x_n)
(EXISTS x | pred)	Bool	pred(x_1) ∨ ... ∨ pred(x_n)
(pred => e_1 [*] e_2)	Any	e_1 if pred else e_2

Constructors (§ 5)

{e_1,...,e_n}	set	with these members
{i:Nat | i<3 | i**2}		of i^2's where i<3
f{e_1 -> e_2}	func	f except = e_2 at arg e_1
f{* -> e}		= e at every arg
(\i:Int | i<3)		lambda (also LAMBDA)
{e_1,...,e_n}	seq	of e's in this order
{i :IN 0 .. 5 || i**2}		{0,1,4,9,16,25}
{i:=0 BY i+1 WHILE i<6||i**2}		same
(e_1,...,e_n)	tuple	of e's in this order
r{f_1:=e_1,...,f_n:=e_n}	record	r except f_1 = e_1 ...

Naming conventions (except in 'Operators')

c	command	op	operator
e	expression	p	proccedure
ex	exception	*Pr*	precedence
f	function, field	q	sequence
g	graph	r	record, relation
i	Int	s	set
m	method	T	type
n	name	x	Any

z_i ith extra argument of a method, or one of several like non-terminals in a rule

§ a section of the Spec reference manual

How to Write a Spec

Figure out what the state is.

Choose the state to make the spec simple and clear, not to match the code.

Describe the actions.

What they do to the state.

What they return.

Helpful hints

Notation is important, because it helps you to think about what's going on.
Invent a suitable vocabulary.

Less is more. Less state is better. Fewer actions are better.

More non-determinism is better, because it allows more implementations.
In distributed systems, replace the separate nodes with non-determinism in the spec.

Pass the coffee-stain test: people should want to read the spec.

I'm sorry I wrote you such a long letter; I didn't have time to write a short one. — Pascal

How to Design an Implementation

Write the spec first.

Dream up the idea of the implementation.

Embody the key idea in the abstraction function.

Check that each implementation action simulates some spec actions.

Add invariants to make this easier. Each action must maintain them.

Change the implementation (or the spec, or the abstraction function) until this works.

Make the implementation correct first, then efficient.

More efficiency means more complicated invariants.

You might need to change the spec to get an efficient implementation.

Measure first before making anything faster.

An efficient program is an exercise in logical brinkmanship. — Dijkstra

4. Spec Reference Manual

Spec is a language for writing specifications and the first few stages of successive refinement towards a practical implementation. As a specification language it includes constructs (quantifiers, backtracking or non-determinism, some uses of atomic brackets) which are impractical in a final implementation; they are there because they make it easier to write clear, unambiguous and suitably general specifications. If you want to write a practical program, avoid them.

This document defines the syntax of the language precisely and the semantics informally**. You should read the** *Introduction to Spec* (handout 3) **before trying to read this manual**. In fact, this manual is intended mainly for reference; rather than reading it carefully, skim through it, and then use the index to find what you need. For a precise definition of the atomic semantics read *Atomic Semantics of Spec*. (handout 9). Handout 17 on *Formal Concurrency* gives the non-atomic semantics semi-formally.

1. Overview

Spec is a notation for writing specifications for a discrete system. What do we mean by a specification? It is the allowed sequences of transitions of a state machine. So Spec is a notation for describing sequences of transitions of a state machine.

Expressions and commands

The Spec language has two essential parts:

An *expression* describes how to compute a value as a function of other values, either constants or the current values of state variables.

A *command* describes possible transitions, or changes in the values of the state variables.

Both are based on the *state*, which in Spec is a mapping from names to values. The names are called state variables or simply variables: in the examples below they are `i` and `j`.

There are two kinds of commands:

An *atomic* command describes a set of possible transitions. For instance, the command `<< i := i + 1 >>` describes the transitions `i=1→i=2, i=2→i=3`, etc. (Actually, many transitions are summarized by `i=1→i=2`, for instance, `(i=1, j=1)→(i=2, j=1)` and `(i=1, j=15)→(i=2, j=15)`). If a command allows more than one transition from a given state we say it is *non-deterministic*. For instance, the command, `<< i := 1 [] i := i + 1 >>` allows the transitions `i=2→i=1` and `i=2→i=3`. More on this in *Atomic Semantics of Spec*.

A *non-atomic* command describes a set of sequences of states. More on this in *Formal Concurrency*.

A sequential program, in which we are only interested in the initial and final states, can be described by an atomic command.

Spec's notation for commands, that is, for changing the state, is derived from Edsger Dijkstra's guarded commands (E. Dijkstra, *A Discipline of Programming*, Prentice-Hall, 1976) as extended by Greg Nelson (G. Nelson, A generalization of Dijkstra's calculus, *ACM TOPLAS* **11**, 4, Oct. 1989, pp 517-561). The notation for expressions is derived from mathematics.

Organizing a program

In addition to the expressions and commands that are the core of the language, Spec has four other mechanisms that are useful for organizing your program and making it easier to understand.

A *routine* is a named computation with parameters (passed by value). There are four kinds:

A *function* is an abstraction of an expression.

An *atomic procedure* is an abstraction of an atomic command.

A general procedure is an abstraction of a non-atomic command.

A *thread* is the way to introduce concurrency.

A *type* is a stylized assertion about the set of values that a name can assume. A type is also an easy way to group and name a collection of routines, called its *methods*, that operate on values in that set.

An *exception* is a way to report an unusual outcome.

A *module* is a way to structure the name space into a two-level hierarchy. An identifier `i` declared in a module `m` is known as `i` in `m` and as `m.i` throughout the program. A *class* is a module that can be instantiated many times to create many objects.

A Spec program is some global declarations of variables, routines, types, and exceptions, plus a set of modules each of which declares some variables, routines, types, and exceptions.

Outline

This manual describes the language bottom-up:
- Lexical rules
- Types
- Expressions
- Commands
- Modules

At the end there are two sections with additional information:
- Scope rules
- Built-in methods for set, sequence, and routine types.

There is also an index. The *Introduction to Spec* has a one-page language summary.

2. Grammar rules

Nonterminal symbols are in lower case; terminal symbols are punctuation other than `::=`, or are quoted, or are in upper case.

Alternative choices for a nonterminal are on separate lines.

`symbol*` denotes zero of more occurrences of `symbol`.

The symbol `empty` denotes the empty string.

If `x` is a nonterminal, the nonterminal `xList` is defined by

```
xList       ::= x
                x , xList
```

A comment in the grammar runs from `%` to the end of the line; this is just like Spec itself.

A [n] in a comment means that there is an explanation in a note labeled [n] that follows this chunk of grammar.

3. Lexical rules

The symbols of the language are literals, identifiers, keywords, operators, and the punctuation `( ) [ ] { } , ; : . | << >> := => -> [] [*]`. Symbols must not have embedded white space. They are always taken to be as long as possible.

A *literal* is a decimal number such as `3765`, a quoted character such as `'x'`, or a double-quoted string such as `"Hello\n"`.

An *identifier* (`id`) is a letter followed by any number of letters, underscores, and digits followed by any number of `'` characters. Case is significant in identifiers. By convention type and procedure identifiers begin with a capital letter. An identifier may not be the same as a keyword. The *predefined* identifiers `Any, Bool, Char, Int, Nat, Null, String, true, false`, and `nil` are declared in every program. The meaning of an identifier is established by a declaration; see section 8 on scope for details. Identifiers cannot be redeclared.

By convention *keywords* are written in upper case, but you can write them in lower case if you like; the same strings with mixed case are not keywords, however. The keywords are

ALL	APROC	AS	BEGIN	BY	CLASS
CONST	DO	END	ENUM	EXCEPT	EXCEPTION
EXISTS	EXPORT	FI	FUNC	HAVOC	IF
IN	IS	LAMBDA	MODULE	OD	PROC
RAISE	RAISES	RET	SEQ	SET	SKIP
SUCHTHAT	THREAD	TYPE	VAR	WHILE	WITH

An *operator* is any sequence of the characters `!@#$^&*-+=:.<>?/\|~` except the sequences `: . | << >> := => ->` (these are punctuation), or one of the keyword operators `AS`, `IN`, and `IS`.

A comment in a Spec program runs from a `%` outside of quotes to the end of the line. It does not change the meaning of the program.

4. Types

A type defines a set of values; we say that a value `v` has type `T` if `v` is in `T`'s set. The sets are not disjoint, so a value can belong to more than one set and therefore can have more than one type. In addition to its value set, a type also defines a set of routines (functions or procedures) called its *methods*; a method normally takes a value of the type as its first argument.

An expression has exactly one type, determined by the rules in section 5; the result of the expression has this type unless it is an exception.

The picky definitions given on the rest of this page are the basis for Spec's type-checking. You can skip them on first reading, or if you don't care about type-checking.

About unions: If the expression `e` has type `T` we say that `e` has a routine type `W` if `T` is a routine type `W` or if `T` is a union type and exactly one type `W` in the union is a routine type. Under corresponding conditions we say that `e` has a sequence or set type, or a record type with a field `f`.

Two types are *equal* if their definitions are the same (that is, have the same parse trees) after all type names have been replaced by their definitions and all `WITH` clauses have been discarded. Recursion is allowed; thus the expanded definitions might be infinite. Equal types define the same value set. Ideally the reverse would also be true, but type equality is meant to be decided by a type checker, whereas the set equality is intractable.

A type `T` *fits* a type `U` if the type-checker thinks they may have some values in common. This can only happen if they have the same structure, and each part of `T` fits the corresponding part of `U`. 'Fits' is an equivalence relation. Precisely, `T` fits `U` if:

`T = U`.

`T` is `T' SUCHTHAT F` or `(... + T' + ...)` and `T'` fits `U`, or vice versa. There may be no values in common, but the type-checker can't analyze the `SUCHTHAT` clauses to find out.

`T` and `U` are tuples of the same length and each component of `T` fits the corresponding component of `U`.

`T` and `U` are record types, and for every `decl id: T'` in `T` there is a corresponding `decl id: U'` in `U` such that `T'` fits `U'`, or vice versa.

`T=T1->T2 RAISES EXt` and `U=U1->U2 RAISES EXu`, or one or both `RAISES` are missing, and `T1` fits `U1` and `T2` fits `U2`. Similar rules apply for `PROC` and `APROC` types.

`T=SET T'` and `U=SET U'` and `T'` fits `U'`.

`T = Int->T'` or `SEQ T'` and `U = SEQ U'` and `T'` fits `U'`.

`T` *includes* `U` if the same conditions apply with "fits" replaced by "includes", all the "vice versa" clauses dropped, and in the `->` rule "`T1` fits `U1`" replaced by "`U1` includes `T1` and `EXt` is a superset of `EXu`". If `T` includes `U` then `T`'s value set includes `U`'s value set; again, the reverse is intractable.

An expression `e` fits a type `U` in state `s` if `e`'s type fits `U` and the result of `e` in state `s` has type `U` or is an exception; in general this can only be checked at runtime unless `U` includes `e`'s type. The check that `e` fits `T` is required for assignment and routine invocation; together with a few other checks it is called *type-checking*. The rules for type-checking are given in sections 5 and 6.

```
type         ::= name                          % name of a type
                 "Any"                         % every value has this type
                 "Null"                        % with value set {nil}
                 "Bool"                        % with value set {true, false}
                 "Char"                        % like an enumeration
                 "String"                      % = SEQ Char
                 "Int"                         % integers
                 "Nat"                         % naturals: non-negative integers
                 SEQ type                      % sequence [1]
                 SET type                      % set
                 ( typeList )                  % tuple; (T) is the same as T
                 [ declList ]                  % record with declared fields
                 ( union )                     % union of the types
                 aType -> type raises          % function [2]
                 APROC aType returns raises    % atomic procedure
                 PROC  aType returns raises    % non-atomic procedure
                 type WITH { methodDefList }   % attach methods to a type [3]
                 type SUCHTHAT primary         % restrict the value set [4]
                 IN exp                        % = T SUCHTHAT (\ t: T | t IN exp)
                                               % where exp's type has an IN method
                 id [ typeList ] . id          % type from a module [5]

name         ::= id . id                       % the first id denotes a module
                 id                            % short for m.id if id is declared
                                               % in the current module m, and for
                                               % Global.id if id is declared globally
                 type . id                     % the id method of type
decl         ::= id : type                     % id has this type
                 id                            % short for id: Id [6]

union        ::= type  + type
                 union + type

aType        ::= ()
                 type
returns      ::= empty                         % only for procedures
                 -> type
raises       ::= empty
                 RAISES exceptionSet           % the exceptions it can return

exceptionSet ::= { exceptionList }             % a set of exceptions
                 name                          % declared as an exception set
                 exceptionSet \/ exceptionSet  % set union
                 exceptionSet - exceptionSet   % set difference
exception    ::= id                            % means "id"

method       ::= id
                 stringLiteral                 % the string must be an operator
                                               % other than "=" or "#" (see section 3)
methodDef    ::= method := name                % name is a routine
```


The ambiguity of the type grammar is resolved by taking `->` to be right associative and giving `WITH` and `RAISES` higher precedence than `->`.

[1] A `SEQ T` is just a function from `{0, 1, ..., size-1}` to `T`. That is, it is short for

```
(Int->T) SUCHTHAT (\ f: Int->T | (EXISTS size: Int |
                                    (ALL i: Int | f!i = (i IN 0 .. size-1)))
                   WITH { see section 9 }.
```

This means that invocation, `!`, and `*` work for a sequence just as they do for any function. In addition, there are many other useful operators on sequences; see section 9. The `String` type is just `SEQ Char`; there are `String` literals, defined in section 5.

[2] A `T->U` value is a partial function from a state and a value of type `T` to a value of type `U`. A `T->U RAISES xs` value is the same except that the function may raise the exceptions in `xs`.

[3] We say `m` is a *method* of `T` defined by `f`, and denote `f` by `T.m`, if

> `T = T' WITH {..., m := f, ...}` and `m` is an identifier or is `"op"` where op is an operator (the construct in braces is a `methodDefList`), or
>
> `T = T' WITH { methodDefList }`, `m` is not defined in `methodDefList`, and `m` is a method of `T'` defined by `f`, or
>
> `T = (... + T' + ...)`, `m` is a method of `T'` defined by `f`, and there is no other type in the union with a method `m`.

There are two special forms for invoking methods: `e1 infixOp e2` or `prefixOp e`, and `e1.id(e2)` or `e.id` or `e.id()`. They are explained in notes [1] and [3] to the expression grammar in the next section. This notation may be familiar from object-oriented languages. Unlike many such languages, Spec makes no provision for varying the method in each object, though it does allow inheritance and overriding.

A method doesn't have to be a routine, though the special forms won't type-check unless the method is a routine. Any method `m` of `T` can be referred to by `T.m`.

[4] In `T SUCHTHAT f`, `f` is a predicate on `T`'s, that is, a function `(T -> Bool)`. The type `T SUCHTHAT f` has the same methods as `T`, and its value set is the values of `T` for which `f` is true. See section 5 for `primary`.

[5] If a type is defined by `m[typeList].id` and `m` is a parameterized module, the meaning is `m'.id` where `m'` is defined by `MODULE m' = m[typeList] END m'`. See section 7 for a full discussion of this kind of type.

[6] `Id` is the `id` of a type, obtained from `id` by dropping trailing `'` characters and digits, and capitalizing the first letter or all the letters (it's an error if these capitalizations yield different identifiers that are both known at this point).

5. Expressions

An expression is a partial function from states to results; results are values or exceptions. That is, an expression computes a result for a given state. The state is a function from names to values. This state is supplied by the command containing the expression in a way explained later. The meaning of an expression (that is, the function it denotes) is defined informally in this section. The meanings of invocations and lambda function constructors are somewhat tricky, and the informal explanation here is supplemented by a formal account in *Atomic Semantics of Spec*. Because expressions don't have side effects, the order of evaluation of operands is irrelevant (but see [5] and [13]).

Every expression has a type. The result of the expression is a member of this type if it is not an exception. This property is guaranteed by the *type-checking* rules, which require an expression used as an argument, the right hand side of an assignment, or a routine result to fit the type of the formal, left hand side, or routine range (see section 4 for the definition of 'fit'). In addition, expressions appearing in certain contexts must have *suitable* types: in `e1(e2)`, `e1` must have a routine type; in `e1+e2`, `e1` must have a type with a `"+"` method, etc. These rules are given in detail in the rest of this section. A union type is suitable if exactly one of the members is suitable. Also, if `T` is suitable in some context, so are `T WITH {...}` and `T SUCHTHAT f`.

An expression can be a literal, a variable known in the scope that contains the expression, or a function invocation. The form of an expression determines both its type and its result in a state:

`literal` has the type and value of the literal.

`name` has the declared type of `name` and its value in the current state, `state("name")`. The form `T.m` (where `T` denotes a type) is also a name; it denotes the `m` method of `T`. Note that if `name` is `id` and `id` is declared in the current module `m`, then it is short for `m.id`.

invocation `f(e)`: `f` must have a function (not procedure) type `U->T RAISES EX` or `U->T` (note that a sequence is a function), and `e` must fit `U`; then `f(e)` has type `T`. In more detail, if `f` has result `rf` and `e` has type `U'` and result `re`, then `U'` must fit `U` (checked statically) and `re` must have type `U` (checked dynamically if `U'` involves a union or `SUCHTHAT`; if the dynamic check fails the result is a fatal error). Then `f(e)` has type `T`.

If either `rf` or `re` is undefined, so is `f(e)`. Otherwise, if either is an exception, that exception is the result of `f(e)`; if both are, `rf` is the result.

If both `rf` and `re` are normal, the result of `rf` at `re` can be:

A normal value, which becomes the result of `f(e)`.

An exception, which becomes the result of `f(e)`. If `rf` is defined by a function body that loops, the result is a special looping exception that you cannot handle.

Undefined, in which case `f(e)` is undefined and the command containing it fails (has no outcome) — failure is explained in section 6.

A function invocation in an expression never affects the state. If the result is an exception, the containing command has an exceptional outcome; for details see section 6.

The other forms of expressions (`e.id`, `constructors`, prefix and infix operators, combinations, and quantifications) are all syntactic sugar for function invocations, and their results are obtained by the rule used for invocations. There is a small exception for conditionals [5] and for the conditional logical operators `/\`,`\/`, and `==>` that are defined in terms of conditionals [13].


```
exp         ::= primary
                prefixOp exp                % [1]
                exp infixOp exp             % [1]
                infixOp : exp               % exp's elements combined by op [2]
                exp IS type                 % (EXISTS x: type | exp = x)
                exp AS type                 % error unless (exp IS type) [14]

primary     ::= literal
                primary . id                % method invocation [3] or record field
                primary arguments           % function invocation
                constructor
                ( exp )
                ( quantif declList | pred ) % /\:{d | p} for ALL, \/ for EXISTS [4]
                ( pred => exp1 [*] exp2 )   % if pred then exp1 else exp2 [5]
                ( pred => exp1 )            % undefined if pred is false

literal     ::= intLiteral                  % sequence of decimal digits
                charLiteral                 % 'x', x a printing character
                stringLiteral               % "xxx", with \ escapes as in C

arguments   ::= ( expList )                 % the arg is the tuple (expList)
                ( )

constructor ::= { }                         % empty function/sequence/set [6]
                { expList }                 % sequence/set constructor [6]
                ( expList )                 % tuple constructor
                name    { }                 % name denotes a func/seq/set type [6]
                name    { expList }         % name denotes a seq/set/record type [6]
                primary { fieldDefList  }   % record constructor [7]
                primary { exp -> result }   % function or sequence constructor [8]
                primary { *   -> result }   % function constructor [8]
                ( LAMBDA signature = cmd )  % function with the local state [9]
                ( \ declList | exp )        % short for (LAMBDA(d)->T=RET exp) [9]
                { declList   | pred | exp }  % set constructor [10]
                { seqGenList | pred | exp }  % sequence constructor [11]

fieldDef    ::= id := exp

result      ::= empty                       % the function is undefined
                exp                         % the function yields exp
                RAISE exception             % the function yields exception

seqGen      ::= id := exp BY exp WHILE exp  % sequence generator [11]
                id :IN exp

pred        ::= exp                         % predicate, of type Bool
quantif     ::= ALL
                EXISTS
```

		(precedence)	*argument/result types*		*operation*
infixOp	::= **	% (8)	(Int, Int)->Int		exponentiate
	*	% (7)	(Int, Int)->Int		multiply
		%	(T->U, U->V)->(T->V)	[12]	function composition
	/	% (7)	(Int, Int)->Int		divide
	//	% (7)	(Int, Int)->Int		remainder
	+	% (6)	(Int, Int)->Int		add
		%	(SEQ T, SEQ T)->SEQ T	[12]	concatenation
		%	(T->U, T->U)->(T->U)	[12]	function overlay
	-	% (6)	(Int, Int)->Int		subtract
		%	(SET T, SET T)->SET T	[12]	set difference;
		%	(SEQ T, SEQ T)->SEQ T	[12]	multiset difference
	!	% (6)	(T->U, T)->Bool	[12]	function is defined
	!!	% (6)	(T->U, T)->Bool	[12]	func has normal value
	..	% (5)	(Int, Int)->SEQ Int	[12]	subrange
	<=	% (4)	(Int, Int)->Bool		less than or equal
		%	(SET T, SET T)->Bool	[12]	subset
		%	(SEQ T, SEQ T)->Bool	[12]	prefix
	<	% (4)	(T, T)->Bool, T with <=		less than
		%	e1<e2 = (e1<=e2 /\ e1#e2)		
	>	% (4)	(T, T)->Bool, T with <=		greater than
		%	e1>e2 = e2<e1		
	>=	% (4)	(T, T)->Bool, T with <=		greater or equal
		%	e1>=e2 = e2<=e1		
	=	% (4)	(Any, Any)->Bool	[1]	equal
	#	% (4)	(Any, Any)->Bool		not equal
		%	e1#e2 = ~ (e1=e2)		
	<<=	% (4)	(SEQ T, SEQ T)->Bool	[12]	non-contiguous sub-seq
	IN	% (4)	(T, SET T)->Bool	[12]	membership
	/\	% (2)	(Bool, Bool)->Bool	[13]	conditional and
		%	(SET T, SET T)->SET T	[12]	intersection
	\/	% (1)	(Bool, Bool)->Bool	[13]	conditional or
		%	(SET T, SET T)->SET T	[12]	union
	==>	% (0)	(Bool, Bool)->Bool	[13]	conditional implies
	op	% (5)	not one of the above	[1]	
prefixOp	::= -	% (6)	Int->Int		negation
	~	% (3)	Bool->Bool		complement
	op	% (5)	not one of the above	[1]	

The ambiguity of the expression grammar is resolved by taking the `infixOps` to be left associative and using the indicated precedences for the `prefixOps` and `infixOps` (with 8 for `IS` and `AS` and 5 for `:` or any operator not listed); higher numbers correspond to tighter binding. The precedence is determined by the operator symbol and doesn't depend on the operand types.

[1] The meaning of `prefixOp e` is `T."prefixOp"(e)`, where `T` is `e`'s type, and of `e1 infixOp e2` is `T1."infixOp"(e1, e2)`, where `T1` is `e1`'s type. The built-in types `Int` (and `Nat` with the same operations), `Bool`, sequences, sets, and functions have the operations given in the grammar. Section 9 on built-in methods specifies the operators for built-in types other than `Int` and `Bool`. Special case: `e1 IN e2` means `T2."IN"(e1, e2)`, where `T2` is `e2`'s type.

Note that the `=` operator does not require that the types of its arguments agree, since both are `Any`. Also, `=` and `#` cannot be overridden by `WITH`. To define your own abstract equality, use a different operator such as `"=="`.

