
Spec is a language for writing specifications and the first few stages of successive refinement towards a practical implementation. As a specification language it includes constructs (quantifiers, backtracking or non-determinism, some uses of atomic brackets) which are impractical in a final implementation; they are there because they make it easier to write clear, unambiguous and suitably general specifications. If you want to write a practical program, avoid them.

This document defines the syntax of the language precisely and the semantics informally. You should read the Introduction to Spec (handout 3) before trying to read this manual. In fact, this manual is intended mainly for reference; rather than reading it carefully, skim through it, and then use the index to find what you need. For a precise definition of the atomic semantics read the forthcoming handout Atomic Semantics of Spec. The handout on Formal Concurrency gives the non-atomic semantics semi-formally.

1. Overview

Spec is a notation for writing specifications for a discrete system. What do we mean by a specification? It is the allowed sequences of transitions of a state machine. So Spec is a notation for describing sequences of transitions of a state machine.

Expressions and commands

The Spec language has two essential parts:

An expression describes how to compute a value as a function of other values, either constants or the current values of state variables.

A command describes possible transitions, or changes in the values of the state variables.

Both are based on the state, which in Spec is a mapping from names to values. The names are called state variables or simply variables: in the examples below they are i and j.

There are two kinds of commands:

An atomic command describes a set of possible transitions. For instance, the command $i := i + 1$ describes the transitions $i=1 \rightarrow i=2$, $i=2 \rightarrow i=3$, etc. (Actually, many transitions are summarized by $i=1 \rightarrow i=2$, for instance, $(i=1, j=1) \rightarrow (i=2, j=1)$ and $(i=1, j=15) \rightarrow (i=2, j=15)$). If a command allows more than one transition from a given state we say it is non-deterministic. For instance, the command $i := i + 1$ allows the transitions $i=2 \rightarrow i=1$ and $i=2 \rightarrow i=3$. More on this in Atomic Semantics of Spec.

A non-atomic command describes a set of sequences of states. More on this in Formal Concurrency.

A sequential program, in which we are only interested in the initial and final states, can be described by an atomic command.


Organizing a program

In addition to the expressions and commands that are the core of the language, Spec has four other mechanisms that are useful for organizing your program and making it easier to understand.

A routine is a named computation with parameters (passed by value). There are four kinds:

A function is an abstraction of an expression.

An atomic procedure is an abstraction of an atomic command.

A general procedure is an abstraction of a non-atomic command.

A thread is the way to introduce concurrency.

A type is a stylized assertion about the set of values that a name can assume. A type is also an easy way to group and name a collection of routines, called its methods, that operate on values in that set.

An exception is a way to report an unusual outcome.

A module is a way to structure the name space into a two-level hierarchy. An identifier declared in a module $m$ is known as $i$ in $m$ and as $m.i$ throughout the program. A class is a module that can be instantiated many times to create many objects.

A Spec program is some global declarations of variables, routines, types, and exceptions, plus a set of modules each of which declares some variables, routines, types, and exceptions.

Outline

This manual describes the language bottom-up:

Lexical rules
Types
Expressions
Commands
Modules

At the end there are two sections with additional information:

Scope rules
Built-in methods for set, sequence, and routine types.

There is also an index. The Introduction to Spec has a one-page language summary.
2. Grammar rules

Nonterminal symbols are in lower case; terminal symbols are punctuation other than ::=, or are quoted, or are in upper case.

Alternative choices for a nonterminal are on separate lines.

symbol* denotes zero or more occurrences of symbol.

The symbol empty denotes the empty string.

If x is a nonterminal, the nonterminal xList is defined by

xList ::= x , xList

A comment in the grammar runs from % to the end of the line; this is just like Spec itself.

A [n] in a comment means that there is an explanation in a note labeled [n] that follows this chunk of grammar.

3. Lexical rules

The symbols of the language are literals, identifiers, keywords, operators, and the punctuation { } [ ] ( ) , : . | < > := => -> [ ] *. Symbols must not have embedded white space. They are always taken to be as long as possible.

A literal is a decimal number such as 3765, a quoted character such as 'x', or a double-quoted string such as "Hello\n".

An identifier (id) is a letter followed by any number of letters, underscores, and digits followed by any number of characters. Case is significant in identifiers. By convention type and procedure identifiers begin with a capital letter. An identifier may not be the same as a keyword. The predefined identifiers Any, Bool, Char, Int, Nat, Null, String, true, false, and nil are declared in every program. The meaning of an identifier is established by a declaration; see section 8 on scope for details. Identifiers cannot be redeclared.