[2] The `exp` must have type `SEQ T` or `SET T`. The value is the elements of `exp` combined into a single value by `infixOp`, which must be associative and have an identity, and must also be commutative if `exp` is a set. Thus

```
+ : {i: Int | 0<i /\ i<5 | i**2} = 1 + 4 + 9 + 16 = 30,
```

and if `s` is a sequence of strings, `+ : s` is the concatenation of the strings. For another example, see the definition of quantifications in [4]. Note that the entire set is evaluated; see [10].

[3] Methods can be invoked by dot notation.

The meaning of `e.id` or `e.id()` is `T.id(e)`, where `T` is `e`'s type.

The meaning of `e1.id(e2)` is `T.id(e1, e2)`, where `T` is `e1`'s type.

Section 9 on built-in methods gives the methods for built-in types other than `Int` and `Bool`.

[4] A quantification is a conjunction (if the quantifier is `ALL`) or disjunction (if it is `EXISTS`) of the `pred` with the `id`'s in the `declList` bound to every possible value (that is, every value in their types); see section 4 for `decl`. Precisely, `(ALL d | p) = /\ : {d | p}` and `(EXISTS d | p) = \/ : {d | p}`. All the expressions in these expansions are evaluated, unlike `e2` in the expressions `e1 /\ e2` and `e1 \/ e2` (see [10] and [13]).

[5] A conditional `(pred => e1 [*] e2)` is not exactly an invocation. If `pred` is true, the result is the result of `e1` even if `e2` is undefined or exceptional; if `pred` is false, the result is the result of `e2` even if `e1` is undefined or exceptional. If `pred` is undefined, so is the result; if `pred` raises an exception, that is the result. If `[*] e2` is omitted and `pred` is false, the result is undefined.

[6] In a `constructor {expList}` each `exp` must have the same type `T`, the type of the `constructor` is `(SEQ T + SET T)`, and its value is the sequence containing the values of the `exp`s in the given order, which can also be viewed as the set containing these values.

If `expList` is empty the type is the union of all function, sequence and set types, and the value is the empty sequence or set, or a function undefined everywhere. If desired, these constructors can be prefixed by a `name` denoting a suitable set or sequence type.

A constructor `T{e1, ..., en}`, where `T` is a record type `[f1: T1, ..., fn: Tn]`, is short for a record constructor (see [7]) `T{f1:=e1, ..., fn:=en}`.

[7] The `primary` must have a record type, and the constructor has the same type as its `primary` and denotes the same value except that the fields named in the `fieldDefList` have the given

values. Each value must fit the type declared for its `id` in the record type. The `primary` may also denote a record type, in which case any fields missing from the `fieldDefList` are given arbitrary (but deterministic) values. Thus if `R=[a: Int, b: Int]`, `R{a := 3, b := 4}` is a record of type `R` with `a=3` and `b=4`, and `R{a := 3, b := 4}{a := 5}` is a record of type `R` with `a=5` and `b=4`. If the record type is qualified by a `SUCHTHAT`, the fields get values that satisfy it, and the constructor is undefined if that's not possible.

[8] The `primary` must have a function or sequence type, and the constructor has the same type as its `primary` and denotes a value equal to the value denoted by the `primary` except that it maps the argument value given by `exp` (which must fit the domain type of the function or sequence) to `result` (which must fit the range type if it is an `exp`). For a function, if `result` is `empty` the constructed function is undefined at `exp`, and if `result` is `RAISE exception`, then `exception` must be in the `RAISES` set of `primary`'s type. For a sequence `result` must not be `empty` or `RAISE`, and `exp` must be in `primary.dom` or the constructor expression is undefined.

In the `*` form the `primary` must be a function type or a function, and the value of the constructor is a function whose result is `result` at every value of the function's domain type (the type on the left of the `->`). Thus if `F=(Int->Int)` and `f=F{*->0}`, then `f` is zero everywhere and `f{4->1}` is zero except at `4`, where it is `1`. If this value doesn't have the function type, the constructor is undefined; this can happen if the type has a `SUCHTHAT` clause. For example, the type can't be a sequence.

[9] A `LAMBDA constructor` is a statically scoped function definition. When it is invoked, the meaning of the body is determined by the local state when the `LAMBDA` was evaluated and the global state when it is invoked; this is ad-hoc but convenient. See section 7 for `signature` and section 6 for `cmd`. The `returns` in the `signature` may not be `empty`. Note that a function can't have side effects.

The form `(\ declList | exp)` is short for `(LAMBDA (declList) -> T = RET exp)`, where `T` is the type of `exp`. See section 4 for `decl`.

[10] A set `constructor { declList | pred | exp }` has type `SET T`, where `exp` has type `T` in the current state augmented by `declList`; see section 4 for `decl`. Its value is a set that contains `x` iff `(EXISTS declList | pred /\ x = exp)`. Thus

```
{i: Int | 0<i /\ i<5 | i**2} = {1, 4, 9, 16}
```

and both have type `SET Int`. If `pred` is omitted it defaults to `true`. If `| exp` is omitted it defaults to the last `id` declared:

```
{i: Int | 0<i /\ i<5} = {1, 2, 3, 4 }
```

Note that if `s` is a set or sequence, `IN s` is a type (see section 4), so you can write a constructor like `{i :IN s | i > 4}` for the elements of `s` greater than 4. This is shorter and clearer than

```
{i | i IN s /\ i > 4}
```

If there are any values of the declared `id`'s for which `pred` is undefined, or `pred` is true and `exp` is undefined, then the result is undefined. If nothing is undefined, the same holds for exceptions; if more than one exception is raised, the result exception is an arbitrary choice among them.

[11] A sequence constructor `{ seqGenList | pred | exp }` has type `SEQ T`, where `exp` has type `T` in the current state augmented by `seqGenList`, as follows. The value of

```
{x1 := e01 BY e1 WHILE p1, ... , xn := e0n BY en WHILE pn | pred | exp}
```


is the sequence which is the value of `result` produced by the following program. Here `exp` has type `T` and `result` is a fresh identifier (that is, one that doesn't appear elsewhere in the program). There's an informal explanation after the program.

```
VAR x2 := e02, ..., xn := e0n, result := T{}, x1 := e01 |
    DO p1 => x2 := e2; p2 => ... => xn := en; pn =>
        IF pred => result := result + {exp} [*] SKIP FI;
        x1 := e1
    OD
```

However, `e0i` and `ei` are not allowed to refer to `xj` if `j > i`. Thus the `n` sequences are unrolled in parallel until one of them ends, as follows. All but the first are initialized; then the first is initialized and all the others computed, then all are computed repeatedly. In each iteration, once all the `xi` have been set, if `pred` is true the value of `exp` is appended to the result sequence; thus `pred` serves to filter the result. As with set constructors, an omitted `pred` defaults to `true`, and an omitted `| exp` defaults to `| xn`. An omitted `WHILE pi` defaults to `WHILE true`. An omitted `:= e0i` defaults to

```
:= {x: Ti | true}.choose
```

where `Ti` is the type of `ei`; that is, it defaults to an arbitrary value of the right type.

The generator `xi :IN ei` generates the elements of the sequence `ei` in order. It is short for

```
j := 0 BY j + 1 WHILE j < ei.size, xi BY ei(j)
```

where `j` is a fresh identifier. Note that if the `:IN` isn't the first generator then the first element of `ei` is skipped, which is probably not what you want. Note that `:IN` in a sequence constructor overrides the normal use of `IN s` as a type (see [10]).

Undefined and exceptional results are handled the same way as in set constructors.

Examples

`{i := 0 BY i+1 WHILE i <= n}`	`= 0..n = {0, 1, ..., n}`
`(r := head BY r.next WHILE r # nil | | r.val}`	the `val` fields of a list starting at `head`
`{x :IN s, sum := 0 BY sum + x}`	partial sums of `s`
`{x :IN s, sum := 0 BY sum + x}.last`	`+ : s`, the last partial sum
`{x :IN s, rev := {} BY {x} + rev}.last`	reverse of `s`
`{x :IN s | | f(x)}`	`s * f`
`{i :IN 1..n | i // 2 # 0 | i * i}`	squares of odd numbers `<= n`
`{i :IN 1..n, iter := e BY f(iter)}`	$\{f(e), f^2(e), ..., f^n(e)\}$

[12] These operations are defined in section 9.

[13] The conditional logical operators are defined in terms of conditionals:

```
e1 \/  e2 = (  e1 => true  [*] e2 )
e1 /\  e2 = ( ~e1 => false [*] e2 )
e1 ==> e2 = ( ~e1 => true  [*] e2 )
```

Thus the second operand is not evaluated if the value of the first one determines the result.

[14] `AS` changes only the type of the expression, not its value. Thus if `(exp IS type)` the value of `(exp AS type)` is the value of `exp`, but its type is `type` rather than the type of `exp`.

6. Commands

A command changes the state (or does nothing). Recall that the state is a mapping from names to values; we denote it by `state`. Commands are non-deterministic. An atomic command is one that is inside `<<...>>` brackets.

The meaning of an atomic command is a set of possible transitions (that is, a relation) between a state and an outcome (a state plus an optional exception); there can be any number of outcomes from a given state. One possibility is a looping exceptional outcome. Another is no outcomes. In this case we say that the atomic command *fails*; this happens because all possible choices within it encounter a false guard or an undefined invocation.

If a subcommand fails, an atomic command containing it may still succeed. This can happen because it's one operand of `[]` or `[*]` and the other operand succeeds. If can also happen because a non-deterministic construct in the language that might make a different choice. Leaving exceptions aside, the commands with this property are `[]`and `VAR` (because it chooses arbitrary values for the new variables). If we gave an operational semantics for atomic commands, this situation would correspond to backtracking. In the relational semantics that we actually give (in *Atomic Semantics of Spec*), it corresponds to the fact that the predicate defining the relation is the "or" of predicates for the subcommands. Look there for more discussion of this point.

A non-atomic command defines a collection of possible transitions, roughly one for each `<<...>>` command that is part of it. If it has simple commands not in atomic brackets, each one also defines a possible transition, except for `assignments` and `invocations`. An `assignment` defines two transitions, one to evaluate the right hand side, and the other to change the value of the left hand side. An `invocation` defines a transition for evaluating the arguments and doing the call and one for evaluating the result and doing the return, plus all the transitions of the body. These rules are somewhat arbitrary and their details are not very important, since you can always write separate commands to express more transitions, or atomic brackets to express fewer transitions. The motivation for the rules is to have as many transitions as possible, consistent with the idea that an expression is evaluated atomically.

A complete collection of possible transitions defines the possible sequences of states or histories; there can be any number of histories from a given state. A non-atomic command still makes choices, but it does not backtrack and therefore can have histories in which it gets stuck, even though in other histories a different choice allows it to run to completion. For the details, see handout 17 on formal concurrency.


```
cmd          ::= SKIP                          % [1]
                 HAVOC                         % [1]
                 RET                           % [2]
                 RET exp                       % [2]
                 RAISE exception               % [9]
                 CRASH                         % [10]
                 invocation                    % [3]
                 assignment                    % [4]

                 cmd              []  cmd      % or [5]
                 cmd              [*] cmd      % else [5]
                 pred             =>  cmd      % guarded cmd: if pred then cmd [5]
                 VAR declInitList  |  cmd      % variable introduction [6]
                 cmd               ;  cmd      % sequential composition
                 cmd EXCEPT handler             % handle exception [9]

                 << cmd >>                      % atomic brackets [7]
                 BEGIN cmd END                  % just brackets
                 IF cmd FI                      % just brackets [5]
                 DO cmd OD                      % repeat until cmd fails [8]

invocation   ::= primary arguments              % primary has a routine type [3]

assignment   ::= lhs         := exp             % state := state{name -> exp} [4]
                 lhs infixOp := exp             % short for lhs := lhs infixOp exp
                 lhs         := invocation      % of a PROC or APROC
                 ( lhsList ) := exp             % exp a tuple that fits lhsList
                 ( lhsList ) := invocation
lhs          ::= name                          % defined in section 4
                 lhs . id                       % record field [4]
                 lhs arguments                  % function  [4]

declInit     ::= decl                          % initially any value of the type [6]
                 id : type := exp               % initially exp, which must fit type [6]
                 id        := exp               % short for id: T := exp, where
                                                %  T is the type of exp

handler      ::= exceptionSet => cmd           % [9]. See section 4 for exceptionSet
```

The ambiguity of the command grammar is resolved by taking the command composition operations `;`, `[]`, and `[*]` to be left-associative and `EXCEPT` to be right associative, and giving `[]` and `[*]` lowest precedence, `=>` and `|` next (to the right only, since their left operand is an `exp`), `;` next, and `EXCEPT` highest precedence.

[1] The empty command and `SKIP` make no change in the state. `HAVOC` produces an arbitrary outcome from any state; if you want to specify undefined behavior when a precondition is not satisfied, write `~precondition => HAVOC`.

[2] A `RET` may only appear in a routine body, and the `exp` must fit the result type of the routine. The `exp` is omitted iff the `returns` of the routine's `signature` is empty.

[3] For `arguments` see section 5. The argument are passed by value, that is, assigned to the formals of the procedure A function body cannot invoke a `PROC` or `APROC`; together with the rule for assignments (see [7]) this ensures that it can't affect the state. An atomic command can invoke an `APROC` but not a `PROC`. A command is atomic iff it is `<< cmd >>`, a subcommand of an

atomic command, or one of the simple commands SKIP, HAVOC, RET, or RAISE. The type-checking rule for invocations is the same as for function invocations in expressions.

[4] You can only assign to a name declared with VAR or in a signature. In an assignment the exp must fit the type of the lhs, or there is a fatal error. In a function body assignments must be to names declared in the signature or the body, to ensure that the function can't have side effects.

An assignment to a left hand side that is not a name is short for assigning a constructor to a name. In particular,

```
lhs(arguments) := exp is short for lhs := lhs{arguments->exp}, and
lhs . id       := exp is short for lhs := lhs{id := exp}.
```

These abbreviations are expanded repeatedly until lhs is a name.

In an assignment the right hand side may be an invocation (of a procedure) as well as an ordinary expression (which can only invoke a function). The meaning of `lhs := exp` or `lhs := invocation` is to first evaluate the exp or do the invocation and assign the result to a temporary variable v, and then do `lhs := v`. Thus the assignment command is not atomic unless it is inside `<<...>>`.

If the left hand side of an assignment is a `(lhsList)`, the exp must be a tuple of the same length, and each component must fit the type of the corresponding lhs. Note that you cannot write a tuple constructor that contains procedure invocations.

[5] A guarded command fails if the result of pred is undefined or false. It is equivalent to cmd if the result of pred is true. A pred is just a Boolean exp; see section 4.

`S1 [] S2` chooses one of the Si to execute. It chooses one that doesn't fail. Usually S1 and S2 will be guarded. For example,
`x=1 => y:=0 [] x> 1 => y:=1` sets y to 0 if x=1, to 1 if x>1, and has no outcome if x<1. But
`x=1 => y:=0 [] x>=1 => y:=1` might set y to 0 or 1 if x=1.

`S1 [*] S2` is the same as S1 unless S1 fails, in which case it's the same as S2.

`IF ... FI` are just command brackets, but it often makes the program clearer to put them around a sequence of guarded commands, thus:

```
IF   x < 0 => y := 3
[]   x = 0 => y := 4
[*]           y := 5
FI
```

[6] In a VAR the unadorned form of declInit initializes a new variable to an arbitrary value of the declared type. The := form initializes a new variable to exp. Precisely,

```
VAR id: T := exp | c
```

is equivalent to

```
VAR id: T | id := exp; c
```

The exp could also be a procedure invocation, as in an assignment.

Several declInits after VAR is short for nested VARs. Precisely,

```
VAR declInit ,     declInitList  | cmd
```

is short for

```
VAR declInit | VAR declInitList | cmd
```

This is unlike a module, where all the names are introduced in parallel.

[7] In an atomic command the atomic brackets can be used for grouping instead of `BEGIN ... END`; since the command can't be any more atomic, they have no other meaning in this context.

[8] Execute cmd repeatedly until it fails. If cmd never fails, the result is a looping exception that doesn't have a name and therefore can't be handled. Note that this is *not* the same as failure.

[9] Exception handling is as in Clu, but a bit simplified. Exceptions are named by literal strings (which are written without the enclosing quotes). A module can also declare an identifier that denotes a set of exceptions. A command can have an attached exception handler, which gets to look at any exceptions produced in the command (by RAISE or by an invocation) and not handled closer to the point of origin. If an exception is not handled in the body of a routine, it is raised by the routine's invocation.

An exception ex must be in the RAISES set of a routine r if either `RAISE ex` or an invocation of a routine with ex in its RAISES set occurs in the body of r outside the scope of a handler for ex.

[10] CRASH stops the execution of any current invocations in the module other than the one that executes the CRASH, and discards their local state. The same thing happens to any invocations outside the module from within it. After CRASH, no procedure in the module can be invoked from outside until the routine that invokes it returns. CRASH is meant to be invoked from within a special Crash procedure in the module that models the effects of a failure.

7. Modules

A program is some global declarations plus a set of modules. Each module contains variable, routine, exception, and type declarations.

Module definitions can be parameterized with mformals after the module id, and a parameterized module can be instantiated. Instantiation is like macro expansion: the formal parameters are replaced by the arguments throughout the body to yield the expanded body. The parameters must be types, and the body must type-check without any assumptions about the argument that replaces a formal other than the presence of a WITH clause that contains all the methods mentioned in the formal parameter list (that is, formals are treated as distinct from all other types).

Each module is a separate scope, and there is also a Global scope for the identifiers declared at the top level of the program. An identifier id declared at the top level of a non-parameterized module m is short for m.id when it occurs in m. If it appears in the exports, it can be denoted by m.id anywhere. When an identifier id that is declared globally occurs anywhere, it is short for Global.id. Global cannot be used as a module id.

An exported id must be declared in the module. If an exported id has a WITH clause, it must be declared in the module as a type with at least those methods, and only those methods are accessible outside the module; if there is no WITH clause, all its methods and constructors are accessible. This is Spec's version of data abstraction.

```
program      ::= toplevel* module* END

module       ::= modclass id mformals exports = body END id

modclass     ::= MODULE
                 CLASS                          % [4]

exports      ::= EXPORT exportList
export       ::= id
                 id WITH {methodList}           % see section 4 for method

mformals     ::= empty
                 [ mfpList ]
mfp          ::= id                             % module formal parameter
                 id WITH { declList }           % see section 4 for decl

body         ::= toplevel*                      % id must be the module id
                 id [ typeList ]                % instance of parameterized module

toplevel     ::= VAR declInit*                  % declares the decl ids [1]
                 CONST declInit*                % declares the decl ids as constant
                 routineDecl                    % declares the routine id
                 EXCEPTION exSetDecl*           % declares the exception set ids
                 TYPE typeDecl*                 % declares the type ids and any
                                                % ids in ENUMs

routineDecl  ::= FUNC   id signature =  cmd     % function
                 APROC  id signature =<<cmd>>   % atomic procedure
                 PROC   id signature =  cmd     % non-atomic procedure
                 THREAD id signature =  cmd     % one thread for each possible
                                                % invocation of the routine [2]
signature    ::= ( declList ) returns raises    % see section 4 for returns
                 ( )          returns raises    %  and raises

exSetDecl    ::= id = exceptionSet              % see section 4 for exceptionSet

typeDecl     ::= id = type                      % see section 4 for type
                 id = ENUM [ idList ]           % a value is one of the id's [3]
```

[1] The "`:= exp`" in a `declInit` (defined in section 6) specifies an initial value for the variable. The `exp` is evaluated in a state in which each variable used during the evaluation has been initialized, and the result must be a normal value, not an exception. The `exp` sees all the names known in the scope, not just the ones that textually precede it, but the relation "used during evaluation of initial values" on the variables must be a partial order so that initialization makes sense. As in an `assignment`, the `exp` may be a procedure invocation as well as an ordinary expression. It's a fatal error if the `exp` is undefined or the invocation fails.

[2] Instead of being invoked by the client of the module or by another procedure, a thread is automatically invoked in parallel once for every possible value of its arguments. The thread is named by the `id` in the declaration together with the argument values. So

```
VAR sum := 0, count := 0
THREAD P(i: Int) = i IN 0 .. 9 =>
  VAR t | t := F(i); <<sum := sum + t>>; <<count := count + 1>>
```


adds up the values of `F(0) ... F(9)` in parallel. It creates a thread `P(i)` for every integer `i`; the threads `P(0), ..., P(9)` for which the guard is true invoke `F(0), ..., F(9)` in parallel and total the results in `sum`. When `count = 10` the total is complete.

A thread is the only way to get an entire program to do anything (except evaluate initializing expressions, which could have side effects), since transitions only happen as part of some thread.

[3] The `id`'s in the list are declared in the module; their type is the `ENUM` type. There are no operations on enumeration values except the ones that apply to all types: equality, assignment, and routine argument and result communication.

[4] A class is shorthand for a module that declares a convenient object type. The next few paragraphs specify the shorthand, and the last one explains the intended usage.

If the class `id` is `Obj`, the module `id` is `ObjMod`. Each variable declared in a top level `VAR` in the class becomes a field of the `ObjRec` record type in the module. The module exports only a type `Obj` that is also declared globally. `Obj` indexes a collection of state records of type `ObjRec` stored in the module's `objs` variable, which is a function `Obj->ObjRec`. `Obj`'s methods are all the names declared at top level in the class except the variables, plus the `new` method described below; the exported `Obj`'s methods are all the ones that the class exports plus `new`.

To make a class routine suitable as a method, it needs access to an `ObjRec` that holds the state of the object. It gets this access through a `self` parameter of type `Obj`, which it uses to refer to the object state `objs(self)`. To carry out this scheme, each routine in the module, unless it appears in a `WITH` clause in the class, is 'objectified' by giving it an extra `self` parameter of type `Obj`. In addition, in a routine body every occurrence of a variable `v` declared at top level in the class is replaced by `objs(self).v` in the module, and every invocation of an objectified class routine gets `self` as an extra first parameter.

The module also gets a synthesized and objectified `StdNew` procedure that adds a state record to `objs`, initializes it from the class's variable initializations (rewritten like the routine bodies), and returns its `Obj` index; this procedure becomes the `new` method of `Obj` unless the class already has a `new` routine.

A class cannot declare a `THREAD`.

The effect of this transformation is that a variable `obj` of type `Obj` behaves like an object. The state of the object is `objs(obj)`. The invocation `obj.m` or `obj.m(x)` is short for `ObjMod.m(obj)` or `ObjMod.m(obj, x)` by the usual rule for methods, and it thus invokes the method `m`; in `m`'s body each occurrence of a class variable refers to the corresponding field in `obj`'s state. `obj.new()` returns a new and initialized `Obj` object. The following example shows how a class is transformed into a module.

```
CLASS Obj EXPORT T1, f, p, … =

TYPE T1 = … WITH {add:=AddT}
CONST c := …

VAR  v1:T1:=ei, v2:T2:=pi(v1), …

FUNC f(p1: RT1, …) = … v1 …
PROC p(p2: RT2, …) = … v2 …
FUNC AddT(t1, t2) = …
…

END Obj
```

```
MODULE ObjMod EXPORT Obj WITH {T1, f, p, new } =

TYPE T1 = … WITH {add:=AddT}
CONST c := …

TYPE ObjRec = [v1: T1, v2: T2, …]
     Obj = Int WITH {T1, c, f:=f, p:=p,
                     AddT:=AddT, …, new:=StdNew}
VAR  objs: Obj -> ObjRec := {}

FUNC f(self: Obj, p1: RT1, …) = … objs(self).v1 …
PROC p(self: Obj, p2: RT2, …) = … objs(self).v2 …
FUNC AddT(t1, t2) = …       % in T1's WITH, so not objectified
…
PROC StdNew(self: Obj) -> Obj =
  VAR obj: Obj | ~ obj IN objs.dom =>
    objs(obj)    := ObjRec{};
    objs(obj).v1 := ei;
    objs(obj).v2 := pi(objs(obj).v1);
    …;
    RET obj

END ObjMod

TYPE Obj = ObjMod.Obj
```

In abstraction functions and invariants we also write `obj.n` for field `n` in `obj`'s state, that is, for `ObjMod.objs(obj).n`.

8. Scope

The declaration of an identifier is known throughout the smallest scope in which the declaration appears (redeclaration is not allowed). This section summarizes how scopes work in Spec; terms defined before section 7 have pointers to their definitions. A scope is one of

> the whole `program`, in which just the predefined (section 3), module, and globally declared identifiers are declared;
>
> a `module`;
>
> the part of a `routineDecl` or `LAMBDA` expression (section 5) after the `=`;
>
> the part of a `VAR declInit | cmd` command after the `|` (section 6);
>
> the part of a constructor or quantification after the first `|` (section 5).
>
> a record `type` or `methodDefList` (section 4);

An identifier is declared by

> a module `id`, `mfp`, or `toplevel` (for types, exception sets, `ENUM` elements, and named routines),
>
> a `decl` in a record `type` (section 4), `| constructor` or quantification (section 5), `declInit` (section 6), routine `signature`, or `WITH` clause of a `mfp`, or
>
> a `methodDef` in the `WITH` clause of a `type` (section 4).

An identifier may not be declared in a scope where it is already known. An occurrence of an identifier `id` always refers to the declaration of `id` which is known at that point, except when `id` is being declared (precedes a `:`, the `=` of a `toplevel`, the `:=` of a record constructor, or the `:=` or `BY` in a `seqGen`), or follows a dot. There are four cases for dot:

> `moduleId . id` — the `id` must be exported from the basic module `moduleId`, and this expression denotes the meaning of `id` in that module.
>
> `record . id` — the `id` must be declared as a field of the record type, and this expression denotes that field of `record`. In an `assignment`'s `lhs` see [7] in section 6 for the meaning.
>
> `typeId . id` — the `typeId` denotes a type, `id` must be a method of this type, and this expression denotes that method.
>
> `primary . id` — the `id` must be a method of `primary`'s type, and this expression, together with any following arguments, denotes an invocation of that method; see [2] in section 5 on expressions.

If `id` refers to an identifier declared by a `toplevel` in the current module `m`, it is short for `m.id`. If it refers to an identifier declared by a `toplevel` in the `program`, it is short for `Global.id`. Once these abbreviations have been expanded, every name in the state is either global (contains a dot and is declared in a `toplevel`), or local (does not contain a dot and is declared in some other way).

Exceptions look like identifiers, but they are actually string literals, written without the enclosing quotes for convenience. Therefore they do not have scope.

9. Built-in methods

Some of the type constructors have built-in methods, among them the operators defined in the expression grammar. The built-in methods for types other than `Int` and `Bool` are defined below. Note that these are not complete definitions of the types; they do not include the constructors.

Sets

A set has methods for

computing union, intersection, and set difference, and adding or removing an element, testing for membership and subset,

choosing (deterministically) a single element from a set, or a sequence with the same members, or a maximum or minimum element, and turning a set into its characteristic predicate (the inverse is the predicate's `set` method).

We define these operations using a module that represents a set by its characteristic predicate. Precisely, `SET T` behaves as though it were `Set[T].S`, where

```
MODULE Set[T] EXPORT S =

TYPE S = Any->Bool SUCHTHAT (\ s | (ALL any | s(any) ==> (any IS T)))
% Defined everywhere so that type inclusion will work; see section 4.
                    WITH {"\/":=Union, "/\":=Intersection, "-":=Difference,
                          "IN":=In, "<=":=Subset, choose:=Choose, seq:=Seq,
                           pred:=Pred, perms:=Perms, fsort:=FSort, sort:=Sort,
                           fmax:=FMax, fmin:=FMin, max:=Max, min:=Min}

FUNC Union(s1, s2)->S        = RET (\ t | s1(t) \/  s2(t))  % s1 \/ s2
FUNC Intersection(s1, s2)->S = RET (\ t | s1(t) /\  s2(t))  % s1 /\ s2
FUNC Difference(s1, s2)->S   = RET (\ t | s1(t) /\ ~s2(t))  % s1 - s2
FUNC In(s, t)->Bool          = RET s(t)                     % t IN s
FUNC Subset(s1, s2)->Bool    = RET (ALL t| s1(t) ==> s2(t)) % s1 <= s2
FUNC Size(s)->Int            =                              % s.size
     VAR t | s(t) => RET Size(s-{t}) + 1 [*] RET 0
FUNC Choose(s)->T            = VAR t | s(t) => RET t        % s.choose
% Not really, since VAR makes a non-deterministic choice,
% but choose makes a deterministic one. It is undefined if s is empty.
FUNC Seq(s)->SEQ T           =                              % s.seq
% Defined only for finite sets.
     RET {q: SEQ T | q.set = s /\ q.size = s.size}.choose
FUNC Pred(s)->(T->Bool)      = RET s                        % s.pred
% s.pred is just s. pred is for symmetry with seq, set, etc.

FUNC Perms(s)->SET SEQ T         = RET s.seq.perms          % s.perms
FUNC FSort(s, f: (T,T)->Bool)->S = RET s.seq.fsort(f)       % s.fsort(f); f is compare
FUNC Sort(s)->S                  = RET s.seq.sort           % s.sort; only if T has <=
FUNC FMax(s, f: (T,T)->Bool)->T  = RET s.fsort(f).last      % s.fmax(f); a max under f
FUNC FMin(s, f: (T,T)->Bool)->T  = RET s.fsort(f).head      % s.fmin(f); a min under f
FUNC Max(s)->T                   = RET s.fmax(T."<=")       % s.max; only if T has <=
FUNC Min(s)->T                   = RET s.fmin(T."<=")       % s.min; only if T has <=
% Note that these functions are undefined if s is empty. If there are extremal
% elements not distinguished by f or "<=", they make an arbitrary deterministic choice.

END Set
```


There are constructors `{}` for the empty set, `{e1, e2, ...}` for a set with specific elements, and `{declList | pred | exp}` for a set whose elements satisfy a predicate. These constructors are described in [6] and [10] of section 5. Note that `{t | p}.pred = (\ t | p)`, and similarly `(\ t | p).set = {t | p}`.

Functions

The function types `T->U` and `T->U RAISES XS` have methods for

composition, overlay, inverse, and restriction;

testing whether a function is defined at an argument and whether it produces a normal (non-exceptional) result at an argument, and for the domain and range;

converting a function to a relation (the inverse is the relation's `func` method).

In other words, they behave as though they were `Function[T, U].F`, where (making allowances for the fact that `XS` and `V` are pulled out of thin air):

```
MODULE Function[T, U] EXPORT F =

TYPE F = T->U RAISES XS WITH {"*":=Compose, "+":=Overlay,
                              inv:=Inverse, restrict:=Restrict,
                              "!":=Defined, "!!":=Normal,
                              dom:=Domain, rng:=Range, rel:=Rel}
     R = (T, U) -> Bool

FUNC Compose(f, g: U -> V) -> (T -> V) = RET (\ t | g(f(t)))

FUNC Overlay(f1, f2) -> F = RET (\ t | (f2!t => f2(t) [*] f1(t)))
% (f1 + f2) is f2(x) if that is defined, otherwise f1(x)

FUNC Inverse(f) -> (U -> T) = RET f.rel.inv.func
FUNC Restrict(f, s: SET T) -> F = RET (\ t | (t IN s => f(t)))

FUNC Defined(f, t)->Bool =
    IF f(t)=f(t) => RET true [*] RET false FI EXCEPT XS => RET true

FUNC Normal(f, t)->Bool =
    IF f(t)=f(t) => RET true [*] RET false FI EXCEPT XS => RET false

FUNC Domain(f) -> SET T = RET {t | f!t}
FUNC Range (f) -> SET U = RET {t | f!!t | f(t)}

FUNC Rel(f) -> R = RET (\ t, u | f(t) = u)

END Function
```

Note that there are constructors `{}` for the function undefined everywhere, `T{* -> result}` for a function of type `T` whose value is `result` everywhere, and `f{exp -> result}` for a function which is the same as `f` except at `exp`, where its value is `result`. These constructors are described in [6] and [8] of section 5. There are also lambda constructors for defining a function by a computation, described in [9] of section 5.

A total function `T->Bool` is a predicate and has an additional method to compute the set of `T`'s that satisfy the predicate (the inverse is the set's `pred` method). In other words, a predicate behaves as though it were `Predicate[T].P`, where

```
MODULE Predicate[T] EXPORT P =

TYPE P = T -> Bool WITH {set:=Set}
FUNC Set(p) -> SET T = RET {t | p(t)}
END Predicate
```

A predicate with `T = (T0, U0)` is a relation and has additional methods to turn it into a function, a total function, or a function to sets of `U0`'s, and to get its domain and range, invert it or compose it (overriding the methods for a function). In other words, it behaves as though it were `Relation[T0, U0].R`, where (making allowances for the fact that `V` is pulled out of thin air in `Compose`):

```
MODULE Relation[T, U] EXPORT R =

TYPE R = (T, U) -> Bool WITH {func:=Func, totalF:=TotalFunc, setF:=SetFunc,
                              dom:=Domain, rng :=Range,
                              inv:=Inverse, "*":=Compose}

FUNC Func(r) -> (T -> U) =
% The result function is defined at t iff r relates t to a single u.
     RET (\ t | (r.setF(t).size = 1 => r.setF(t).choose))

FUNC TotalFunc(r) -> (T -> (U + Null)) =
% The result function is defined everywhere, returning some related U, or nil if there is none.
     RET (\ t | (r.setF(t) # {}     => r.setF(t).choose [*] nil))
FUNC SetFunc(r) -> (T -> SET U) = RET (\ t | {u | r(t, u)})
% The result function is defined everywhere, returning the set of related U's.

FUNC Domain(r) -> SET T = RET {t, u | r(t, u) | t}
FUNC Range (r) -> SET U = RET {t, u | r(t, u) | u}

FUNC Inverse(r) -> ((U, T) -> Bool) = RET (\ u, t | r(t, u))

FUNC Compose(r: R, s: (U, V)->Bool) -> (T, V)->Bool =
     RET (\ t, v | (EXISTS u | r(t, u) /\ s(u, v)) )

END Relation
```

A relation with `T = U` is a graph and has additional methods to test whether a sequence of `T`'s is a path in the graph and to compute the transitive closure . In other words, it behaves as though it were `Graph[T].G`, where

```
MODULE Graph[T] EXPORT G =

TYPE G = (T, T) -> Bool WITH {isPath:=IsPath, closure:=TransitiveClosure }
     P = SEQ T

FUNC IsPath(g, p) = RET (ALL i :IN p.dom - {0} | g(p(i-1), p(i)))
% Any p of size <= 1 is a path by this definition.
FUNC TransitiveClosure(g) -> G = RET (\ t1, t2 |
     (EXISTS p | p.size > 1 /\ p.head = t1 /\ p.last = t2 /\ g.isPath(p) ))

END Graph
```


Sequences

A function is called a sequence if its domain is a finite set of consecutive `Int`'s starting at 0, that is, if it has type

```
Q = Int -> T SUCHTHAT (\ q | (EXISTS size: Int | q.dom = (0 .. size-1).set))
```

We denote this type (with the methods defined below) by `SEQ T`. A sequence inherits the methods of the function (though it overrides `+`), and it also has methods for

- detaching or attaching the first or last element,
- extracting a segment of a sequence, concatenating two sequences, or finding the size,
- making a sequence with all elements the same
- making a sequence into a tuple or set (`rng` also makes it into a set),
- testing for empty, prefix, or sub-sequence (not necessarily contiguous),
- lexical comparison, permuting, and sorting,
- treating a sequence as a multiset with operations to:
  - count the number of times an element appears, test membership and multiset equality, take differences, and remove an element (`"+"` or `"\/"` is union and `addl` adds an element).

All these operations are undefined if they use out-of-range subscripts, except that an empty sub-sequence is defined regardless of the subscripts.

We define the sequence methods with a module. Precisely, `SEQ T` is `Sequence[T].Q`, where:

```
MODULE Sequence[T] EXPORTS Q =

TYPE I          =  Int
     Q          =  (I -> T)
                   SUCHTHAT (\ q | (ALL i | q!i = (0 <= i /\ i < q.size)))
                   WITH { size:=Size, sub:=Sub, "+":=Concatenate,
                          head:=Head, tail:=Tail, addh:=AddHead, remh:=Tail,
                          last:=Last, reml:=RemoveLast, addl:=AddLast,
                          seg:=Seg, fill:=Fill, tuple:=Tuple,
                          isEmpty:=IsEmpty, "<=":=Prefix, "<<=":=SubSeq,
                          lexLE:=LexLE, perms:=Perms,
                          fsorter:=FSorter, fsort:=FSort, sort:=Sort,

                     % These methods treat a sequence as a multiset (or bag).
                          count:=Count, "IN":=In, "==":=EqElem,
                          "\/":=Concatenate, "-":=Diff, set:=Q.rng }

FUNC Size(q)-> Int  = RET q.dom.size

FUNC Sub(q, i1, i2) -> Q =                              % q.sub(i1, i2); yields
     RET ({0, i1}.max .. {i2, q.size-1}.min) * q        % {q(i1),...,q(i2)}

FUNC Concatenate(q1, q2) -> Q = VAR q |                 % q1 + q2
     q.sub(0, q1.size-1) = q1 /\ q.sub(q1.size, q.size-1) = q2 => RET q

FUNC Head(q) -> T = RET q(0)                            % q.head; first element
FUNC Tail(q) -> Q =                                     % q.tail; all but first
     q.size > 0 => RET q.sub(1, q.size-1)
FUNC AddHead(q, t) -> Q = RET {t} + q                   % q.addh(t)
```

```
FUNC Last(q) -> T = RET q(q.size-1)                    % q.last; last element
FUNC RemoveLast(q) -> Q =                              % q.reml; all but last
     q # {} => RET q.sub(0, q.size-2)
FUNC AddLast(q, t) -> Q = RET q + {t}                  % q.addl(t)

FUNC Seg(q, i, n: I) -> Q = RET q.sub(i, i+n-1)        % q.seg(i,n); n T's from q(i)

FUNC Fill(t, n: I) -> Q = RET {i :IN 0 .. n-1 | | t}   % yields i copies of t

FUNC IsEmpty(q) -> Bool = RET (q = {})

FUNC Prefix(q1, q2) -> Bool =                          % q1 <= q2
     RET (EXISTS q | q1 + q = q2)

FUNC SubSeq(q1, q2) -> Bool =                          % q1 <<= q2
% Are q1's elements in q2 in the same order, not necessarily contiguously.
     RET (EXISTS p: SET Int | p <= q2.dom /\ q1 = p.seq.sort * q2)

FUNC LexLE(q1, q2, f: (T,T)->Bool) -> Bool =           % q1.lexLE(q2, f); f is <=
% Is q1 lexically less than or equal to q2. True if q1 is a prefix of q2,
% or the first element in which q1 differs from q2 is less.
     RET    q1 <= q2
         \/ (EXISTS i :IN q1.dom /\ q2.dom |   q1.sub(0, i-1) = q2.sub(0, i-1)
                                           /\ q1(i) # q2(i)) /\ f(q1(i), q2(i))

FUNC Perms(q)->SET Q =                                 % q.perms
     RET {q' | (ALL t | q.count(t) = q'.count(t))}

FUNC FSorter(q, f: (T,T)->Bool)->SEQ Int =             % q.fsorter(f); f is <=
% The permutation that sorts q stably. Note: can't use min to define this, since min is defined using sort.
     VAR ps := {p :IN q.dom.seq.perms                  % all perms that sort q
               | (ALL i :IN (q.dom - {0}) | f((p*q)(i-1), (p*q)(i))) } |
       VAR p0 :IN ps |                                 % the one that reorders the least
          (ALL p :IN ps | p0.lexLE(p, Int."<=")) => RET p0

FUNC FSort(q, f: (T,T)->Bool) -> Q =                   % q.fsort(f); f is <= for the sort
     RET q.fsorter(f) * q
FUNC Sort(q)->Q = RET q.fsort(T.”<=”)                  % q.sort; only if T has <=

FUNC Count(q, t)->Int = RET {t' :IN q | t' = t}.size   % q.count(t)

FUNC In(t, q)->Bool = RET (q.count(t) # 0)             % t IN q
FUNC EqElem(q1, q2) -> Bool = RET q1 IN q2.perms       % q1 == q2; equal as multisets

FUNC Diff(q1, q2) -> Q =                               % q1 - q2
     RET {q | (ALL t | q.count(t) = {q1.count(t) - q2.count(t), 0}.max)}.choose

END Sequence
```

We can't program `Tuple` in Spec, but it is defined as follows. If `q: SEQ T`, then `q.tuple` is a tuple of `q.size` `T`'s, the first equal to `q(0)`, the second equal to `q(1)`, and so forth. For the inverse, if `u` is a tuple of `T`'s, then `u.seq` is a `SEQ T` such that `u.seq.tuple = u`. If `u` is a tuple in which not all the elements have the same declared type, then `u.seq` is a `SEQ Any` such that `u.seq.tuple = u`.

`Int` has a method `..` for making sequences: `i .. j = {i, i+1, ..., j-1, j}`. If `j < i`, `i .. j = {}`. You can also write `i .. j` as `{k := i BY k + 1 WHILE k <= j}`; see [11] in section 5. `Int` also has a `seq` method: `i.seq = 0 .. i-1`.

There is a constructor `{e1, e2, ...}` for a sequence with specific elements and a constructor `{}` for the empty sequence. There is also a constructor `q{e1 -> e2}`, which is equal to `q` except at `e1` (and undefined if `e1` is out of range). For the constructors see [6] and [8] of section 5. To generate a sequence there are constructors `{x :IN q | pred | exp}` and `{x := e1 BY e2 WHILE pred1 | pred2 | exp}`. For these see [11] of section 5.

To map each element `t` of `q` to `f(t)` use function composition `q * f`. Thus if `q: SEQ Int`, `q * (\ i: Int | i*i)` yields a sequence of squares. You can also write this `{i :IN q | | i*i}`.

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6.826—Principles of Computer Systems 2000

5. Examples of Specifications and Implementations

This handout is a supplement for the first two lectures. It contains several example specifications and implementations, all written using Spec.

Section 1 contains a specification for sorting a sequence. Section 2 contains two specifications and one implementation for searching for an element in a sequence. Section 3 contains specifications for a read/write memory. Sections 4 and 5 contain implementations for a read/write memory based on caching and hashing, respectively. Finally, Section 6 contains an implementation based on replicated copies.

1. Sorting

The following specification describes the behavior required of a program that sorts sets of some type `T` with a `"<="` comparison method. We do not assume that `"<="` is antisymmetric; in other words, we can have `t1 <= t2` and `t2 <= t1` without having `t1 = t2`, so that `"<="` is not enough to distinguish values of `T`. For instance, `T` might be the record type `[name:String, salary: Int]` with `"<="` comparison of the `salary` field. Several `T`'s can have different `name`s but the same `salary`.

```
APROC Sort(s: SET T) -> SEQ T = <<
    VAR q: SEQ T | (ALL i: T | s.count(i) = q.count(i)) /\ Sorted(b) => RET b >>
```

This specification uses the auxiliary function `Sorted`, defined as follows.

```
FUNC Sorted(q: SEQ T) -> Bool = RET (ALL i :IN q.dom – {0} |  q(i-1) <= q(i))
```

If we made `Sort` a `FUNC` rather than a `PROC`, what would be wrong?[1] What could we change to make it a `FUNC`?

We could have written this more concisely as

```
APROC Sort(s: SET T) -> SEQ T =
    << VAR q :IN a.perms | Sorted(q) => RET q >>
```

using the `perms` method for sets that returns a set of sequences that contains all the possible permutations of the set.

2. Searching

Search specification

We begin with a specification for a procedure to search an array for a given element. Again, this is an `APROC` rather than a `FUNC` because there can be several allowable results for the same inputs.

[1] Hint: a `FUNC` can't have side effects and must be deterministic (return the same value for the same arguments).


```
APROC Search(q: SEQ T, x: T) -> Int RAISES {NotFound} =
    << IF  VAR i: Int | (0 <= i /\ i < q.size /\ q(i) = x) => RET i
       [*] RAISE NotFound
       FI >>
```

Or, equivalently but slightly more concisely:

```
APROC Search(q: SEQ T, x: T) -> Int RAISES {NotFound} =
    << IF VAR i :IN q.dom | q(i) = x => RET i [*] RAISE NotFound FI >>
```

Sequential search implementation

Here is an implementation of the `Search` specification given above. It uses sequential search, starting at the first element of the input sequence.

```
APROC SeqSearch(q: SEQ T, x: T) -> Int RAISES {NotFound} = << VAR i := 0 |
    DO i < q.size => IF q(i) = x => RET i [*] i + := 1 FI OD; RAISE NotFound >>
```

Alternative search specification

Some searching algorithms, for example, binary search, assume that the input argument sequence is sorted. Such algorithms require a different specification, one that expresses this requirement.

```
APROC Search1(q: SEQ T, x: T) -> Int RAISES {NotFound} = <<
    IF  ~Sorted(q) => HAVOC
    [*] VAR i :IN q.dom | q(i) = x => RET i
    [*] RAISE NotFound
    FI >>
```

You might consider writing the specification to raise an exception when the array is not sorted:

```
APROC Search2(q: SEQ T, x: T) -> Int RAISES {NotFound, NotSorted} = <<
    IF  ~Sorted(q) => RAISE NotSorted
    ...
```

This is not a good idea. The whole point of binary search is to obtain O(log *n*) time performance (for a sorted input sequence). But any implementation of the `Search2` specification requires an O(*n*) check, even for a sorted input sequence, in order to verify that the input sequence is in fact sorted.

This is a simple but instructive example of the difference between defensive programming and efficiency. If `Search` were part of an operating system interface, it would be intolerable to have `HAVOC` as a possible transition, because the operating system is not supposed to go off the deep end no matter how it is called (though it might be OK to return the wrong answer if the input isn't sorted; what would that spec be?). On the other hand, the efficiency of a program often depends on assumptions that one part of it makes about another, and it's appropriate to express such an assumption in a spec by saying that you get `HAVOC` if it is violated. We don't care to be more specific about what happens because we intend to ensure that it doesn't happen. Obviously a program written in this style will be more prone to undetected or obscure errors than one that checks the assumptions, as well as more efficient.

3. Read/write memory

The simplest form of read/write memory is a single read/write register, say of type `D` (for data), with arbitrary initial value. The following Spec module describes this:

```
MODULE Register [D] EXPORT Read, Write =

VAR x: D                                                    % arbitrary initial value

APROC Read() -> D = << RET x  >>
APROC Write(d)    = << x := d >>

END Register
```

Now we give a specification for a simple addressable memory with elements of type `D`. This is like a collection of read/write registers, one for each address in a set `A`. In other words, it's a function from addresses to data values. For variety, we include new `Reset` and `Swap` operations in addition to `Read` and `Write`.

```
MODULE Memory [A, D] EXPORT Read, Write, Reset, Swap =

TYPE M =  A -> D
VAR  m := Init()

APROC Init() -> M  = << VAR m' | (ALL a | m'!a) => RET m' >>
% Choose an arbitrary function that is defined everywhere.

FUNC  Read(a) -> D = << RET m(a)  >>
APROC Write(a, d)  = << m(a) := d >>

APROC Reset(d)     = << m := M{* -> d} >>
% Set all memory locations to d.

APROC Swap(a, d) -> D = << VAR d' := m(a) | m(a) := d; RET d' >>
% Set location a to the input value and return the previous value.

END Memory
```

The next three sections describe implementations of `Memory`.