By convention keywords are written in upper case, but you can write them in lower case if you like; the same strings with mixed case are not keywords, however. The keywords are:

<table>
<thead>
<tr>
<th>ALL</th>
<th>APROC</th>
<th>AS</th>
<th>BEGIN</th>
<th>BY</th>
<th>CLASS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONST</td>
<td>DO</td>
<td>END</td>
<td>ENUM</td>
<td>EXCEPTION</td>
<td>EXCEPT</td>
</tr>
<tr>
<td>EXISTS</td>
<td>EXPORT</td>
<td>FI</td>
<td>FUNC</td>
<td>HAVOC</td>
<td>IF</td>
</tr>
<tr>
<td>IN</td>
<td>IS</td>
<td>LAMBDA</td>
<td>MODULE</td>
<td>OD</td>
<td>PROC</td>
</tr>
<tr>
<td>RAISE</td>
<td>RAISES</td>
<td>RET</td>
<td>SEQ</td>
<td>SET</td>
<td>SKIP</td>
</tr>
<tr>
<td>SUCHTHAT</td>
<td>THREAD</td>
<td>TYPE</td>
<td>VAR</td>
<td>WHILE</td>
<td>WITH</td>
</tr>
</tbody>
</table>

An operator is any sequence of the characters !@#$%^&*-+=:.<>?/\|~ except the sequences : . | < > := => -> (these are punctuation), or one of the keyword operators AS, IN, and IS.

A comment in a Spec program runs from a \ outside of quotes to the end of the line. It does not change the meaning of the program.

4. Types

A type defines a set of values; we say that a value v has type T if v is in T's set. The sets are not disjoint, so a value can belong to more than one set and therefore can have more than one type.

In addition to its value set, a type also defines a set of routines (functions or procedures) called its methods; a method normally takes a value of the type as its first argument.

An expression has exactly one type, determined by the rules in section 5; the result of the expression has this type unless it is an exception.

The picky definitions given on the rest of this page are the basis for Spec's type-checking. You can skip them on first reading, or if you don't care about type-checking.

About unions: If the expression e has type T we say that e has a routine type W if W is a routine in T's set. The symbol empty denotes the empty string.

An expression e fits a type U if the type-checker thinks they may have some values in common. This can only happen if they have the same structure, and each part of e fits the corresponding part of U.

A type T fits a type U if the type-checker thinks they may have some values in common. This can only happen if they have the same structure, and each part of T fits the corresponding part of U.

A type T includes a type U if the same conditions apply with "fits" replaced by "includes", all the "vice versa" clauses dropped, and in the => rule "T fits U" replaced by "U includes T". The types T and U are tuples of the same length and each component of T fits the corresponding component of U.

Two types are equal if their definitions are the same (that is, have the same parse trees) after all type names have been replaced by their definitions and all WITH clauses have been discarded.

Equal types define the same value set. Ideally the reverse would also be true, but type equality is meant to be decided by a type checker, whereas the set equality is intractable.

A type T fits a type U if if the type-checker thinks they may have some values in common. This can only happen if they have the same structure, and each part of T fits the corresponding part of U.

A type T includes a type U if the same conditions apply with "fits" replaced by "includes", all the "vice versa" clauses dropped, and in the => rule "T fits U" replaced by "U includes T". The types T and U are tuples of the same length and each component of T fits the corresponding component of U.

A symbol denotes zero of more occurrences of symbol*.

symbol denotes zero or more occurrences of symbol.

The picky definitions given on the rest of this page are the basis for Spec's type-checking. You can skip them on first reading, or if you don't care about type-checking.

About unions: If the expression e has type T we say that e has a routine type W if W is a routine in T's set. The symbol empty denotes the empty string.

An expression e fits a type U if the type-checker thinks they may have some values in common. This can only happen if they have the same structure, and each part of e fits the corresponding part of U.

A type T includes a type U if the same conditions apply with "fits" replaced by "includes", all the "vice versa" clauses dropped, and in the => rule "T fits U" replaced by "U includes T". The types T and U are tuples of the same length and each component of T fits the corresponding component of U.

Two types are equal if their definitions are the same (that is, have the same parse trees) after all type names have been replaced by their definitions and all WITH clauses have been discarded.