4. Write-back cache implementation

Our first implementation is based on two memory mappings, a main memory `m` and a *write-back cache* `c`. The implementation maintains the invariant that the number of addresses at which `c` is defined is constant. A real cache would probably maintain a weaker invariant, perhaps bounding the number of addresses at which `c` is defined.

```
MODULE WBCache [A, D] EXPORT Read, Write, Reset, Swap =
% implements Memory

TYPE M          =  A -> D
     C          =  A -> D

CONST Csize     :  Int := ...                          % cache size

VAR  m          := InitM()
     c          := InitC()

APROC InitM() -> M = << VAR m' | (ALL a | m'!a)      => RET m' >>
% Returns a M with arbitrary values.

APROC InitC() -> C = << VAR c' | c'.dom.size = CSize => RET c' >>
% Returns a C that has exactly CSize entries defined, with arbitrary values.

APROC Read(a) -> D = << Load(a); RET c(a) >>
APROC Write(a, d) = << IF ~c!a => FlushOne() [*] SKIP FI; c(a) := d >>
% Makes room in the cache if necessary, then writes to the cache.

APROC Reset(d) = <<...>>                               % exercise for the reader

APROC Swap(a, d) -> D = << VAR d' | Load(a); d' := c(a); c(a) := d; RET d' >>

% Internal procedures.

APROC Load(a) = << IF ~c!a => FlushOne(); c(a) := m(a) [*] SKIP FI >>
% Ensures that address a appears in the cache.

APROC FlushOne() =
% Removes one (arbitrary) address from the cache, writing the data value back to main memory if necessary.
    << VAR a | c!a => IF Dirty(a) => m(a) := c(a) [*] SKIP FI; c := c{a -> } >>

FUNC Dirty(a) -> Bool = RET c!a /\ c(a) # m(a)
% Returns true if the cache is more up-to-date than the main memory.

END WBCache
```

The following Spec function is an abstraction function mapping a state of the `WBCache` module to a state of the `Memory` module. It's written to live inside the module. It says that the contents of location a is `c(a)` if `a` is in the cache, and `m(a)` otherwise.

```
FUNC AF() -> M = RET (\ a | c!a => c(a) [*] m(a) )
```

5. Hash table implementation

Our second implementation of `Memory` uses a hash table for the representation.

```
MODULE HashMemory [A WITH {hf: A->Int}, D] EXPORT Read, Write, Reset, Swap =
% Implements Memory.
% The module expects that the hash function A.hf is total and that its range is 0 .. n for some n.

TYPE Pair       =  [a, d]
     B          =  SEQ Pair                          % Bucket in hash table
     HashT      =  SEQ B

VAR  nb         := NumB()                            % Number of Buckets
     m          := HashT.fill(B{}, nb)               % Memory hash table; initially empty
     default    :  D                                 % arbitrary default value

APROC Read(a) -> D = << VAR b := m(a.hf), i: Int |
     i := FindEntry(a, b) EXCEPT NotFound => RET default ; RET b(i).d >>

APROC Write(a, d) = << VAR b := DeleteEntry(a, m(a.hf)) |
     m(a.hf) := b + {Pair{a, d}} >>

APROC Reset(d) = << m := HashT.fill(B{}, nb); default := d >>

APROC Swap(a, d) -> D = << VAR d' | d' := Read(a); Write(a, d); RET d' >>

% Internal procedures.

FUNC NumBs() -> Int =
% Returns the number of buckets needed by the hash function; havoc if the hash function is not as expected.
     IF VAR n: Nat | A.hf.rng = (0 .. n).set => RET n + 1 [*] HAVOC FI

APROC FindEntry(a, b) -> Int RAISES (NotFound) =
% If a appears in a pair in b, returns the index of some pair containing a; otherwise raises NotFound.
     << VAR i :IN b.dom | b(i).a = a => RET i [*] RAISE NotFound >>

APROC DeleteEntry(a, b) -> B << VAR i: Int |
% Removes some pair with address a from b, if any exists.
     i := FindEntry(a, b) EXCEPT NotFound => RET b ;
     RET b.sub(0, i-1) + b.sub(i+1, b.size-1) >>

END HashMemory
```

Note that `FindEntry` and `DeleteEntry` are `APROC`s because they are not deterministic when given arbitrary `b` arguments.

The following is a key invariant that holds between invocations of the operations of `HashMemory`:

```
FUNC Inv() -> Bool = RET
     (  nb > 0
     /\ m.size = nb
     /\ (ALL a | a.hf IN m.dom)
     /\ (ALL i :IN m.dom, p :IN m(i).rng | p.a.hf = i)
     /\ (ALL a | { j :IN m(a.hf).dom | m(a.hf)(j).a = a }.size <= 1) )
```

This says that the number of buckets is positive, that the hash function maps all addresses to actual buckets, that a pair containing address `a` appears only in the bucket at index `a.hf` in `m`, and

that at most one pair for an address appears in the bucket for that address. Note that these conditions imply that in any reachable state of `HashMemory`, each address appears in at most one pair in the entire memory.

The following Spec function is an abstraction function between states of the `HashMemory` module and states of the `Memory` module:

```
FUNC AF() -> M = RET
     (LAMBDA(a) -> D =
          IF  VAR i :IN m.dom, p :IN m(i).rng | p.a = a => RET p.d
          [*] RET default
          FI)
```

That is, the data value for address `a` is any value associated with address `a` in the hash table; if there is none, the data value is the default value. Spec says that a function is undefined at an argument if its body can yield more than one result value. The invariants given above ensure that the `LAMBDA` is actually single-valued for all the reachable states of `HashMemory`.

Of course `HashMemory` is not a fully detailed implementation. Its main deficiency is that it doesn't explain how to maintain the variable-length bucket sequences, which is usually done with a linked list. However, the implementation does capture all the essential details.

6. Replicated copies

Our final implementation is based on some number $k \geq 1$ of copies of each memory location. Initially, all copies have the same default value. A `Write` operation only modifies an arbitrary *majority* of the copies. A `Read` reads an arbitrary majority, and selects and returns the most recent of the values it sees. In order to allow the `Read` to determine which value is the most recent, each `Write` records not only its value, but also a sequence number.

For simplicity, we just show the module for a single read/write register. The constant `k` determines the number of copies.

```
MODULE MajorityRegister [D] =                        % implements Register

CONST k          =  5

TYPE N           =  Nat
     Kint        =  IN 1 .. k                        % ints between 1 and k
     Maj         =  SET KInt                         % all majority subsets of Kint
                      SUCHTHAT (\m: Maj | m.size>k/2)

TYPE P           =  [D, seqno: N]                    % Pair
     M           =  KInt -> P                        % Memory
     S           =  SET P

VAR  default     :  D
     m           := M{* -> P{d := default, seqno := 0}}

APROC Read() -> D = << RET ReadPair().d >>

APROC Write(d) = << VAR i: Int, maj |
% Determines the highest sequence number i, then writes d paired with i+1 to some majority maj of the copies.
```

```
    i := ReadPair().seqno;
    DO VAR j :IN maj | m(j).seqno # i+1 => m(j) := P{d := d, seqno := i+1} OD >>
```

% Internal procedures.

```
APROC ReadPair() -> P = << VAR s := ReadMaj () |
```
% Returns a pair with the largest sequence number from some majority of the copies.
```
    VAR p :IN s | p.seqno = {p' :IN s | | p'.seqno}.max => RET p >>

APROC ReadMaj () -> S = << VAR maj | RET { i :IN maj | | m(i) } >>
```
% Returns the set of pairs belonging to some majority of the copies.

```
END MajorityRegister
```

We could have written the body of `ReadPair` as

```
    << VAR s := ReadMaj() | RET s.fmax((\ p1, p2 | p1.seqno <= p2.seqno)) >>
```

except that `fmax` always returns the same maximal `p` from the same `s`, whereas the `VAR` in `ReadPair` chooses one non-deterministically.

The following is a key invariant for `MajorityRegister`.

```
FUNC Inv(m: M) -> Bool = RET
       (ALL p :IN m.rng, p' :IN m.rng | p.seqno = p'.seqno ==> p.d = p'.d)
    /\ (EXISTS maj | (ALL i :IN maj, p :IN m.rng | m(i).seqno >= p.seqno)))
```

The first conjunct says that any two pairs having the same sequence number also have the same data. The second conjunct says that the highest sequence number appears in some majority of the copies.

The following Spec function is an abstraction function between states of the `MajorityRegister` module and states of the `Register` module.

```
FUNC AF() -> D = RET m.rng.fmax((\ p1, p2 | p1.seqno <= p2.seqno)).d
```

That is, the abstract register data value is the data component of a copy with the highest sequence number. Again, because of the invariants, there is only one `p.d` that will be returned.

.

6. Abstraction Functions and Invariants

This handout describes the main techniques used to prove correctness of state machines: abstraction functions and invariant assertions. We demonstrate the use of these techniques for some of the `Memory` examples from handout 5.

Throughout this handout, we consider modules all of whose externally invocable procedures are `APROC`s. We assume that the body of each such procedure is executed all at once. Also, we do not consider procedures that modify global variables declared outside the module under consideration.

Modules as state machines

Our methods apply to an arbitrary state machine or automaton. In this course, however, we use a Spec module to define a state machine. Each state of the automaton designates values for all the variables declared in the module. The initial states of the automaton consist of initial values assigned to all the module's variables by the Spec code. The transitions of the automaton correspond to the invocations of `APROC`s together with their result values.

An *execution fragment* of a state machine is a sequence of the form s_0, π_1, s_1, π_2, …, where each s is a state, each π is a label for a transition (an invocation of a procedure), and each consecutive (s_i, π_{i+1}, s_{i+1}) triple follows the rules specified by the Spec code. (We do not define these rules here—wait for the lecture on atomic semantics.) An *execution* is an execution fragment that begins in an initial state.

The π_i are labels for the transitions; we often call them *actions*. When the state machine is written in Spec, each transition is generated by some atomic command, and we can use some unambiguous identification of the command for the action. At the moment we are studying sequential Spec, in which every transition is the invocation of an exported atomic procedure. We use the name of the procedure, the arguments, and the results as the label.

Figure 1 shows some of the states and transitions of the state machine for the `Memory` module with `A = IN 1 .. 4`, and Figure 2 does likewise for the `WBCache` module with `Csize = 2`. The arrow for each transition is labeled by its π_i, that is, by the procedure name, arguments, and result.

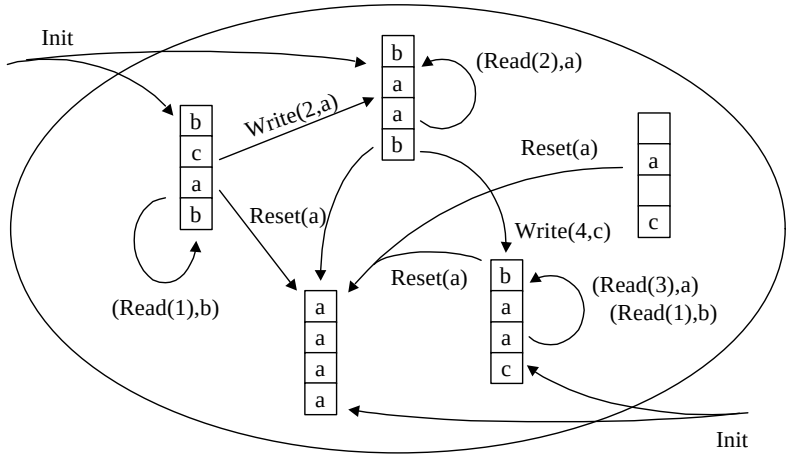


Figure 1: Part of the `Memory` state machine

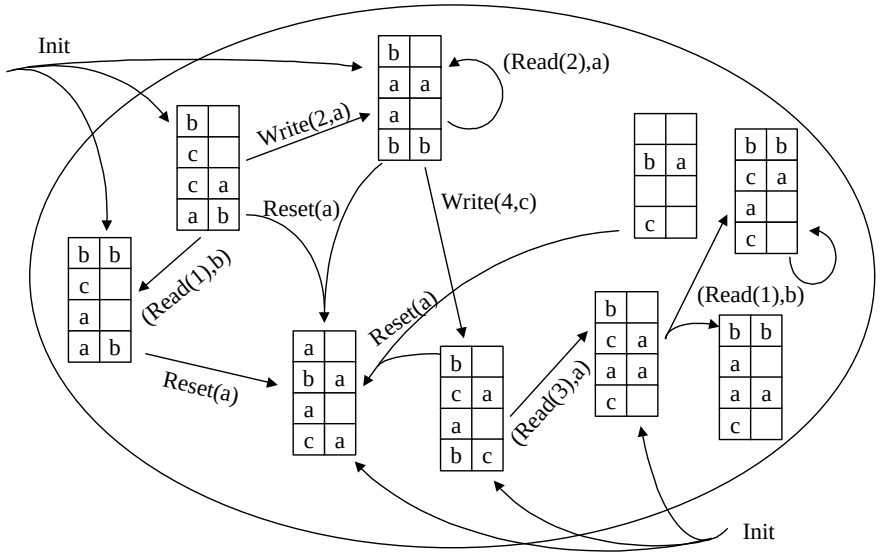


Figure 2: Part of the `WBCache` state machine

External behavior

Usually, a client of a module is not interested in all aspects of its execution, but only in some kind of external behavior. Here, we formalize the external behavior as a set of *traces*. That is, from an execution (or execution fragment) of a module, we discard both the states and the internal actions, and extract the *trace*. This is the sequence of labels π_i for external actions (that is, invocations of exported routines) that occur in the execution (or fragment). Then the external behavior of the module is the set of traces that are obtained from all of its executions.

It's important to realize that in going from the execution to the trace we are discarding a great deal of information. First, we discard all the states, keeping only the actions or labels. Second, we discard all the internal actions, keeping only the external ones. Thus the only information we keep in the trace is the behavior of the state machine at its external interface. This is appropriate, since we want to study state machines that have the same behavior at the external interface; we shall see shortly exactly what we main by 'the same' here. Two machines can have the same traces even though they have very different state spaces.

In the sequential Spec that we are studying now, a module only makes a transition when an exported routine is invoked, so all the transitions appear in the trace. Later, however, we will introduce modules with internal transitions, and then the distinction between the executions and the external behavior will be important.

For example, the set of traces generated by the `Memory` module includes the following trace:

```
(Reset(d),)
(Read(a1),d)
(Write(a2,d'))
(Read(a2),d')
```

However, the following trace is not included if `d # d'`:

```
(Reset(d))
(Read(a1),d')                    should have returned d
(Write(a2,d'))
(Read(a2),d)                     should have returned d'
```

In general, a trace is included in the external behavior of `Memory` if every `Read(a)` or `Swap(a, d)` operation returns the last value written to `a` by a `Write`, `Reset` or `Swap` operation, or returned by a `Read` operation; if there is no such previous operation, then `Read(a)` or `Swap(a, d)` returns an arbitrary value.

Implements relation

In order to understand what it means for one state machine to implement another one, it is helpful to begin by considering what it means for one atomic procedure to implement another. The meaning of an atomic procedure is a relation between an initial state just before the procedure starts (sometimes called a 'pre-state') and a final state just after the procedure has finished (sometimes called a 'post-state'). This is often called an 'input-output relation'. For example, the relation defined by a square-root procedure is that the post-state is the same as the pre-state, except that the square of the procedure result is close enough to the argument. This meaning makes sense for an arbitrary atomic procedure, not just for one in a trace.

We say that procedure *P* implements spec *S* if the relation defined by *P* (considered as a set of ordered pairs of states) is a subset of the relation defined by *S*. This means that *P* never does anything that S couldn't do. However, P doesn't have to do everything that S can do. An implementation of square root is probably deterministic and always returns the same result for a given argument. Even though the spec allows several results (all the ones that are within the specified tolerance), we don't require an implementation to be able to produce all of them; instead we are satisfied with one.

Actually this is not enough. The definition we have given allows *P*'s relation to be empty, that is, it allows *P* not to terminate. This is usually called 'partial correctness'. In addition, we usually want to require that *P*'s relation be total on the domain of *S*; that is, *P* must produce some result whenever *S* does. The combination of partial correctness and termination is usually called 'total correctness'.

If we are only interested in external behavior of a procedure that is part of a stateless module, the only state we care about is the arguments and results of the procedure. In this case, a transition is completely described by a single entry in a trace, such as `(Read(a1),d)`.

Now we are ready to consider modules with state. Our idea is to generalize what we did with pairs of states described by single trace entries to sequences of states described by longer traces. Suppose that *T* and *S* are any modules that have the same external interface (set of procedures that are exported and hence may be invoked externally). In this discussion, we will often refer to *S* as the *specification* module and *T* as the *implementation*. Then we say that *T implements S* if every trace of *T* is also a trace of *S*. That is, the set of traces generated by *T* is a subset of the set of traces generated by *S*.

This says that any external behavior of the implementation *T* must also be an external behavior of the spec *S*. Another way of looking at this is that we shouldn't be able to tell by looking at the implementation that we aren't looking at the spec, so we have to be able to explain every behavior of *T* as a possible behavior of *S*.

The reverse, however, is not true. We do not insist that the implementation must exhibit every behavior allowed by the spec. In the case of the simple memory the spec is completely deterministic, so the implementations cannot take advantage of this freedom. In general, however, the spec may allow lots of behaviors and the implementation choose just one. The spec for sorting, for instance, allows any sorted sequence as the result of `Sort`; there may be many such sequences if the ordering relation is not total. The implementation will usually be deterministic and return exactly one of them, so it doesn't exhibit all the behavior allowed by the spec.

Safety and liveness

Just as with procedures, this subset requirement is not strong enough to satisfy our intuitive notion of implementation. In particular, it allows the set of traces generated by *T* to be empty; in other word, the implementation might do nothing at all, or it might do some things and then stop. As we saw, the analog of this for a simple sequential procedure is non-termination. Usually we want to say that the implementation of a procedure should terminate, and similarly we want to say that the implementation of a module should keep doing things. More generally, when we

have concurrency we usually want the implementation to be *fair*, that is, to eventually service all its clients, and more generally to eventually make any transition that continues to be enabled.

It turns out that any external behavior (that is, any set of traces) can be described as the intersection of two sets of traces, one defined by a *safety* property and the other defined by a *liveness* property.[1] A safety property says that in the trace nothing bad ever happens, or more precisely, that no bad transition occurs in the trace. It is analogous to partial correctness for a stateless procedure; a state machine that never makes a bad transition can define any safety property. If a trace doesn't satisfy a safety property, you can always find this out by looking at a finite prefix of the trace, in particular, at a prefix that includes the first bad transition.

A liveness property says that in the trace something good *eventually* happens. It is analogous to termination for a stateless procedure. You can never tell that a trace doesn't have a liveness property by looking at a finite prefix, since the good thing might happen later. A liveness property cannot be defined by a state machine. It is usual to express liveness properties in terms of *fairness*, that is, in terms of a requirement that if some transition stays enabled continuously it eventually occurs (weak fairness), or that if some transition stays enabled intermittently it eventually occurs (strong fairness).

With a few exceptions, we don't deal with liveness in this course. There are two reasons for this. First, it is usually not what you want. Instead, you want a result within some time bound, which is a safety property, or you want a result with some probability, which is altogether outside the framework we have set up. Second, liveness proofs are usually hard.

Abstraction functions and simulation

The definition of 'implements' as inclusion of external behavior is a sound formalization of our intuition. It is difficult to work with directly, however, since it requires reasoning about infinite sets of infinite sequences of actions. We would like to have a way of proving that *T* implements *S* that allows us to deal with one of *T*'s actions at a time. Our method is based on *abstraction functions*.

An abstraction function maps each state of the implementation *T* to a state of the specification *S*. For example, each state of the `WBCache` module gets mapped to a state of the `Memory` module. The abstraction function explains how to interpret each state of the implementation as a state of the specification. For example, Figure 3 depicts part of the abstraction function from `WBCache` to `Memory`. Here is its definition in Spec, copied from handout 5.

```
FUNC AF() -> M = RET (\ a | c!a => c(a) [*] m(a) )
```

[1] B. Alpern and F. Schneider. Recognizing safety and liveness. *Distributed Computing* **2**, 3 (1987), pp 117-126.

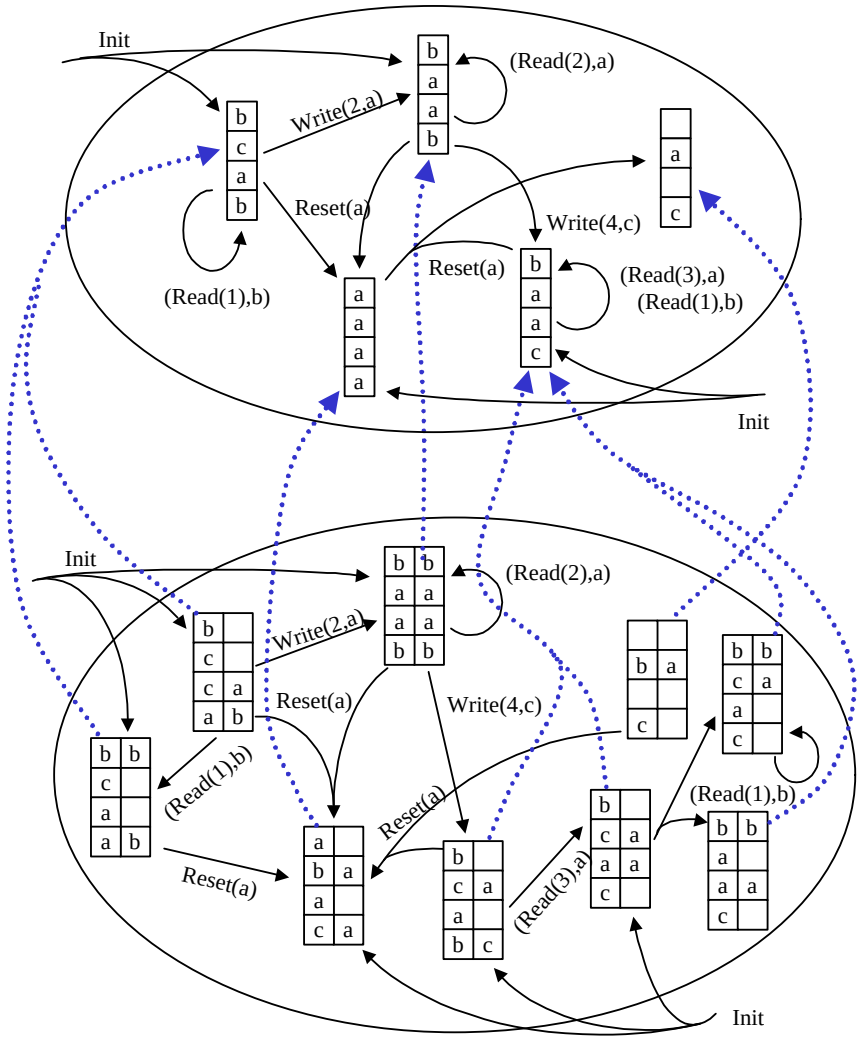


Figure 3: Abstraction function for `WBCache`

You might think that an abstraction function should map the other way, from states of the specification to states of the implementation, explaining how to represent each state of the specification. This doesn't work, however, because there is usually more than one way of representing each state of the specification. For example, for the `WBCache` implementation of `Memory`, if an address is in the cache, then the value stored for that address in memory does not matter. There are also choices about which addresses appear in the cache. Thus, many states of the implementation can represent the same state of the specification. In other words, the abstraction function is many-to-one.

An abstraction function F is required to satisfy the following two conditions.

1. If t is any initial state of T, then $F(t)$ is an initial state of S.
2. If t is a reachable state of T and (t, π, t') is a step of T, then there is a step of S from $F(t)$ to $F(t')$, having the same trace.

Condition 2 says that T *simulates* S; every step of T faithfully copies a step of S. It is stated in a particularly simple way, forcing the given step of T to simulate a single step of S. That is enough for the special case we are considering right now. Later, when we consider concurrent invocations for modules, we will generalize condition 2 to allow any number of steps of S rather than just a single step.

The diagram in Figure 4 represents condition 2. The dashed arrows represent the abstraction function F, and the solid arrows represent the transitions; if the lower (double) solid arrow exists in the implementation, the upper (single) solid arrow must exist in the specification. The diagram is sometimes called a "commutative diagram" because if you start at the lower left and follow arrows, you will end up in the same state regardless of which way you go.

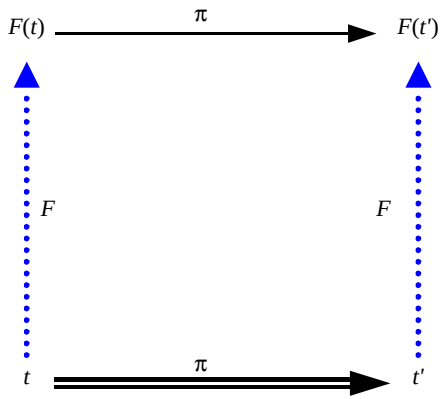


Figure 4: Commutative diagram for correctness

An abstraction function is important because it can be used to show that one module implements another:

Theorem 1: If there is an abstraction function from T to S, then T implements S, i.e., every trace of T is a trace of S.

Note that this theorem applies to both finite and infinite traces.

Proof: (Sketch) Let β be any trace of T, and let α be any execution of T that generates trace β. Use Conditions 1 and 2 above to construct an execution α' of S with the same trace. That is, if t is the initial state of α, then let $F(t)$ be the initial state of α'. For each step of α in turn, use Condition 2 to add a corresponding step to α'.

More formally, this proof is an induction on the length of the execution. Condition 1 gives the basis: any initial state of T maps to an initial state of S. Condition 2 gives the inductive step: if we have an execution of T of length n that simulates an execution of S, any next step by T simulates a next step by S, so any execution of T of length n+1 simulates an execution of S.

We would like to have an inverse of Theorem 1: if every trace of T is a trace of S, then there is an abstraction function that shows it. This is not true for the simple abstraction functions and simulations we have defined here. Later on, in handout 8, we will generalize them to a simulation method that is strong enough to prove that T implements S whenever that is true.

Invariants

An *invariant* of a module is any property that is true of all *reachable* states of the module, i.e., all states that can be reached in executions of the module (starting from initial states). Invariants are important because condition 2 for an abstraction function requires us to show that the implementation simulates the spec from every reachable state, and the invariants characterize the reachable states. It usually isn't true that the implementation simulates the spec from every state.

Here are examples of invariants for the `HashMemory` and `MajorityRegister` modules, written in Spec and copied from handout 5.

```
FUNC HashMemory.Inv(nb: Int, m: HashT, default: D) -> Bool = RET
     (  nb > 0
     /\ m.size = nb
     /\ (ALL a | a.hf IN m.dom)
     /\ (ALL i :IN m.dom, p :IN m(i).rng | p.a.hf = i)
     /\ (ALL a | { j :IN m(a.hf) | m(a.hf)(j).a = a }.size <= 1) )

FUNC MajorityRegister.Inv(m: M) -> Bool = RET
        (ALL p :IN m.rng, p' :IN m.rng | p.seqno = p'.seqno ==> p.d = p'.d)
     /\ (EXISTS maj | (ALL i :IN maj, p :IN m.rng | m(i).seqno >= p.seqno)))
```

For example, for the `HashMemory` module, the invariant says (among other things) that a pair containing address `a` appears only in the appropriate bucket `a.hf`, and that at most one pair for an address appears in the bucket for that address.

The usual way to prove that a property P is an invariant is by induction on the length of finite executions leading to the states in question. That is, we must show the following:

(Basis, length = 0) *P* is true in every initial state.

(Inductive step) If (*t*, π, *t*') is a transition and *P* is true in *t*, then *P* is also true in *t*'.

Not all invariants are proved directly by induction, however. It is often better to prove invariants in groups, starting with the simplest invariants. Then the proofs of the invariants in the later groups can assume the invariants in the earlier groups.

Example: We sketch a proof that the property `MajorityRegister.Inv` is in fact an invariant.

Basis: In any initial state, a single (arbitrarily chosen) default value *d* appears in all the copies, along with the `seqno` 0. This immediately implies both parts of the invariant.

Inductive step: Suppose that (*t*, π, *t*') is a transition and Inv is true in *t*. We consider cases based on π. If π is an invocation or response, or the body of a `Read` procedure, then the step does not affect the truth of Inv. So it remains to consider the case where π is a `Write`, say `Write(d)`.

In this case, the second part of the invariant for *t* (i.e., the fact that the highest `seqno` appears in more than half the copies), together with the fact that the `Write` reads a majority of the copies, imply that the `Write` obtains the highest `seqno`, say *i*. Then the new `seqno` that the `Write` chooses must be the new highest `seqno`. Since the `Write` writes *i*+1 to a majority of the copies, it ensures the second part of the invariant. Also, since it associates the same `d` with the `seqno` *i*+1 everywhere it writes, it preserves the first part of the invariant.

Proofs using abstraction functions

Example: We sketch a proof that the function `WBCache.AF` given above is an abstraction function from `WBCache` to `Memory`. In this proof, we get by without any invariants.

For Condition 1, suppose that *t* is any initial state of `WBCache`. Then `AF`(*t*) is some (memory) state of `Memory`. But all memory states are allowable in initial states of `Memory`. Thus, `AF`(*t*) is an initial state of `Memory`, as needed. For Condition 2, suppose that *t* and `AF`(*t*) are states of `WBCache` and `Memory`, respectively, and suppose that (*t*, π, *t*') is a step of `WBCache`. We consider cases, based on π.

For example, suppose π is `Read(a)`. Then the step of `WBCache` may change the cache and memory by writing a value back to memory. However, these changes don't change the corresponding abstract memory. Therefore, the memory correspondence given by `AF` holds after the step. It remains to show that both `Read`s give the same result. This follows because:

The `Read(a)` in `WBCache` returns the value *t*.`c(a)` if it is defined, otherwise *t*.`m(a)`.

The `Read(a)` in `Memory` returns the value of `AF`(*t*).`m(a)`.

The value of `AF`(*t*).`m(a)` is equal to *t*.`c(a)` if it is defined, otherwise *t*.`m(a)`. This is by the definition of `AF`.

For another example, suppose π is `Write(a,d)`. Then the step of `WBCache` writes value `d` to location `a` in the cache. It may also write some other value back to memory. Since writing a value back does not change the corresponding abstract state, the only change to the abstract state

is that the value in location `a` is changed to `d`. On the other hand, the effect of `Write(a,d)` in `Memory` is to change the value in location `a` to `d`. It follows that the memory correspondence, given by `AF`, holds after the step.

We leave the other cases, for the other types of operations, to the reader. It follows that `AF` is an abstraction function from `WBCache` to `Memory`. Then Theorem 1 implies that `WBCache` implements `Memory`, in the sense of trace set inclusion.

Example: Here is a similar analysis for `MajorityRegister`, using `MajorityRegister.AF` as the abstraction function.

```
FUNC AF() -> D = RET m.rng.fmax((\ p1, p2 | p1.seqno <= p2.seqno)).d
```

This time we depend on the invariant `MajorityRegister.Inv`. Suppose π is `Read(a)`. No state changes occur in either module, so the only thing to show is that the return values are the same in both modules. In `MajorityRegister`, the `Read` collects a majority of values and returns a value associated with the highest `seqno` from among that majority. By the invariant that says that the highest `seqno` appears in a majority of the copies, it must be that the `Read` in fact obtains the highest `seqno` that is present in the system. That is, the `Read` in `MajorityRegister` returns a value associated with the highest `seqno` that appears in state *t*.

On the other hand, the `Read` in `Register` just returns the value of the single variable `x` in state *s*. Since `AF`(*t*) = *s*, it must be that *s*.`x` is a value associated with the highest `seqno` in *t*. But the uniqueness invariant says that there is only one such value, so this is the same as the value returned by the `Read` in `MajorityRegister`.

Now suppose π is `Write(d)`. Then the key thing to show is that `AF`(*t*') = *s*'. The majority invariant implies that the `Write` in `MajorityRegister` sees the highest `seqno` `i` and thus `i+1` is the new highest `seqno`. It writes `(i+1, d)` to a majority of the copies. On the other hand, the `Write` in `Register` just sets `x` to `d`. But clearly `d` is a value associated with the largest `seqno` after the step, so `AF`(*t*') = *s*' as required.

It follows that `AF` is an abstraction function from `MajorityRegister` to `Register`. Then Theorem 1 implies that `MajorityRegister` implements `Register`.

7. Disks and File Systems

Motivation

The two lectures on disks and file systems are intended to show you a number of things:

Some semi-realistic examples of specs.

Many important implementation techniques for file systems.

Some of the tradeoffs between a simple spec and an efficient implementation.

Examples of abstraction functions and invariants.

Encoding: a general technique for representing arbitrary types as byte sequences.

How to model crashes.

Transactions: a general technique for making big actions atomic.

There are a lot of ideas here. After you have read this handout and listened to the lectures, it's a good idea to go back and reread the handout with this list of themes in mind.

Outline of topics

We give the specifications of disks and files in the `Disk` and `File` modules, and we discuss a variety of implementation issues:

Crashes

Disks

Files

Caching and buffering of disks and files

Representing files by trees and extents

Allocation

Encoding and decoding

Directories

Transactions

Redundancy

Crashes

The specs and implementations here are without concurrency. However, they do allow for crashes. A crash can happen between any two atomic commands. Thus the possibility of crashes introduces a limited kind of concurrency.

When a crash happens, the volatile global state is reset, but the stable state is normally unaffected. We express precisely what happens to the global state as well as how the module recovers by including a `Crash` procedure in the module. When a crash happens:

1. The `Crash` procedure is invoked. It need not be atomic.
2. If the `Crash` procedure does a `CRASH` command, the execution of the current invocations (if any) stop, and their local state is discarded; the same thing happens to any invocations outside the module from within it. After `CRASH`, no procedure in the module can be invoked from outside until `Crash` returns.
3. The `Crash` procedure may do other actions, and eventually it returns.
4. Normal operation of the module resumes; that is, external invocations are now possible.

You can tell which parts of the state are volatile by looking at what `Crash` does; it will reset the volatile variables.

Because crashes are possible between any two atomic commands, atomicity is important for any operation that involves a change to stable state.

The meaning of a Spec program with this limited kind of concurrency is that each atomic command corresponds to a transition. A hidden piece of state called the program counter keeps track of what transitions are enabled next: they are the atomic commands right after the program counter. There may be several if the command after the program counter has `[]` as its operator. In addition, a crash transition is always possible; it resets the program counter to a null value from which no transition is possible until some external routine is invoked and then invokes the `Crash` routine.

If there are non-atomic procedures in the spec with many atomic commands, it can be rather difficult to see the consequences of a crash. It is therefore clearer to write a spec with as much atomicity as possible, making it explicit exactly what unusual transitions are possible when there's a crash. We don't always follow this style, but we give some examples of it, notably at the end of the section on disks.

Disks

Essential properties of a disk:

Storage is stable across crashes (we discuss error models for disks in the `Disk` spec).
It's organized in blocks, and the only atomic update is to write one block.
Random access is about 100k times slower than random access to RAM (10 ms vs. 100 ns)
Sequential access is 10-50 times slower than sequential access to RAM (20 MB/s vs. 200-1000 MB/s)
Costs 50 times less than RAM ($.01 MB vs. $1/MB from the back of a PC magazine) in January 2000.
MTBF 1 million hours = 100 years.

Performance numbers:

Blocks of .5k - 4k bytes
20 MB/sec sequential, sustained (more with parallel disks)
3 ms average rotational delay (10000 rpm = 6 ms rotation time)
7 ms average seek time; 3 ms minimum

It takes 10 ms to get anything at all from a random place on the disk. In another 10 ms you can transfer 200 KB. Hence the cost to get 200 KB is only twice the cost to get 1 byte. By reading from several disks in parallel (called *striping* or *RAID*) you can easily increase the transfer rate by a factor of 5-10.

Performance techniques:

Avoid disk operations: use caching
Do sequential operations: allocate contiguously, prefetch, write to log
Write in background (write-behind)

A spec for disks

The following module describes a disk `Dsk` as a function from a `DA` to a disk block `DB`, which is just a sequence of `DBSize` bytes. The `Dsk` function can also yield `nil`, which represents a permanent read error. The module is a class, so you can instantiate as many `Disk`s as needed. The state is one `Dsk` for each `Disk`. There is a `New` method for making a new disk; think of this as ordering a new disk drive and plugging it in. An extent `E` represents a set of consecutive disk addresses. The main routines are the `read` and `write` methods of `Disk`: `read`, which reads an extent, and `write`, which writes `n` disk blocks worth of data sequentially to the extent `E{da, n}`. The write is not atomic, but can be interrupted by a failure after each single block is written.

Usually a spec like this is written with a concurrent thread that introduces permanent errors in the recorded data. Since we haven't discussed concurrency yet, in this spec we introduce the errors in `reads`, using the `AddErrors` procedure. An error sets a block to `nil`, after which any read that includes that block raises the exception `error`. Strictly speaking this is illegal, since `read` is a function and therefore can't call the procedure `AddErrors`. When we learn about concurrency we can move `AddErrors` to a separate thread; in the meantime we take the liberty, since it would be a real nuisance for `read` to be a procedure rather than a function.

Since neither Spec nor our underlying model deals with probabilities, we don't have any way to say how likely an error is. We duck this problem by making `AddErrors` completely non-deterministic; it can do anything from introducing no errors (which we must hope is the usual case) to clobbering the entire disk. Characterizing errors would be quite tricky, since disks usually have at least two classes of error: failures of single blocks and failures of an entire disk. However, any user of this module must assume something about the probability and distribution of errors.

Transient errors are less interesting because they can be masked by retries. We don't model them, and we also don't model errors reported by `writes`. Finally, a realistic error model would include the possibility that a block that reports a read error might later be readable after all.


```
CLASS Disk EXPORT Byte, Data, DA, E, DBSize, read, write, size, check, Crash  =

TYPE Byte       =  IN 0 .. 255
     Data       =  SEQ Byte
     DA         =  Nat                                  % Disk block Address
     DB         =  SEQ Byte                             % Disk Block
                   SUCHTHAT (\db| db.size = DBSize)
     Blocks     =  SEQ DB
     E          =  [da, size: Nat]                      % Extent, in disk blocks
                   WITH {das:=EToDAs, "IN":=(\ e, da | da IN e.das)}
     Dsk        =  DA -> (DB + Null)                    % a DB or nil (error) for each DA

CONST DBSize    := 1024

VAR  disk       :  Dsk

APROC new(size: Int) -> Disk = <<                       % overrides StdNew
     VAR dsk | dsk.dom = size.seq.set =>                % size blocks, arbitrary contents
          self := StdNew(); disk := dsk; RET self >>

FUNC read(e) -> Data RAISES {notThere, error} =
     check(e); AddErrors();
     VAR dbs := e.das * disk |                          % contents of the blocks in e
          IF nil IN dbs => RAISE error [*] RET BToD(dbs) FI

PROC write(da, data) RAISES {notThere} =                % fails if data not a multiple of DBsize
     VAR blocks := DToB(data), i := 0 |
     % Atomic by block, and in order
          check(E{da, blocks.size});
          DO blocks!i => WriteBlock(da + i, blocks(i)); i + := 1 OD

APROC WriteBlock(da, db) = << disk(da) := db >>         % the atomic update. PRE: disk!da

FUNC size() -> Int = RET disk.dom.size

APROC check(e) RAISES {notThere} =                      % every DA in e is in disk.dom
     << e.das.set <= disk.dom => RET [*] RAISE notThere >>

PROC Crash() = CRASH                                    % no global volatile state

FUNC EToDAs(e) -> SEQ DA = RET e.da .. e.da+e.size-1    % e.das

% Internal routines

% Functions to convert between Data and Blocks.
FUNC BToD(blocks) -> Data   = RET + : blocks
FUNC DToB(data  ) -> Blocks = VAR blocks | BToD(blocks) = data => RET blocks
% Undefined if data.size is not a multiple of DBsize

APROC AddErrors() =                                     % clobber some blocks
     << DO RET [] VAR da :IN disk.dom | disk(da) := nil OD >>

END Disk
```

This module doesn't worry about the possibility that a disk may fail in such a way that the client can't tell whether a write is still in progress; this is a significant problem in fault tolerant systems that want to allow a backup processor to start running a disk as soon as possible after the primary fails.

Many disks do not guarantee the order in which blocks are written (why?) and thus do not implement this spec, but instead one with a weaker `write`:

```
PROC writeUnordered(da, data) RAISES {notThere} =
  VAR blocks := DToB(data) |
    % Atomic by block, in arbitrary order; assumes no concurrent writing.
    check(E{da, blocks.size});
    DO VAR i | blocks(i) # disk(da + i) => WriteBlock(da + i, blocks(i)) OD
```

In both specs `write` establishes `blocks = E{da, blocks.size}.das * disk`, which is the same as `data = read(E{da, blocks.size})`, and both change each disk block atomically. `writeUnordered` says nothing about the order of changes to the disk, so after a crash any subset of the blocks being written might be changed; `write` guarantees that the blocks changed are a prefix of all the blocks being written. (`writeUnordered` would have other differences from `write` if concurrent access to the disk were possible, but we have ruled that out for the moment.)

Clarifying crashes

In this spec, what happens when there's a crash is expressed by the fact that `write` is not atomic and changes the disk one block at a time in the atomic `WriteBlock`. We can make this more explicit by making the occurrence of a crash visible inside the spec in the value of the `crashed` variable. To do this, we modify `Crash` so that it temporarily makes `crashed` true, to give `write` a chance to see it. Then `write` can be atomic; it writes all the blocks unless `crashed` is true, in which case it writes some prefix; this will happen only if `write` is invoked between the `crashed := true` and the `CRASH` commands of `Crash`. To describe the changes to the disk neatly, we introduce an internal function `NewDisk` that maps a `dsk` value into another one in which disk blocks at `da` are replaced by corresponding blocks defined in `bs`.

Again, this wouldn't be right if there were concurrent accesses to `Disk`, since we have made all the changes atomically, but it gives the possible behavior if the only concurrency is in crashes.

```
VAR  crashed     : Bool := false

...

APROC write(da, data) RAISES {notThere} = <<          % fails if data not a multiple of DBsize
    VAR blocks := DToB(data) |
        check(E{da, blocks.size});
        IF   crashed =>                               % if crashed, write some prefix
            VAR i | i < blocks.size => blocks := blocks.sub(0, i)
        [*]  SKIP FI;
        disk := NewDisk(disk, da, blocks)
    >>

FUNC NewDisk(dsk, da, bs: (Int -> DB)) -> Dsk =       % result is dsk overwritten with bs at da
    RET dsk + (\ da' | da' - da) * bs
```



```
PROC Crash() = crashed := true; CRASH; crashed := false
```

For unordered writes we need only a slight change, to write an arbitrary subset of the blocks if there's a crash, rather than a prefix:

```
        IF   crashed =>                               % if crashed, write some subset
            VAR w: SET I | w <= blocks.dom => blocks := blocks.restrict(w)
```

Files

This section gives a variety of specifications for files. Implementations follow in later sections.

We treat a file as just a sequence of bytes. Files have names, and for now we confine ourselves to a single directory that maps names to files. We call the name a 'path name' `PN` with an eye toward later introducing multiple directories, but for now we just treat the path name as a string without any structure. We package the operations on files as methods of `PN`. The main methods are `read` and `write`; we define the latter initially as `WriteAtomic`, and later introduce less atomic variations `Write` and `WriteUnordered`. There are also boring operations that deal with the size and with file names.

```
MODULE File EXPORT PN, Byte, Data, X, F, Crash =

TYPE PN          =  String                            % Path Name
                    WITH {read:=Read, write:=WriteAtomic, size:=GetSize,
                          setSize:=SetSize, create:=Create, remove:=Remove,
                          rename:=Rename}
     I           =  Int
     Byte        =  IN 0 .. 255
     Data        =  SEQ Byte
     X           =  Nat                               % byte-in-file indeX
     F           =  Data                              % File

     Dir         =  PN -> F                           % Directory

VAR  dir         := Dir{}                             % undefined everywhere
```

Note that the only state of the spec is `dir`, since files are only reachable through `dir`.

There are tiresome complications in `Write` caused by the fact that the arguments may extend beyond the end of the file. These can be handled by imposing preconditions (that is, writing the spec to do `HAVOC` when the precondition isn't satisfied), by raising exceptions, or by defining some sensible behavior. This spec takes the third approach; `NewFile` computes the desired contents of the file after the write. So that it will work for unordered writes as well, it handles sparse `data` by choosing an arbitrary `data'` that agrees with `data` where `data` is defined. Compare it with `Disk.NewDisk`.

```
FUNC Read(pn, x, i) -> Data = RET dir(pn).seg(x, i)
% Returns as much data as available, up to i bytes, starting at x.

APROC WriteAtomic(pn, x, data) = << dir(pn) := NewFile(dir(pn), x, data) >>
```

```
FUNC NewFile(f0, x, data: Int -> Byte) -> F =
% f is the desired final file. Fill in space between f0 and x with zeros, and undefined data elements arbitrarily.
  VAR z := data.dom.max, z0 := f0.size , f, data' |
       data'.size = z /\ data'.restrict(data.dom) = data
    /\ f.size = {z0, x+z}.max
    /\ (ALL i |    ( i IN 0   .. {x, z0}.min-1 ==> f(i) = f0(i)      )
              /\  ( i IN z0  .. x-1           ==> f(i) = 0          )
              /\  ( i IN x   .. x+z-1         ==> f(i) = data'(i-x) )
              /\  ( i IN x+z .. z0-1          ==> f(i) = f0(i) )    )
    => RET f

FUNC GetSize(pn) -> X = RET dir(pn).size

APROC SetSize(pn, x) = << VAR z := pn.size |
    IF   x <= z => << dir(pn) := pn.read(0, z) >>      % truncate
    [*]  pn.write(z, F.fill(0, x - z + 1))             % handles crashes like write
    FI >>

APROC Create(pn) = << dir(pn) := F{} >>
APROC Remove(pn) = << dir := dir{pn -> } >>
APROC Rename(pn1, pn2) = << dir(pn2) := dir(pn1); Remove(pn1) >>

PROC Crash() = SKIP                                    % no volatile state or non-atomic changes

END File
```

`WriteAtomic` changes the entire file contents at once, so that a crash can never leave the file in an intermediate state. This would be quite expensive in most implementations. For instance, consider what is involved in making a write of 20 megabytes to an existing file atomic; certainly you can't overwrite the existing disk blocks one by one. For this reason, real file systems don't implement `WriteAtomic`. Instead, they change the file contents a little at a time, reflecting the fact that the underlying disk writes blocks one at a time. Later we will see how an atomic `Write` could be implemented in spite of the fact that it takes several atomic disk writes. In the meantime, here is a more realistic spec for `Write` that writes the new bytes in order. It is just like `Disk.write` except for the added complication of extending the file when necessary, which is taken care of in `NewFile`.

```
APROC Write(pn, x, data) = <<
    IF   crashed =>                                    % if crashed, write some prefix
         VAR i | i < data.size => data := data.sub(0, i)
    [*] SKIP FI;
    dir(pn) := NewFile(dir(pn), x, data) >>

PROC Crash() = crashed := true; CRASH; crashed := false
```

This spec reflects the fact that only a single disk block can be written atomically, so there is no guarantee that all of the data makes it to the file before a crash. At the file level it isn't appropriate to deal in disk blocks, so the spec promises only bytewise atomicity. An actually implementation would probably make changes one page at a time, so it would not exhibit all the behavior allowed by the spec. There's nothing wrong with this, as long as the spec is restrictive enough to satisfy its clients.

`Write` does promise, however, that `f(i)` is changed no later than `f(i+1)`. Some file systems make no ordering guarantee; for them the following `WriteUnordered` is appropriate; it is just like `Disk.writeUnordered`.

```
APROC WriteUnordered(pn, x, data) = <<
    IF   crashed =>                                    % if crashed, write some subset
         VAR w: SET I | w <= data.dom => data := data.restrict(w)
    [*] SKIP FI;
    dir(pn) := NewFile(dir(pn), x, data) >>
```

Notice that although writing a file is not atomic, `File`'s directory operations are atomic. This corresponds to the semantics that file systems usually attempt to provide: if there is a failure during a `Create`, `Remove`, or `Rename`, the operation is either completed or not done at all, but if there is a failure during a `Write`, any amount of the data may be written. The other reason for making this choice in the spec is simple: with the abstractions available there's no way to express any sensible intermediate state of a directory operation other than `Rename` (of course a sloppy implementation might leave the directory scrambled, but that has to count as a bug; think what it would look like in the spec).

The spec we gave for `SetSize` made it as atomic as `write`. The following spec for `SetSize` is unconditionally atomic; this might be appropriate because an atomic `SetSize` is easier to implement than a general atomic `Write`:

```
APROC SetSize(pn, x) = << dir(pn) := (dir(pn) + F.fill(0, x)).seg(0, x) >>
```

Here is another version of `NewFile`, written in a more operational style just for comparison. It is a bit shorter, but less explicit about the relation between the initial and final states.

```
FUNC NewFile(f0, x, data: Int -> Byte) -> F = VAR z0 := f0.size, data' |
    data'.size = data.dom.max =>
        data' := data' + data;
        RET   (x > z0 => f0 + F.fill(0, x - z0) [*] f0.sub(0, x - 1))
            + data'
            + f0.sub(f.size, z0-1)
```

Our `File` spec is missing some things that are important in real file systems:

Access control: permissions or access control lists on files, ways of defaulting these when a file is created and of changing them, an identity for the requester that can be checked against the permissions, and a way to establish group identities.

Multiple directories. We will discuss this when we talk about naming.

Quotas, and what to do when the disk fills up.

Multiple volumes or file systems.

Backup. We will discuss this near the end of this handout when we describe the copying file system.

Cached and buffered disks

The simplest way to decouple the file system client from the slow disk is to provide a cached and write buffered implementation of the `Disk` abstraction; then the file system implementation need not change. The basic ideas are very similar to the ideas for cached memory, although for the disk we preserve the order of writes. We didn't do this for the memory because we didn't worry about failures.

Failures add complications; in particular, the spec must change, since buffering writes means that some writes may be lost if there is a crash. Furthermore, the client needs a way to ensure that its writes are actually stable. We therefore need a new spec `BDisk`. To get it, we add to `Disk` a variable `oldDisks` that remembers the previous states that the disk might revert to after a crash (note that this is not necessarily all the previous states) and code to use `oldDisks` appropriately. `BDisk.write` no longer needs to test `crashed`, since it's now possible to lose writes even if the crash happens after the `write`.

```
CLASS BDisk EXPORT ..., sync  =                          % write-buffered disk

TYPE ...
CONST ...
VAR  disk        :  Dsk                                  % as in Disk
     oldDisks    :  SET Dsk := {}

...

APROC write(da, data) RAISES {notThere} = <<             % fails if data not a multiple of DBsize
  << VAR blocks := DToB(data) |
        check(E{da, blocks.size});
        disk := NewDisk(disk, da, blocks);
        oldDisks \/ := {i | i < blocks.size |
                        NewDisk(disk, da, blocks.sub(0, i))};
        Forget()
     >>

FUNC NewDisk(dsk, da, bs: (Int -> DB)) -> Dsk =          % result is dsk overwritten with bs at da
     RET dsk + (\ da' | da' – da) * bs

PROC sync() = oldDisks := {}                             % make disk stable

PROC Forget() = VAR ds: SET Dsk | oldDisks - := ds
% Discards an arbitrary subset of the remembered disk states.

PROC Crash() = CRASH; << VAR d :IN oldDisks | disk := d; sync() [*] SKIP >>

END BDisk
```

`Forget` is there so that we can write an abstraction function for an implementation that doesn't defer all its disk writes until they are forced by `Sync`. A write that actually changes the disk needs to change `oldDisks`, because `oldDisks` contains the old state of the disk block being overwritten, and there is nothing in the state of the implementation after the write from which to compute that old state. Later we will study a better way to handle this problem: history variables or multi-valued mappings. They complicate the implementation rather than the spec, which is preferable. Furthermore, they do not affect the performance of the implementation at all.

A weaker spec would revert to a state in which any subset of the writes has been done. For this, change the assignment to `oldDisks` in `write`, along the lines we have seen before.

```
oldDisks \/ := {w: SET I | w <= blocks.dom |
                NewDisk(disk, da, blocks.restrict(w))};
```

The module `BufferedDisk` below is an implementation of `BDisk`. It copies newly written data into the cache and does the writes later, preserving the original order so that the state of the disk after a crash will always be the state at some time in the past. In the absence of crashes this implements `Disk` and is completely deterministic. We keep track of the order of writes with a `queue` variable, instead of keeping a `dirty` bit for each cache entry as we did for cached memory. If we didn't do the writes in order, there would be many more possible states after a crash, and it would be much more difficult for a client to use this module. Many real disks have this unpleasant property, and many real systems deal with it by ignoring it.

A striking feature of this implementation is that it uses the same abstraction that it implements, namely `BDisk`. The implementation of `BDisk` that it uses we call `UDisk` (`U` for 'underlying'). We think of it as a 'physical' disk, and of course it is quite different from `BufferedDisk`: it contains SCSI controllers, magnetic heads, etc. A module that implements the same interface that it uses is sometimes called a *filter* or a *stackable module*. A Unix filter is a familiar example that uses and implements the byte stream interface. We will see many other examples of this in the course.

Invocations of `UDisk` are in bold type, so you can easily see how the module depends on the lower-level implementation of `BDisk`.

```
CLASS BufferedDisk                                       % implements BDisk
     EXPORT Byte, Data, DA, E, DBSize, read, write, size, check, sync, Crash =

TYPE % Data, DA, DB, Blocks, E as in Disk
     I           =  Int
     J           =  Int

     Queue       =  SEQ DA                               % data is in cache

CONST
     cacheSize   := 1000
     queueSize   := 50

VAR  udisk       :  Disk
     cache       :  DA -> DB := {}
     queue       := Queue{}

% ABSTRACTION FUNCTION bdisk.disk = udisk.disk + cache
% ABSTRACTION FUNCTION bdisk.oldDisks =
     { q: Queue | q <= queue | udisk.disk + cache.restrict(q.set) }

% INVARIANT queue.set <= cache.dom                       % if queued then cached
% INVARIANT queue.size = queue.set.size                  % no duplicates in queue
% INVARIANT cache.dom.size <= cacheSize                  % cache not too big
% INVARIANT queue.size <= queueSize                      % queue not too big

APROC new(size: Int) -> BDisk = <<                       % overrides StdNew
     self := StdNew(); udisk := udisk.new(size); RET self >>
```

```
PROC read(e) -> Data RAISES {notThere} =
% We could make provision for read-ahead, but do not.
     check(e);
     VAR data := Data{}, da := e.da, upTo := e.da + e.size |
          DO da < upTo =>
               IF  cache!da => data + := cache(da); da + := 1
               [*] %  read as many blocks from disk as possible
                    VAR i := RunNotInCache(da, upTo),
                        buffer := udisk.read(E{da, i}),
                        k := MakeCacheSpace(i) |
                         % k blocks will fit in cache; add them.
                         DO VAR j :IN k.seq | ~ cache!(da + j) =>
                                cache(da + j) := udisk.DToB(buffer)(j)
                         OD;
                         data + := buffer; da + := i
               FI
          OD; RET data

PROC write(da, data) RAISES {notThere} =
  VAR blocks := udisk.DToB(data) |
     check(E{da, blocks.size});
     DO VAR i :IN queue.dom | queue(i) IN da .. da+size-1 => FlushQueue(i) OD;
     % Do any previously buffered writes to these addresses. Why?
     VAR j := MakeCacheSpace(blocks.size), i := 0 |
          IF  j < blocks.size => udisk.write(da, data)
               % Don't cache if the write is bigger than the cache.
          [*]  DO blocks!i =>
                    cache(da+i) := blocks(i); queue + := {da+i}; i + := 1
               OD
          FI

PROC Sync() = FlushQueue(queue.size - 1)

PROC Crash() = CRASH; cache := {}; queue := {}

FUNC RunNotInCache(da, upTo: DA) -> I =
     RET {i | da + i <= upTo /\ (ALL j :IN i.seq | ~ cache!(da + j)}.max

PROC MakeCacheSpace(i) -> Int =
% Make room for i new blocks in the cache; returning min(i, the number of blocks now available).
% May flush queue entries.
% POST: cache.dom.size + result <= cacheSize
     . . .

PROC FlushQueue(i) = VAR q := queue.sub(0, i) |
% Write queue entries 0 .. i and remove them from queue.
% Should try to combine writes into the biggest possible writes
     DO q # {} => udisk.write(q.head, 1); q := q.tail OD;
     queue := queue.sub(i + 1, queue.size - 1)

END BufferedDisk
```

This code keeps the cache as full as possible with the most recent data, except for gigantic writes. It would be easy to change it to make non-deterministic choices about which blocks to keep in the cache, or to take advice from the client about which blocks to keep. The latter would require changing the interface to accept the advice, of course.

Note that the only state of `BDisk` that this module can actually revert to after a crash is the one in which none of the queued writes has been done. You might wonder, therefore, why the body of the abstraction function for `BDisk.oldDisks` has to involve `queue`. Why can't it just be `{udisk.disk}`? The reason is that when the internal procedure `FlushQueue` does a write, it changes the state that a crash reverts to, and there's no provision in the `BDisk` spec for adding anything to `oldDisks` except during `write`. So `oldDisks` has to include all the states that the disk can reach after a sequence of 'internal' writes, that is, writes done in `FlushQueue`. And this is just what the abstraction function says.

Building other kinds of disks

There are other interesting and practical ways to implement a disk abstraction on top of a 'base' disk. Some examples that are used in practice:

Mirroring: use two base disks of the same size to implement a single disk of that size, but with much greater availability and twice the read bandwidth, by doing each write to both base disks.

Striping: use *n* base disks to implement a single disk *n* times as large and with *n* times the bandwidth, by reading and writing in parallel to all the base disks

RAID: use *n* base disks of the same size to implement a single disk *n*-1 times as large and with *n*-1 times the bandwidth, but with much greater availability, by using the *n*th disk to store the exclusive-or of the others. Then if one disk fails, you can reconstruct its contents from the others.

Snapshots: use 'copy-on-write' to implement an ordinary disk and some number of read-only 'snapshots' of its previous state.

Buffered files

We need to make changes to the `File` spec if we want the option to implement it using buffered disks without doing too many `sync`s. One possibility is do a `bdisk.sync` at the end of each `write`. This spec is not what most systems implement, however, because it's too slow. Instead, they implement a version of `File` with the following additions. This version allows the data to revert to any previous state since the last `Sync`. The additions are very much like those we made to `Disk` to get `BDisk`. For simplicity, we don't change `oldDirs` for operations other than `write` and `setSize` (well, except for truncation); real systems differ in how much they buffer the other operations.

```
MODULE File EXPORT ..., Sync =

TYPE ...
VAR  dir         := Dir{}
     oldDirs     :  SET Dir := {}
...

APROC Write(pn, x, byte) = << VAR f0 := dir(pn) |
     dir(pn) := NewFile(f0, x, data);
     oldDirs \/ := {i | i < data.size |
                   dir{pn -> NewFile(f0, x, data.sub(0, i))}} >>

APROC Sync() = << oldDirs := {} >>

PROC Crash() = CRASH; << VAR d :IN oldDirs => dir := d; Sync() [*] SKIP >>

END File
```

Henceforth we will use `File` to refer to the modified module. Since we are not giving an implementation, we leave out `Forget` for simplicity.

Many file systems do their own caching and buffering. They usually loosen this spec so that a crash resets each file to some previous state, but does not necessarily reset the entire system to a previous state. (Actually, of course, real file systems usually don't have a spec, and it is often very difficult to find out what they can actually do after a crash.)

```
MODULE File2 EXPORT ..., Sync =

TYPE ...
     OldFiles    =  PN -> SET F

VAR  dir         := Dir{}
     oldFiles    := OldFiles{* -> {}}

...

APROC Write(pn, x, byte) = << VAR f0 := dir(pn) |
     dir(pn) := NewFile(f0, x, data);
     oldFiles(pn) \/ := {i | i < data.size | NewFile(f0, x, data.sub(0, i))} >>

APROC Sync() = << oldFiles:= OldFiles{* -> {}} >>

PROC Crash() =
     CRASH;
     << VAR dir' |    dir'.dom = dir.dom
                  /\ (ALL pn :IN dir.dom | dir'(pn) IN oldFiles(pn) \/ {dir(pn)})
        => dir := dir' >>

END File
```

A picky point about Spec: A function constructor like `(\ pn | {dir(pn)})` is no good as a value for `oldFiles`, because the value of the global variable `dir` in that constructor is not captured when the constructor is evaluated. Instead, this function uses the value of `dir` when it is invoked. This is a little weird, but it is usually very convenient. Here it is a pain; we avoid the problem by using a *local* variable `d` whose value *is* captured when the constructor is evaluated in `SnapshotDir`.

A still weaker spec allows `dir` to revert to a state in which any subset of the byte writes has been done, except that the files still have to be sequences. By analogy with unordered `BDisk`, we change the assignment to `oldFiles` in `Write`.

```
     oldFiles(pn) \/ := {w: SET i | w <= data.dom |
                         NewFile(f0, x, data.restrict(w))} >>
```

Implementing files

The main issue is how to represent the bytes of the file on the disk so that large reads and writes will be fast, and so that the file will still be there after a crash. The former requires using contiguous disk blocks to represent the file as much as possible. The latter requires a representation for `Dir` that can be changed atomically. In other words, the file system state has type `PN -> SEQ Byte`, and we have to find a disk representation for the `SEQ Byte` that is efficient, and one for the function that is robust. This section addresses the first problem.

The simplest approach is to represent a file by a sequence of disk blocks, and to keep an *index* that is a sequence of the `DA`'s of these blocks. Just doing this naively, we have

```
TYPE F           =  [das: SEQ DA, size: N]          % Contents and size in bytes
```

The abstraction function to the spec says that the file is the first `f.size` bytes in the disk blocks pointed to by `c`. Writing this as though both `File` and its implementation `FImpl0` had the file `f` as the state, we get

```
File.f = (+ : (FImpl0.f.das * disk.disk)).seg(0, FImpl0.f.size)
```

or, using the `disk.read` method rather than the state of `disk` directly

```
File.f = (+ : {da :IN FImpl0.f.das | | disk.read(E{da, 1})}).seg(0, FImpl0.f.size)
```

But actually the state of `File` is `dir`, so we should have the same state for `FImpl` (with the different representation for `F`, of course), and

```
File.dir = (LAMBDA (pn) -> File.F =
     VAR f := FImpl0.dir(pn) |                    % fails if dir is undefined at pn
          RET (+ : (f.das * disk.disk)).seg(0, f.size)
```

We need an invariant that says the blocks of each file have enough space for the data.

```
% INVARIANT ( ALL f :IN dir.rng | f.das.size * DBSize >= f.size )
```

Then it's easy to see how to implement `read`:

```
PROC read(pn, x, i) =
     VAR f := dir(pn),
         diskData := + : (da :IN f.das | | disk.read(E{da, 1})},
         fileData := diskData.seg(0, f.size) |
          RET fileData.seg(x, i)
```

To implement `write` we need a way to allocate free `DA`s; we defer this to the next section.

There are two problems with using this representation directly:

1. The index takes up quite a lot of space (with 4 byte DA's and DBSize = 1Kbyte it takes .4% of the disk). Since RAM costs about 50 times as much as disk, keeping it all in RAM will add about 20% to the cost of the disk, which is a significant dollar cost. On the other hand, if the index is not in RAM it will take two disk accesses to read from a random file address, which is significant performance cost.

2. The index is of variable length with no small upper bound, so representing the index on the disk is not trivial either.

To solve the first problem, store Disk.E's in the index rather than DA's. A single extent can represent lots of disk blocks, so the total size of the index can be much less. Following this idea, we would represent the file by a sequence of Disk.E's, stored in a single disk block if it isn't too big or in a file otherwise. This recursion obviously terminates. It has the drawback that random access to the file might become slow if there are many extents, because it's necessary to search them linearly to find the extent that contains byte x of the file.

To solve the second problem, use some kind of tree structure to represent the index. In standard Unix file systems, for example, the index is a structure called an *inode* that contains:

a sequence of 10 DA's (enough for a 10 KB file, which is well above the median file size), followed by

the DA of an *indirect* DB that holds DBSize/4 = 250 or so DA's (enough for a 250 KB file), followed by

the DA of a second-level indirect block that holds the DA's of 250 indirect blocks and hence points to 250^2 = 62500 DA's (enough for a 62 MB file),

and so forth. The third level can address an 16 GB file, which is enough for today's systems.

Thus the inode itself has room for 13 DA's. These systems duck the first problem; their extents are always a single disk block.

We give an implementation that incorporates both extents and trees, representing a file by a generalized extent that is a tree of extents. The leaves of the tree are *basic* extents Disk.E, that is, references to contiguous sequences of disk blocks, which are the units of i/o for disk.read and disk.write. The purpose of such a general extent is simply to define a sequence of disk addresses, and the E.das method computes this sequence so that we can use it in invariants and abstraction functions. The tree structure is there so that the sequence can be stored and modified more efficiently.

An extent that contains a sequence of basic extents is called a *linear* extent. To do fast i/o operations, we need a linear extent which includes just the blocks to be read or written, grouped into the largest possible basic extents so that disk.read and disk.write can work efficiently. Flatten computes such a linear extent from a general extent; the spec for Flatten given below flattens the entire extent for the file and then extracts the smallest segment that contains all the blocks that need to be touched.

Read and Write just call Flatten to get the relevant linear extent and then call disk.read and disk.write on the basic extents; Write may extend the file first, and it may have to read the first and last blocks of the linear extent if the data being written does not fill them, since the disk can only write entire blocks. Extending or truncating a file is more complex, because it requires changing the extent, and also because it requires allocation. Allocation is described in the next section. Changing the extent requires changing the tree.

The tree itself must be represented in disk blocks; methods inspired by B-trees can be used to change it while keeping it balanced. Our implementation shows how to extract information from the tree, but not how it is represented in disk blocks or how it is changed. In standard Unix file systems, changing the tree is fairly simple because a basic extent is always a single disk block in the multi-level indirect block scheme described above.

We give the abstraction function to the simple implementation above. It just says that the das of a file are the ones you get from Flatten.

The code below makes heavy use of function composition to apply some function to each element of a sequence: s * f is {f(s(0)), ..., f(s(s.size-1))}. If f yields an integer or a sequence, the combination + : (s * f) adds up or concatenates all the f(s(i)).

```
MODULE FSImpl =                                          % implements File

TYPE N          =  Nat
     E          =  [c: (Disk.DA + SE), size: N]          % size = # of DA's in e
                   SUCHTHAT (\e | Size(e) = e.size)
                   WITH {das:=EToDAs, le:=EToLE}
     BE         =  E SUCHTHAT (\e| e.c IS Disk.DA)       % Basic Extent
     LE         =  E SUCHTHAT (\e| e.c IS SEQ BE)        % Linear Extent: sequence of BEs
                   WITH {"+":=Cat}
     SE         =  SEQ E                                 % Sequence of Extents: may be tree

     X          =  File.X
     F          =  [e, size: X]                          % size = # of bytes

     PN         =  File.PN                               % Path Name

CONST DBSize    := 1024

VAR  dir        :  File.PN -> F := {}
     disk

% ABSTRACTION FUNCTION File.dir = (LAMBDA (pn) -> File.F = dir!pn =>
% The file is the first f.size bytes in the disk blocks of the extent f.e
     VAR f := dir(pn),
         d := + : {be :IN Flatten(f.e, 0, f.e.size).c | | disk.read(be)} |
          RET d.seg(0, f.size) )

% ABSTRACTION FUNCTION FImpl0.dir = (LAMBDA (pn) -> FImpl0.F =
     VAR f := dir(pn) | RET {be :IN Flatten(f.e, 0, f.e.size).c | | be.c}

FUNC Size(e) -> Int = RET ( e IS BE => e.size [*] + :(e.c * Size) )
% # of DA's reachable from e. Should be equal to e.size.
```

```
FUNC EToDAs(e) -> SEQ DA =                                  % e.das
% The sequence of DA's defined by e. Just for specs.
     RET ( e IS BE => {i :IN e.size.seq | | e.c + i} [*] + :(e.c * EToDAs) )

FUNC EToLE(e) -> LE =                                       % e.le
% The sequence of BE's defined by e.
     RET ( e IS BE => LE{SE{e}, e.size}                    [*] + :(e.c * EToLE ) )

FUNC Cat(le1, le2) -> LE =
% The "+" method of LE. Merge e1 and e2 if possible.
     IF   e1 = {} => RET le2
     []   e2 = {} => RET le1
     []   VAR e1 := le1.c.last, e2 := le2.c.head, se |
          IF   e1.c + e1.size = e2.c =>
               se := le1.c.reml + SE{E{e1.c, e1.size + e2.size}} + le2.c.tail
          [*]  se := le1.c + le2.c
          FI;
          RET LE{se, le1.size + le2.size}
     FI

FUNC Flatten(e, start: N, size: N) -> LE = VAR le0 := e.le, le1, le2, le3 |
% The result le is such that le.das = e.das.seg(start, size);
% This is fewer than size DA's if e gets used up.
% It's empty if start >= e.size.
% This is not a practical implementation; see below.
        le0 = le1 + le2 + le3
     /\ le1.size = {start, e.size}.min
     /\ le2.size = {size, {e.size - start, 0}.max}.min
     => RET le2

...

END FSImpl
```

This version of `Flatten` is not very practical; in fact, it is more like a spec than an implementation. A practical one, given below, searches the tree of extents sequentially, taking the largest possible jumps, until it finds the extent that contains the `start`th `DA`. Then it collects extents until it has gotten `size` `DA`'s. Note that because each `e.size` gives the total number of `DA`'s in `e`, `Flatten` only needs time `log(e.size)` to find the first extent it wants, provided the tree is balanced. This is a standard trick for doing efficient operations on trees: summarize the important properties of each subtree in its root node. A further refinement (which we omit) is to store cumulative sizes in an `SE` so that we can find the point we want with a binary search rather than the linear search in the `DO` loop below; we did this in the editor buffer example of handout 3.

```
FUNC Flatten(e, start: N, size: N) -> LE =
     VAR z := {size, {e.size - start, 0}.max}.min |
          IF  z = 0   => RET E{c := SE{},        size := 0}
          [*] e IS BE => RET E{c := e.c + start, size := z}.le
          [*] VAR se := e.c AS SE, sbe : SEQ BE := {}, at := start, want := z |
               DO want > 0 =>                    % maintain at + want <= Size(se)
                    VAR e1 := se.head, e2 := Flatten(e1, at, want) |
                         sbe := sbe + e2.c; want := want - e2.size;
                         se := se.tail; at := {at - e1.size, 0}.max
               OD;
               RET E{c := sbe, size := z}
```



```
          FI
```

Allocation

We add something to the state to keep track of which disk blocks are free:

```
VAR free: DA -> Bool
```

We want to ensure that a free block is not also part of a file. In fact, to keep from losing blocks, a block should be free iff it isn't in a file or some other data structure such as an inode:

```
PROC IsReachable(da) -> Bool =
     RET ( EXISTS f :IN dir.rng | da IN f.e.das \/ ...

% INVARIANT (ALL da | IsReachable(da) = ~ free(da) )
```

This can't be implemented without some sort of log-like mechanism for atomicity if we want separate representations for `free` and `f.e`, that is, if we want any implementation for `free` other than the brute-force search implied by `IsReachable` itself. The reason is that the only atomic operation we have on the disk is to write a single block, and we can't hope to update the representations of both `free` and `f.e` with a single block write. But `~ IsReachable` is not a satisfactory implementation for `free`, even though it does not require a separate data structure, because it's too expensive — it traces the entire extent structure to find out whether a block is free.

A weaker invariant allows blocks to be lost, but still ensures that the file data will be inviolate. This isn't as bad as it sounds, because blocks will only be lost if there is a crash between writing the allocation state and writing the extent. Also, it's possible to garbage-collect the lost blocks.

```
% INVARIANT (ALL da | IsReachable(da) ==> ~ free(da))
```

A weaker invariant than this would be a disaster, since it would allow blocks that are part of a file to be free and therefore to be allocated for another file.

The usual representation of `free` is a `SEQ Bool` (often called a *bit table*). It can be stored in a fixed-size file that is allocated by magic (so that the implementation of allocation doesn't depend on itself). To reduce the size of `free`, the physical disk blocks may be grouped into larger units (usually called 'clusters') that are allocated and deallocated together.

This is a fairly good scheme. The only problem with it is that the table size grows linearly with the size of the disk, even when there are only a few large files, and concomitantly many bits may have to be touched to allocate a single extent. This will certainly be true if the extent is large, and may be true anyway if lots of allocated blocks must be skipped to find a free one.

The alternative is a tree of free extents, usually implemented as a B-tree with the extent size as the key, so that we can find an extent that exactly fits if there is one. Another possibility is to use the extent address as the key, since we also care about getting an extent close to some existing one. These goals are in conflict. Also, updating the B-tree atomically is complicated. There is no best answer.

Encoding and decoding

To store complicated values on the disk, such as the function that constitutes a directory, we need to encode them into a byte sequence, since `Disk.Data` is `SEQ Byte`. (We also need encoding to send values in messages, an important operation later in the course.) It's convenient to do this with a pair of functions for each type, called `Encode` and `Decode`, which turn a value of the type into a byte sequence and recover the value from the sequence. We package them up into an `EncDec` pair.

```
TYPE D          =  SEQ Byte
     EncDec     =  [enc: Any -> D, dec: D -> Any]     % Encode/Decode pair
                   SUCHTHAT (\ed: EncDec | ( EXISTS T: SET Any |
                                ed.enc.dom = T
                              /\ (ALL t :IN T | dec(enc(t)) = t) ))
```

Other names for 'encode' are 'serialize', 'pickle', and 'marshal'.

A particular `EncDec` works only on values of a single type (represented by the set `T` in the `SUCHTHAT`, since you can't quantify over types in Spec). This means that `enc` is defined exactly on values of that type, and `dec` is the inverse of `enc` so that the process of encoding and then decoding does not lose information. We do *not* assume that `enc` is the inverse of `dec`, since there may be many byte sequences that decode to the same value; for example, if the value is a set, it would be pointless and perhaps costly to insist on a canonical ordering of the encoding. In this course we will generally assume that every type has methods `enc` and `dec` that form an `EncDec` pair.

A type that has other types as its components can have its `EncDec` defined in an obvious way in terms of the `EncDec`'s of the component types. For example, a `SEQ T` can be encoded as a sequence of encoded `T`'s, provided the decoding is unambiguous. A function `T -> U` can be encoded as a set or sequence of encoded `(T, U)` pairs.

A directory is one example of a situation in which we need to encode a sequence of values into a sequence of bytes. A log is another example of this, discussed below, and a stream of messages is a third. It's necessary to be able to parse the encoded byte sequence unambiguously and recover the original values. We can express this idea precisely by saying that a parse is an `EncDec` sequence, a language is a set of parses, and the language is unambiguous if for every byte sequence `d` the language has at most one parse that can completely decode `d`.

```
TYPE M          =  SEQ D                              % for segmenting a D
     P          =  SEQ EncDec                         % Parse
     % A sequence of decoders that parses a D, as defined by IsParse below
     Language   =  SET P

FUNC IsParse(p, d) -> Bool = RET ( EXISTS m |
        + :m = d                                      % m segments d
     /\ m.size = p.size                               % m is the right size
     /\ (ALL i :IN p.dom | (p(i).dec)!m(i)] )         % each p decodes its m

FUNC IsUnambiguous(l: Language) -> Bool = RET (ALL d, p1, p2|
     p1 IN l /\ p2 IN l /\ IsParse(p1, d) /\ IsParse(p2, d) ==> p1 = p2)
```


Of course ambiguity is not decidable in general. The standard way to get an unambiguous language for encodings is to use type-length-value (TLV) encoding, in which the result `d` of `enc(x)` starts with some sort of encoding of `x`'s type, followed by an encoding of `d`'s own length, followed by a `D` that contains the rest of the information the decoder needs to recover `x`.

```
FUNC IsTLV(ed: EncDec) -> Bool =
     RET (ALL x :IN ed.enc.dom | (EXISTS d1, d2, d3 |
             ed.enc(x) = d1 + d2 + d3 /\ EncodeType(x) = d1
          /\ (ed.enc(x).size).enc = d2 ))
```

In many applications there is a grammar that determines each type unambiguously from the preceding values, and in this case the types can be omitted. For instance, if the sequence is the encoding of a `SEQ T`, then it's known that all the types are `T`. If the length is determined from the type it can be omitted too, but this is done less often, since keeping the length means that the decoder can reliably skip over parts of the encoded sequence that it doesn't understand. If desired, the encodings of different types can make different choices about what to omit.

There is an international standard called ASN-1 (for Abstract Syntax Notation) that defines a way of writing a grammar for a language and deriving the `EncDec` pairs automatically from the grammar. Like most such standards, it is rather complicated and often yields somewhat inefficient encodings, but it is fairly widely used.

Another standard way to get an unambiguous language is to encode into S-expressions, in which the encoding of each value is delimited by parentheses, and the type, unless it can be omitted, is given by the first symbol in the S-expression. A variation on this scheme which is popular for Internet Email and Web protocols, is to have a 'header' of the form

```
attribute1: value1
attribute2: value2
...
```

with various fairly ad-hoc rules for delimiting the values that are derived from early conventions for the human-readable headers of Email messages.

In both TLV and S-expression encodings, decoding depends on knowing exactly where the byte sequence starts. This is not a problem for `D`'s coming from a file system, but it is a serious problem for `D`'s coming from a wire or byte stream, since the wire produces a continuous stream of voltages, bits, bytes, or whatever. The process of delimiting a stream of symbols into `D`'s that can be decoded is called *framing*; we will discuss it later in connection with networks.

Directories

Recall that a `Dir` is just a `PN -> F`. We have seen various ways to represent `F`. The simplest implementation relies on an `EncDec` for an entire `Dir`. It represents a `Dir` as a file containing `enc` of the `PN -> F` map as a set of ordered pairs.

There are two problems with this scheme:

- Lookup in a large `Dir` will be slow, since it requires decoding the whole `Dir`. This can be fixed by using a hash table or B-tree. Updating the `Dir` can still be done as in the simple scheme, but this will also be slow. Incremental update is possible, if more complex; it also has atomicity issues.

- If we can't do an atomic file write, then when updating a directory we are in danger of scrambling it if there is a crash during the write. There are various ways to solve this problem. The most general and practical way is to use the transactions explained in the next section.

It is very common to implement directories with an extra level of indirection called an 'inode', so that we have

```
TYPE INo         =  Int                          % Inode Number
     Dir         =  PN -> INo
     INoMap      =  INo -> F

VAR  dir         :  Dir := {}
     inodes      :  INoMap := {}
```

You can see that `inodes` is just like a directory except that the names are `INo`'s instead of `PN`'s. There are three advantages:

Because `INo`'s are integers, they are cheaper to store and manipulate. It's customary to provide an `Open` operation to turn a `PN` into an `INo` (usually through yet another level of indirection called a 'file descriptor'), and then use the `INo` as the argument of `Read` and `Write`.

Because `INo`'s are integers, if `F` is fixed-size (as in the Unix example discussed earlier, for instance) then `inodes` can be represented as an array on the disk that is just indexed by the `INo`.

The enforced level of indirection means that file names automatically get the semantics of pointers or memory addresses: two of them can point to the same file variable.

The third advantage can be extended by extending the definition of `Dir` so that the value of a `PN` can be another `PN`, usually called a "symbolic link".

```
TYPE Dir         =  PN -> (INo + PN)
```

Transactions

We have seen several examples of a general problem: to give a spec for what happens after a crash that is acceptable to the client, and an implementation that satisfies the spec even though it has only small atomic actions at its disposal. In writing to a file, in maintaining allocation information, and in updating a directory, we wanted to make a possibly large state change atomic in the face of crashes during its execution, even though we can only write a single disk block atomically.

The general technique for dealing with this problem is called *transactions*. General transactions make large state changes atomic in the face of arbitrary concurrency as well as crashes; we will discuss this later. For now we confine ourselves to 'sequential transactions', which only take care of crashes. The idea is to conceal the effects of a crash entirely within the transaction abstraction, so that its clients can program in a crash-free world.

The implementation of sequential transactions is based on the very general idea of a *deterministic state machine* that has inputs called *actions* and makes a deterministic transition for every input it sees. The essential observation is that:

> If two instances of a deterministic state machine start in the same state and see the same inputs, they will make the same transitions and end up in the same state.

This means that if we record the sequence of inputs, we can replay it after a crash and get to the same state that we reached before the crash. Of course this only works if we start in the same state, or if the state machine has an 'idempotency' property that allows us to repeat the inputs. More on this below.

Here is the spec for sequential transactions. There's a state that is queried and updated (read and written) by actions. We keep a stable version `ss` and a volatile version `vs`. Updates act on the volatile version, which is reset to the stable version after a crash. A 'commit' action atomically sets the stable state to the current volatile state.

```
MODULE SeqTr [                                   % Sequential Transaction
    V,                                           % Value of an action
    S WITH { s0: ()-> S }                        % State; s0 initially
    ] EXPORT Do, Commit, Crash =

TYPE A           =  S->(V, S)                    % Action

VAR  ss          := S.s0()                       % Stable State
     vs          := S.s0()                       % Volatile State

APROC Do(a) -> V = << VAR v | (v, vs) := a(vs); RET v >>
APROC Commit() = << ss := vs >>
APROC Crash () = << vs := ss >>                  % Abort is the same

END SeqTr
```

In other words, you can do a whole series of actions to the volatile state `vs`, followed by a `Commit`. Think of the actions as reads and writes, or queries and updates. If there's a crash before the `Commit`, the state reverts to what it was initially. If there's a crash after the `Commit`, the state reverts to what it was at the time of the commit. An action is just a function from an initial state to a final state and a result value.

There are many implementation techniques for transactions. Here is the simplest. It breaks each action down into a sequence of *updates*, each one of which can be done atomically; the most common example of an atomic update is a write of a single disk block. The updates also must have an 'idempotency' property discussed later. Given a sequence of `Do`'s, each applying an action, the implementation concatenates the update sequences for the actions in a volatile *log* that is a representation of the actions. `Commit` writes this log atomically to a stable log. Once the stable log is written, `Redo` applies the volatile log to the stable state and erases both logs. `Crash` resets the volatile to the stable log and then applies the log to the stable state to recover the volatile state. It then uses `Redo` to update the stable state and erase the logs. Note that we give `S` a `"+"` method `s + l` that applies a log to a state.

This scheme reduces the problem of implementing arbitrary changes atomically to the problem of atomically writing an arbitrary amount of stuff to a log. This is easier but still not trivial to do efficiently; we discuss it at the end of the section.

```
MODULE LogRecovery [                                  % implements SeqTr
    V,                                                % Value of an action
    S0 WITH { s0: () -> S0 }                          % State
    ] EXPORT Do, Commit, Crash =

TYPE A          =  S->(V, S)                          % Action
    U           =  S -> S                             % atomic Update
    L           =  SEQ U                              % Log
    S           =  S0 WITH { "+":=DoLog }             % State; s+l applies l to s

VAR  ss         := S.s0()                             % Stable State
    vs          := S.s0()                             % Volatile State
    sl          := L{}                                % Stable Log
    vl          := L{}                                % Volatile Log

% ABSTRACTION to SeqTr
    SeqTr.ss = ss + sl
    SeqTr.vs = vs

% INVARIANT vs = ss + vl

FUNC DoLog(s, l) -> S =                               % s+l = DoLog(s, l)
% Apply the updates in l to the state s.
    l={} => RET s [*] RET DoLog((l.head)(s),l.tail))

APROC Do(a) -> V =
% Find an l (a sequence of updates) that has the same effect as a on the current state.
    << VAR v, l | (v, vs + l) = a(vs) =>
          vl := vl + l; vs := vs + l; RET v >>

PROC Commit() = << sl := vl >>; Redo()

PROC Redo() =                                         % replay vl, then clear sl
    DO vl # {} => << ss := ss + vl.head; vl := vl.tail >> OD; << sl := {} >>

PROC Crash() =
    CRASH;
    << vl := {}; vs := S.s0()  >>;                    % crash erases vs, vl
    << vl := sl; vs := ss + vl >>;                    % recovery restores them
    Redo()                                            % and repeats the Redo; this is optional

END LogRecovery
```

For this *redo* crash recovery to work, `l` must have the property that repeatedly applying prefixes of it, followed by the whole thing, has the same effect as applying the whole thing. For example, suppose `l = L{a,b,c,d,e}`. Then `L{a,b,c}, a, a, a,b,c,d, a,b, a,b,c,d,e, a, a,b,c,d,e}` must have the same effect as `l` itself; here we have grouped the prefixes together for clarity. We need this property because a crash can happen while `Redo` is running; the crash reapplies the whole log and runs `Redo` again. Another crash can happen while the second `Redo` is running, and so forth.

This 'hiccup' property follows from 'log idempotence':

```
    s + l + l = s + l                                                    (1)
```

From this we get (recall that < is the 'prefix' predicate for sequences).

```
    k < l ==> (s + k + l = s + l)                                        (2)
```

because `k < l` implies there is a `l'` such that `k + l' = l`, and hence

```
      s + k + l = s + k + (k + l') = (s + k + k) + l'
    = (s + k) + l' = s + (k + l') = s + l
```

From (2) we get the property we want:

```
    IsHiccups(k, l) ==> (s + k + l = s + l)                              (3)
```

where

```
FUNC IsHiccups(k, l) -> Bool =
% k is a sequence of attempts to complete l
    RET    k = {}
        \/ (EXISTS k', l'|    k = k' + l' /\ l' # {} /\ l' <= l
                           /\ IsHiccups(k', l) )
```

because we can keep absorbing the last hiccup `l'` into the final complete `l`. For example, taking some liberties with the notation for sequences:

```
      abcaaabcdabcdeaabcde
    = abcaaabcdababcde + (a + abcde)
    = abcaaabcdababcde + abcde                          by (2)
    = abcaaabcdab + (abcde + abcde)
    = abcaaabcdab + abcde                               by (2)
    = abcaaabcd + (ab + abcde)
    = abcaaabcd + abcde                                 by (2)
```

and so forth.

To prove (3), observe that

```
    IsHiccups(k, l) /\ k # {} ==> k = k' + l' /\ l' <= l /\ IsHiccups(k', l).
```

Hence

```
    s+k+l = (s+k')+l'+l = s+k'+l                        by (2)
```

and `k' < k`. But we have `IsHiccups(k', l)`, so we can proceed by induction until `k' = {}` and we have the desired result.

We can get log idempotence if the `U`'s commute and are idempotent (that is, `u * u = u`), or if they are all writes. More generally, for arbitrary `U`'s we can attach a `UID` to each `U` and record it in `S` when the `U` is applied, so we can tell that it shouldn't be applied again. Calling the original state `SS`, and defining a `meaning` method that turns a `U` record into a function, we have

```
TYPE
    S           =  [ss, tags: SET UID]
    U           =  [uu: SS->SS, tag: UID] WITH { meaning:=Meaning }

FUNC Meaning(u, s)->S =
        u.tag IN s.tags => RET s                        % u already done
    [*] RET S{ (u.uu)(s.ss), s.tags + {u.tag} }
```

If all the `U`'s in `l` have different tags, we get log idempotence. The tags make `U`'s 'testable' in the jargon of transaction processing; after a crash we can test to find out whether a `U` has been done or not. In the standard database implementation each `U` works on one disk page, the tag is the 'log

sequence number', the index of the update in the log, and the update writes the tag on the disk page.

Writing the log atomically

There is still an atomicity problem in this implementation: `Commit` atomically does `<< sl := vl >>`, and the logs can be large. A simple way to use a disk to implement a log that requires this assignment of arbitrary-sized sequences is to keep the size of `sl` in a separate disk block, and to write all the data first, then do a `Sync` if necessary, and finally write the new size. Since `sl` is always empty before this assignment, in this representation it will remain empty until the single `Disk.write` that sets its size. This is a rather wasteful implementation, since it does an extra disk write.

A more efficient implementation writes a 'commit record' at the end of the log, and treats the log as empty unless the commit record is present. Now it's only necessary to ensure that the log can never be mis-parsed if a crash happens while it's being written. An easy way to accomplish this is to write a distinctive 'erased value into each disk block that may become part of the log, but this means that for every disk write to a log block, there will be another write to erase it. To avoid this cost we can use a ring buffer of disk blocks for the log and a sequence number that increments each time the ring buffer wraps around; then a block is 'erased' if its sequence number is not the current one. There's still a cost to initialize the sequence numbers, but it's only paid once. With careful implementation, a single bit of sequence number is enough.

In some applications it's inconvenient to make room in the data stream for a sequence number every `DBsize` bytes. To get around this, use a 'displaced' representation for the log, in which the first data bit of each block is removed from its normal position to make room for the one bit sequence number. The displaced bits are written into their own disk blocks at convenient intervals.

Another approach is to compute a strong checksum for the log contents, write it at the end after all the other blocks are known to be on the disk, and treat the log as empty unless a correct checksum is present. With a good *n*-bit checksum, the probability of mis-parsing is 2^{-n}.

Redundancy

A disk has many blocks. We would like some assurance that the failure of a single block will not damage a large part of the file system. To get such assurance we must record some critical parts of the representation redundantly, so that they can be recovered even after a failure.

The simplest way to get this effect is to record *everything* redundantly. This gives us more: a single failure won't damage *any* part of the file system. Unfortunately, it is expensive. In current systems this is usually done at the disk abstraction, and is called *mirroring* or *shadowing* the disk.

The alternative is to record redundantly only the information whose loss can damage more than one file: extent, allocation, and directory information.

Another approach is to

do all writes to a log,

keep a copy of the log for a long time (by writing it to tape, usually), and

checkpoint the state of the file system occasionally.

Then the current state can be recovered by restoring the checkpoint and replaying the log from the moment of the checkpoint. This method is usually used in large database systems, but not in any file systems that I know of.

We will discuss these methods in more detail near the end of the course.

Copying File Systems

The file system described in `FSImpl` above separates the process of adding `DB`'s to the representation of a file from the process of writing data into the file. A *copying* file system (CFS) combines these two processes into one. It is called a 'log-structured' file system in the literature[1], but as we shall see, the log is not the main idea. A CFS is based on three ideas:

- Use a generational copying garbage collector (called a *cleaner*) to reclaim `DB`'s that are no longer reachable and keep all the free space in a single (logically) contiguous region, so that there is no need for a bit table or free list to keep track of free space.
- Do *all* writes sequentially at one end of this region, so that existing data is never overwritten and new data is sequential.
- Log and cache updates to metadata (the index and directory) so that the metadata doesn't have to be rewritten too often.

A CFS is a very interesting example of the subtle interplay among the ideas of sequential writing, copying garbage collection, and logging. This section describes the essentials of a CFS in detail and discusses more briefly a number of refinements and practical considerations. It will repay careful study.

Here is a picture of a disk organized for a CFS:

```
abc==defgh====ijkl=m=nopqrs-----------------
```

In this picture letters denote reachable blocks, `=`'s denote unreachable blocks that are not part of the free space, and `-`'s denote free blocks (contiguous on the disk viewed as a ring buffer). After the cleaner copies blocks `a-e` the picture is

```
-------fgh====ijkl=m=nopqrsabcde------------
```

because the data `a-e` has been copied to free space and the blocks that used to hold `a-e` are free, together with the two unreachable blocks which were not copied. Then after blocks `g` and `j` are overwritten with new values `G` and `J`, the picture is

```
-------f=h====i=kl=m=nopqrsabcdeGJ----------
```

[1] M. Rosenblum and J. Osterhout, The design and implementation of a log-structured file system, *ACM Transactions on Computer Systems*, **10**, 1, Feb. 1992, pp 26-52.

The new data `G` and `J` has been written into free space, and the blocks that used to hold `g` and `j` are now unreachable. After the cleaner runs to completion the picture is

```
--------------------nopqrsabcdeGJfhiklm----
```

Pros and cons

A CFS has two main advantages:

- All writing is done sequentially; as we know, sequential writes are much faster than random writes. We have a good technique for making disk reads faster: caching. As main memory caches get bigger, more reads hit in the cache and disks spend more of their time writing, so we need a technique to make writes faster.
- The cleaner can copy reachable blocks to anywhere, not just to the standard free space region, and can do so without interfering with normal operation of the system. In particular, it can copy reachable blocks to tape for backup, or to a different disk drive that is faster, cheaper, less full, or otherwise more suitable as a home for the data.

There are some secondary advantages. Since the writes are sequential, they are not tied to disk blocks, so it's easy to write items of various different sizes without worrying about how they are packed into `DB`'s. Furthermore, it's easy to compress the sequential stream as it's being written[2], and if the disk is a RAID you never have to read any blocks to recompute the parity. Finally, there is no bit table or free list of disk blocks to maintain.

There is also one major drawback: unless large amounts of data in the same file are written sequentially, a file will tend to have lots of small extents, which can cause the problems discussed on page 13. In Unix file systems most files are written all at once, but this is certainly not true for databases. Ways of alleviating this drawback are the subject of current research. The cost of the cleaner is also a potential problem, but in practice the cost of the cleaner seems to be small compared to the time saved by sequential writes.

Updating metadata

For the CFS to work, it must update the index that points to the `DB`'s containing the file data on every write and every copy done by the cleaner, not just when the file is extended. And in order to keep the writing sequential, we must handle the new index information just like the file data, writing it into the free space instead of overwriting it. This means that the directory too must be updated, since it points to the index; we write it into free space as well. Only the *root* of the entire file system is written in a fixed location; this root says where to find the directory.

You might think that all this rewriting of the metadata is too expensive, since a single write to a file block, whether existing or new, now triggers three additional writes of metadata: for the index (if it doesn't fit in the directory), the directory, and the root. Previously none of these writes was needed for an existing block, and only the index write for a new block. However, the scheme for logging updates that we introduced to implement transactions can also handle this

[2] M. Burrows et al., On-line compression in a log-structured file system, *Proc. 5th Conference on Architectural Support for Programming Languages and Operating Systems*, Oct. 1992, pp 2-9. This does require some blocking so that the decompressor can obtain the initial state it needs.

problem. The idea is to write the *changes* to the index into a log, and cache the updated index (or just the updates) only in main memory. An example of a logged change is "block 43 of file 'alpha' now has disk address 385672". Later (with any luck, after several changes to the same piece of the index) we write the index itself and log the consequent changes to the directory; again, we cache the updated directory. Still later we write the directory and log the changes to the root. We only write a piece of metadata when:

We run out of main memory space to cache changed metadata, or

The log gets so big (because of many writes) that recovery takes too long.

To recover we replay the *active tail* of the log, starting before the oldest logged change whose metadata hasn't been rewritten. This means that we must be able to read the log sequentially from that point. It's natural to write the log to free space along with everything else. While we are at it, we can also log other changes like renames.

Note that a CFS can use exactly the same directory and index data as an ordinary file system, and in fact exactly the same code for `Read`. To do this we must give up the added flexibility we can get from sequential writing, and write each `DB` of data into a `DB` on the disk. Several implementations have done this (but the simple implementation below does not).

The logged changes serve another purpose. Because a file can only be reached from a single directory entry (or inode), the cleaner need not trace the directory structure in order to find the reachable blocks. Instead, if the block at `da` was written as block `b` of file `f`, it's sufficient to look at the file index and find out whether block `b` of file `f` is still at `da`. But the triple `(b, f, da)` is exactly the logged change. To take advantage of this we must keep the logged change as long as `da` remains reachable since the cleaner needs it (it's called 'segment summary' information in the literature). We don't need to replay it on recovery once its metadata is written out, however, and hence we need the sequential structure of the log only for the active tail.

Existing CFS's use the extra level of naming called inodes that is described on page 19. The inode numbers don't change during writing or copying, so the `PN -> INo` directory doesn't change. The root points to index information for the inodes (called the 'inode map'), which points to inodes, which point to data blocks or, for large files, to indirect blocks which point to data blocks.

Segments

Running the cleaner is fairly expensive, since it has to read and write the disk. It's therefore important to get as much value out of it as possible, by cleaning lots of unreachable data instead of copying lots of data that is still reachable. To accomplish this, divide the disk into *segments*, large enough (say 1 MB or 10 MB) that the time to seek to a new segment is much smaller than the time to read or write a whole segment. Clean each segment separately. Keep track of the amount of unreachable space in each segment, and clean a segment when (unreachable space) * (age of data) exceeds a threshold. Rosenblum and Osterhout explain this rule, which is similar in

spirit to what a generational garbage collector[3] does; the goal is to recover as much free space as possible, without allowing too much unreachable space to pile up in old segments.

Now the free space isn't physically contiguous, so we must somehow link the segments in the active tail together. We also need a table that keeps track for each segment of whether it is free, and if not, what its unreachable space and age are; this is cheap because segments are so large.

Backup

As we mentioned earlier, one of the major advantages of a CFS is that it is easier to back up. There are several reasons for this.

1. You can take a snapshot just by stopping the cleaner from freeing cleaned segments, and then copy the root information and the log to the backup medium, recording the logged data backward from the end of the log.
2. This backup data structure allows a single file (or a small set of files) to be restored in one pass.
3. It's only necessary to copy the log back to the point at which the previous backup started.
4. The disks reads done by backup are sequential and therefore fast. This is an important issue when the file system occupies many terabytes. At the 10 MB/s peak transfer rate of the disk, it takes 10^5 seconds, or a bit more than one day, to copy a terabyte. This means that a small number of disks and tapes running in parallel can do it in a fraction of a day. If the transfer rate is reduced to 1 MB/s by lots of seeks (which is what you get with random seeks if the average block size is 10 KB), the copying time becomes 10 days, which is impractical.
5. If a large file is partially updated, only the updates will be logged and hence appear in the backup.
6. It's easy to merge several incremental backups to make a full backup.

To get these advantages, we have to retain the ordering of segments in the log even after recovery no longer needs it.

There have been several research implementations of CFS's, and at least one commercial one called Spiralog in Digital Equipment Corporation's (now Compaq's) VMS system. You can read a good deal about it at http://www.digital.com/info/DTJM00/.

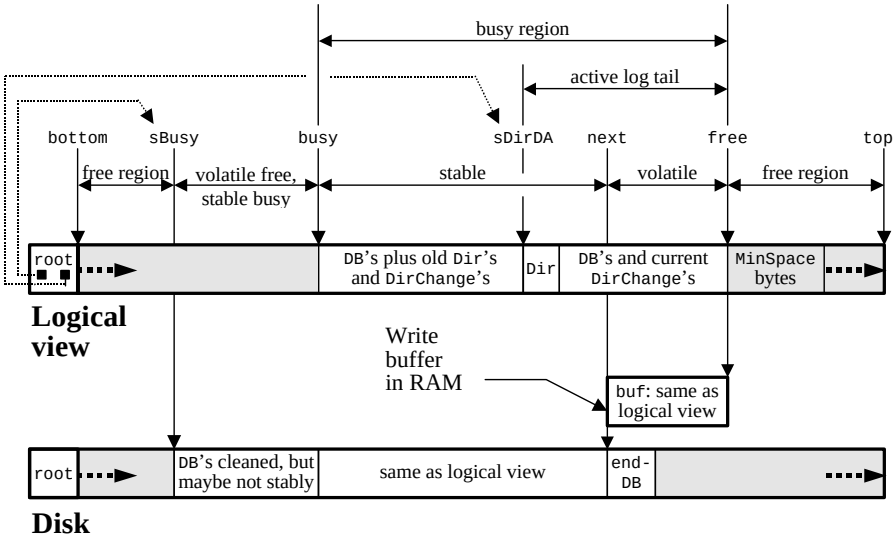
[3] H. Lieberman and C. Hewitt, A real-time garbage collector based on the lifetimes of objects, *Comm. ACM* **26**, 6, June 1983, pp 419-429.

A simple CFS implementation

We give an implementation `CopyingFS` of a CFS that contains all the essential ideas (except for segments, and the rule for choosing which segment to clean), but simplifies the data structures for the sake of clarity. `CopyingFS` treats the disk as a root `DB` plus a ring buffer of bytes. Since writing is sequential this is practical; the only cost is that we may have to pad to the end of a `DB` occasionally in order to do a `Sync`. A `DA` is therefore a byte address on the disk. We could dispense with the structure of disk blocks entirely in the representation of files, just write the `data` of each `File.Write` to the disk, and make a `FSImpl.BE` point directly to the resulting byte sequence on the disk. Instead, however, we will stick with tradition, take `BE = DA`, and represent a file as a `SEQ DA` plus its size.

So the disk consists of a root page, a busy region, and a free region (as we have seen, in a real system both busy and free regions would be divided into segments); see the figure below. The busy region is a sequence of encoded `Item`'s, where an `Item` is either a `Dir` or a `Change` to a `DB` in a file or to the `Dir`. The busy region starts at `busy` and ends just before `free`, which always points to the start of a disk block. We could write `free` into the root, but then making anything stable would require a (non-sequential) write of the root. Instead, the busy region ends with a recognizable `endDB`, put there by `Sync`, so that recovery can find the end of the busy region.

`DirDA` is the address of the latest directory on the disk. The part of the busy region after `dirDA` is the active tail of the log and contains the changes that need to be replayed during recovery to reconstruct the current directory; this arrangement ensures that we start the replay with a `dir` to which it makes sense to apply the changes that follow.



This implementation does bytewise writes that are buffered in `buf` and flushed to the disk only by `Sync`. Hence after a crash the state reverts to the state at the last `Sync`. Without the replay done during recovery by `ApplyLog`, it would revert to the state the last time the root was written; be sure you understand why this is true.

We assume that a sequence of encoded `Item`'s followed by an `endDB` can be decoded unambiguously. See the earlier discussion of writing logs atomically.

Other simplifications:

1. We store the `SEQ DA` that points to the file `DB`'s right in the directory. In real life it would be a tree, along one of the lines discussed in `FSImpl`, so that it can be searched and updated efficiently even when it is large. Only the top levels of the tree would be in the directory.
2. We keep the entire directory in main memory and write it all out as a single `Item`. In real life we would cache parts of it in memory and write out only the parts that are dirty (in other words, that contain changes).
3. We write a data block as part of the log entry for the change to the block, and make the `DA`'s in the file representation point to these log entries. In real life the logged change information would be batched together (as 'segment summary information') and the data written separately, so that recovery and cleaning can read the changes efficiently without having to read the file data as well, and so that contiguous data blocks can be read with a single disk operation and no extra memory-to-memory copying.
4. We allocate space for data in `Write`, though we buffer the data in `buf` rather than writing it immediately. In real life we might cache newly written data in the hope that another adjacent write will come along so that we can allocate contiguous space for both writes, thus reducing the number of extents and making a later sequential read faster.
5. Because we don't have segments, the cleaner always copies items starting at `busy`. In real life it would figure out which segments are most profitable to clean.
6. We run the cleaner only when we need space. In real life, it would run in the background to take advantage of times when the disk is idle, and to maintain a healthy amount of free space so that writes don't have to wait for the cleaner to run.
7. We treat `WriteData` and `WriteRoot` as atomic. In real life we would use one of the techniques for making log writes atomic that are described on page 23.
8. We treat `Init` and `Crash` as atomic, mainly for convenience in writing invariants and abstraction functions.In real life they do several disk operations, so we have to lock out external invocations while they are running.
9. We ignore the possibility of errors.


```
MODULE CopyingFS EXPORTS PN, Sync =                    % implements File, uses Disk

TYPE DA          =  Nat                                % Disk Address in bytes
                    WITH "+":=DAAdd, "-":=DASub}
     LE          =  SEQ DA                             % Linear Extent
     Data        =  File.Data

     X           =  File.X
     F           =  [le, size: X]                      % size = # of bytes

     PN          =  String WITH [...]                  % Path Name
     Dir         =  PN -> F

     Item        =  (DBChange + DirChange + Dir + Pad) % item on the disk
     DBChange    =  [pn, x, db]                        % db is data at x in file pn
     DirChange   =  [pn, dirOp, x]                     % x only for SetSize
     DirOp       =  ENUM[create, delete, setSize]
     Pad         =  [size: X]                          % For filling up a DB;
                                                       % Pad{x}.enc.size = x.

     IDA         =  [item, da]
     SI          =  SEQ IDA                            % for parsing the busy region

     Root        =  [dirDA: DA, busy: DA]              % assume encoding < DBSize

CONST
     DBSize      := Disk.DBSize
     diskSize    := 1000000
     rootDA      := 0
     bottom      := rootDA + DBSize                    % smallest DA outside root
     top         := (DBSize * diskSize) AS DA
     ringSize    := top - bottom
     endDB       := DB{...}                            % starts unlike any Item

VAR                                                    % All volatile; stable data is on disk.
     dir         :  Dir  := {}
     sDirDA      :  DA   := bottom                     % = ReadRoot().dirDA
     sBusy       :  DA   := Bottom                     % = ReadRoot().busy
     busy        :  DA   := bottom
     free        :  DA   := bottom
     next        :  DA   := bottom                     % DA to write buf at
     buf         :  Data := {}                         % waiting to be written
     disk                                              % the disk

ABSTRACTION FUNCTION File.dir = ( LAMBDA (pn) -> File.F =
% The file is the data pointed to by the DA's in its F.
     VAR f := dir(pn), diskData := + :(f.le * ReadOneDB) |
         RET diskData.seg(0, f.size) )

ABSTRACTION FUNCTION File.oldDirs = { SDir(), dir }

INVARIANT 1: ( ALL f :IN dir.rng | f.le.size * DBSize >= f.size )
% The blocks of a file have enough space for the data. From FSImpl.
```

The reason that `oldDirs` doesn't contain any intermediate states is that the stable state changes only in a `Sync`, which shrinks `oldDirs` to just `dir`.

During normal operation we need to have the variables that keep track of the region boundaries and the stable directory arranged in order around the disk ring, and we need to maintain this condition after a crash. Here are the relevant current and post-crash variables, in order (see below for `MinSpace`). The 'post-crash' column gives the value that the 'current' expression will have after a crash.

Current	*Post-crash*	
`busy`	`sBusy`	start of busy region
`sDirDA`	`sDirDA`	most recent stable `dir`
`next`		end of stable busy region
`free`	`next`	end of busy region
`free + minSpace()`	`next + minSpace()`	end of cushion for writes

In addition, the stable busy region should start and end before or at the start and end of the volatile busy region, and the stable directory should be contained in both. Also, the global variables that are supposed to equal various stable variables (their names start with 's') should in fact do so. The analysis that leads to this invariant is somewhat tricky; I hope it's right.

```
INVARIANT 2:
       IsOrdered((SEQ DA){next + MinSpace(), sBusy, busy, sDirDA, next, free,
                          free + MinSpace(), busy})
    /\ EndDA() = next /\ next//DBSize = 0 /\ Root{sDirDA, sBusy} = ReadRoot()
```

Finally,

> The busy region should contain all the items pointed to from `DA`'s in `dir` or in global variables.
>
> The directory on disk at `sDirDA` plus the changes between there and `free` should agree with `dir`.
>
> This condition should still hold after a crash.

```
INVARIANT 3:
    IsAllGood(ParseLog(busy, buf), dir)
    /\ IsAllGood(ParseLog(sBusy, {}), SDir())
```

The following functions are mainly for the invariants, though they are also used in crash recovery. `ParseLog` expects that the disk from `da` to the next `DB` with contents `endDB`, plus `data`, is the encoding of a sequence of `Item`'s, and it returns the sequence `SI`, each `Item` paired with its `DA`. `ApplyLog` takes an `SI` that starts with a `Dir` and returns the result of applying all the changes in the sequence to that `Dir`.

```
FUNC ParseLog(da, data) -> SI = VAR si, end: DA |
% Parse the log from da to the next endDB block, and continue with data.
       + :(si * (\ ida | ida.item.enc) = ReadData(da, end - da) + data
    /\ (ALL n :IN si.dom - {0} |
           si(n).da = si(n-1).da + si(n-1).item.enc.size)
    /\ si.head.da = da
    /\ ReadOneDB(end) = endDB => RET si
```



```
FUNC ApplyLog(si) -> Dir = VAR dir' := si.head.item AS Dir |
% si must start with a Dir. Apply all the changes to this Dir.
    DO VAR item := si.head.item |
        IF  item IS DBChange  => dir'(item.pn).le(item.x/DBSize) := si.head.da
        []  item IS DirChange => dir' := ...          % details omitted
        [*] SKIP                                      % ignore Dir and Pad
        FI; si := si.tail
    OD; RET dir'

FUNC IsAllGood(si, dir') -> Bool = RET
% All dir' entries point to DBChange's and si agrees with dir'
       (ALL da, pn, item | dir'!pn /\ da IN dir'(pn).le /\ IDA{item, da} IN si
                           ==> item IS DBChange)
    /\ ApplyLog(si) = dir'

FUNC SDir() -> Dir = RET ApplyLog(ParseLog(sDirDA), {})
% The Dir encoded by the Item at sDirDA plus the following DirChange's

FUNC EndDA() -> DA = VAR ida := ParseLog(sDirDA).last |
% Return the DA of the first endDB after sDirDA, assuming a parsable log.
    RET ida.da + ida.item.enc.size
```

The minimum free space we need is room for writing out `dir` when we are about to overwrite the last previous copy on the disk, plus the wasted space in a disk block that might have only one byte of data, plus the `endDB`.

```
FUNC MinSpace() -> Int = RET dir.enc.size + (DBSize-1) + DBsize
```

The following `Read` and `Write` procedures are much the same as they would be in `FSImpl`, where we omitted them. They are full of boring details about fitting things into disk blocks; we include them here for completeness, and because the way `Write` handles allocation is an important part of `CopyingFS`. We continue to omit the other `File` procedures like `SetSize`, as well as the handling in `ApplyLog` of the `DirChange` items that they create.

```
PROC Read(pn, x, size: X) -> Data =
    VAR f     := dir(pn),
        size  := {{size, f.size - x}.min, 0}.max,     % the available bytes
        n     := x/DBSize,                            % first block number
        nSize := NumDBs(x, size),                     % number of blocks
        blocks:= n .. n + nSize -1,                   % blocks we need in f.le
        data  := + :(blocks * f.le * ReadItem *       % all data in these blocks
                     (\ item | (item AS DBChange).db)) |
            RET data.seg(x//DBSize, size)             % the data requested

PROC Write(pn, x, data) = VAR f := dir(pn) |
% First expand data to contain all the DB's that need to be written
    data := Data.fill(0, x - f.size) + data;          % add 0's to extend f to x
    x := {x, f.size}.min;                             % and adjust x to match
    IF  VAR y := x//DBSize | y # 0 =>                 % fill to a DB in front
        x := x - y; data := Read(pn, x, y) + data
    [*] SKIP FI;
    IF  VAR y := data.size//DBSize | y # 0 =>         % fill to a DB in back
        data + := Read(pn, x + data.size, DBSize - y)
    [*] SKIP FI;
    % Convert data into DB's, write it, and compute the new f.le
```

```
    VAR blocks := Disk.DToB(data), n := x/DBSize,
        % Extend f.le with 0's to the right length.
        le := f.le + LE.fill(0, x + blocks.size - le.size),
        i := 0 |
            DO blocks!i =>
                le(n + i) := WriteData(DBChange{pn, x, blocks(i)}.enc);
                x + := DBSize; i + := 1
            OD; dir(pn).le := le
```

These procedures initialize the system and handle crashes. `Crash` is somewhat idealized; a more realistic implementation would read the log and apply the changes to `dir` as it reads them, but the logic would be the same.

```
PROC Init() = disk := disk.new(diskSize); WriteDir()   % initially dir is empty

PROC Crash() = <<                                      % atomic for simplicity
    CRASH;
    sDirDA   := ReadRoot().sDirDA; dir := SDir();
    sBusy    := ReadRoot().busy; busy := sBusy;
    free := EndDA(); next := free; buf := {} >>
```

These functions read an item, some data, or a single `DB` from the disk. They are boring. `ReadItem` is somewhat unrealistic, since it just chooses a suitable size for the `item` at `da` so that `Item.dec` works. In real life it would read a few blocks at `DA`, determine the length of the item from the header, and then go back for more blocks if necessary. It reads either from `buf` or from the disk, depending on whether `da` is in the write buffer, that is, between `next` and `free`.

```
FUNC ReadItem(da) -> Item = VAR size: X |
    RET Item.dec( (   DABetween(da, next, free) => buf.seg(da - next, size)
    [*] ReadData(da, size) ) )

FUNC ReadData(da, size: X) -> Data =                   % 1 or 2 disk.read's
    IF   size + da <= top =>                           % Int."+", not DA."+"
        % Read the necessary disk blocks, then pick out the bytes requested.
        VAR data := disk.read(LE{da/DBSize, NumDBs(da, size)}) |
            RET data.seg(da//DBSize, size)
    [*] RET ReadData(da, top - da) + ReadData(bottom, size - (top - da))

PROC ReadOneDB(da) = RET disk.read(LE{da/DBSize, 1}))
```

`WriteData` writes some data to the disk. It is not boring, since it includes the write buffering, the cleaning, and the space bookkeeping. The writes are buffered in `buf`, and `Sync` does the actual disk write. In this module `Sync` is only called by `WriteDir`, but since it's a procedure in `File` it can also be called by the client. When `WriteData` needs space it calls `Clean`, which does the basic cleaning step of copying a single item. There should be a check for a full disk, but we omit it. This check can be done by observing that the loop in `WriteData` advances `free` all the way around the ring, or by keeping track of the available free space. The latter is fairly easy, but `Crash` would have to restore the information as part of its replay of the log.

These write procedures are the only ones that actually write into `buf`. `Sync` and `WriteRoot` below are the only procedures that write the underlying disk.


```
PROC WriteData(data) -> DA =                           % just to buf, not disk
    DO IsFull(data.size) => Clean() OD;
    buf + := data; VAR da := free | free + := data.size; RET da

PROC WriteItem(item) = VAR d := item.enc | buf + := d; free + := d.size
% No check for space because this is only called by Clean, WriteDir.

PROC Sync() =
% Actually write to disk, in 1 or 2 disk.write's (2 if wrapping).
% If we will write past sBusy, we have to update the root.
    IF (sBusy - next) + (free - next) <= MinSpace() => WriteRoot()[*]  SKIP FI;
    % Pad buf to even DB's. A loop because one Pad might overflow current DB.
    DO VAR z := buf.size//DBSize | z # 0 => buf := buf + Pad{DBSize-z}.enc OD;.
    buf := buf + endDB;                                % add the end marker DB
    <<  % atomic for simplicity
    IF  buf.size + next < top => disk.write(next/DBSize, buf)
    [*] disk.write(next  /DBSize, buf.seg(0       , top-next  ));
        disk.write(bottom/DBSize, buf.sub(top-next, buf.size-1))
    FI;
    >>; free := next + buf.size - DBSize; next := free; buf := {}
```

The constraints on using free space are that `Clean` must not cause writes beyond the stable `sBusy` or into a disk block containing `Item`'s that haven't yet been copied. (If `sBusy` is equal to `busy` and in the middle of a disk block, the second condition might be stronger. It's necessary because a write will clobber the whole block.) Furthermore, there must be room to write an `Item` containing `dir`. Invariant 2 expresses all this precisely. In real life, of course, `Clean` would be called in the background, the system would try to maintain a fairly large amount of free space, and only small parts of `dir` would be dirty. `Clean` drops `DirChange`'s because they are recorded in the `Dir` item that must appear later in the busy region.

```
FUNC IsFull(size: X) -> Bool = RET busy - free < MinSpace() + size

PROC Clean() = VAR item := ReadItem(busy) |            % copy the next item
    IF  item IS DBChange /\ dir(item.pn).le(item.x/DBSize) = busy =>
        dir(item.pn).le(item.x/DBSize) := free; WriteItem(item)
    []  item IS Dir /\ da = sDirDA => WriteDir()       % the latest Dir
    [*] SKIP                                           % drop DirChange, Pad
    FI; busy := busy + item.enc.size

PROC WriteDir() =
% Called only from Clean and Init. Could call it more often to speed up recovery
%, after DO busy - free < MinSpace() => Clean() OD to get space.
    sDirDA := free; WriteItem(dir); Sync(); WriteRoot()
```

The remaining utility functions read and write the root, convert byte sizes to `DB` counts, and provide arithmetic on `DA`'s that wraps around from the top to the bottom of the disk. In real life we don't need the arithmetic because the disk is divided into segments and items don't cross segment boundaries; if they did the cleaner would have to do something quite special for a segment that starts with the tail of an item.

```
FUNC ReadRoot() -> Root = VAR root, pad |
     ReadOneDB(rootDA) = root.enc + pad.enc => RET root

PROC WriteRoot() = << VAR pad, db | db = Root{sDirDA, busy}.enc + pad.enc =>
     disk.write(rootDA, db); sBusy := busy >>

FUNC NumDBs(da, size: X) -> Int = RET (size + da//DBSize + DBSize-1)/DBSize
% The number of DB's needed to hold size bytes starting at da.

FUNC DAAdd(da, i: Int) -> DA = RET ((da - bottom + i) // ringSize) + bottom

FUNC DASub(da, i: Int) -> DA = RET ((da - bottom - i) // ringSize) + bottom
% Arithmetic modulo the data region. abs(i) should be < ringSize.

FUNC DABetween(da, da1, da2) -> Bool = RET da = da1 \/ (da2 - da1) < (da1 - da)

FUNC IsOrdered(s: SEQ DA) -> Bool =
     RET (ALL i :IN s.dom - {0, 1} | DABetween(s(i-1), s(i-2), s(i)))

END CopyingFS
```


.