Equal types define the same value set. Ideally the reverse would also be true, but type equality is meant to be decided by a type checker, whereas the set equality is intractable.

A type T fits a type U if if the type-checker thinks they may have some values in common. This can only happen if they have the same structure, and each part of T fits the corresponding part of U.

A type T includes a type U if the same conditions apply with "fits" replaced by "includes", all the "vice versa" clauses dropped, and in the => rule "T fits U" replaced by "U includes T". The types T and U are tuples of the same length and each component of T fits the corresponding component of U.

Two types are equal if their definitions are the same (that is, have the same parse trees) after all type names have been replaced by their definitions and all WITH clauses have been discarded.

Equal types define the same value set. Ideally the reverse would also be true, but type equality is meant to be decided by a type checker, whereas the set equality is intractable.

A symbol denotes zero or more occurrences of symbol.

symbol denotes zero or more occurrences of symbol.

The picky definitions given on the rest of this page are the basis for Spec's type-checking. You can skip them on first reading, or if you don't care about type-checking.

About unions: If the expression e has type T we say that e has a routine type W if W is a routine in T's set. The symbol empty denotes the empty string.

An expression e fits a type U if the type-checker thinks they may have some values in common. This can only happen if they have the same structure, and each part of e fits the corresponding part of U.

A type T includes a type U if the same conditions apply with "fits" replaced by "includes", all the "vice versa" clauses dropped, and in the => rule "T fits U" replaced by "U includes T". The types T and U are tuples of the same length and each component of T fits the corresponding component of U.

Two types are equal if their definitions are the same (that is, have the same parse trees) after all type names have been replaced by their definitions and all WITH clauses have been discarded.

Equal types define the same value set. Ideally the reverse would also be true, but type equality is meant to be decided by a type checker, whereas the set equality is intractable.

A type T fits a type U if if the type-checker thinks they may have some values in common. This can only happen if they have the same structure, and each part of T fits the corresponding part of U.

A type T includes a type U if the same conditions apply with "fits" replaced by "includes", all the "vice versa" clauses dropped, and in the => rule "T fits U" replaced by "U includes T". The types T and U are tuples of the same length and each component of T fits the corresponding component of U.

Two types are equal if their definitions are the same (that is, have the same parse trees) after all type names have been replaced by their definitions and all WITH clauses have been discarded.

Equal types define the same value set. Ideally the reverse would also be true, but type equality is meant to be decided by a type checker, whereas the set equality is intractable.

A type T fits a type U if if the type-checker thinks they may have some values in common. This can only happen if they have the same structure, and each part of T fits the corresponding part of U.

A type T includes a type U if the same conditions apply with "fits" replaced by "includes", all the "vice versa" clauses dropped, and in the => rule "T fits U" replaced by "U includes T". The types T and U are tuples of the same length and each component of T fits the corresponding component of U.

Two types are equal if their definitions are the same (that is, have the same parse trees) after all type names have been replaced by their definitions and all WITH clauses have been discarded.

Equal types define the same value set. Ideally the reverse would also be true, but type equality is meant to be decided by a type checker, whereas the set equality is intractable.
The ambiguity of the type grammar is resolved by taking -> to be right associative and giving with and raises higher precedence than ->.

[1] A $SEQ\ T$ is just a function from $\{0, 1, \ldots, size-1\}$ to $T$. That is, it is short for $(Int->T)$ SUCHTHAT $(\\exists size : Int | (ALL i : Int | f!i = (i IN 0 .. size-1)))$

WITH { (see section 9) }

This means that invocation, $t$, and $v$ work for a sequence just as they do for any function. In addition, there are many other useful operators on sequences; see section 9. The String type is just $SEQ\ Char$; there are String literals, defined in section 5.

[2] A $T\rightarrow U$ value is a partial function from a state and a value of type $T$ to a value of type $U$. A $T\rightarrow U$ RAISES $xs$ value is the same except that the function may raise the exceptions in $xs$.

[3] We say $m$ is a method of $T$ defined by $f$, and denote $f$ by $T.m$, if

$$T = T' WITH \{..., m := f, ...\}$$

and $m$ is an identifier or is "op" where op is an operator (the construct in braces is a methodDefList), or

$$T = T' WITH \{methodDefList\}, m is not defined in methodDefList, and m is a method of $T'$ defined by $f$, or$$T = (... + T' + ...), m is a method of $T'$ defined by $f$, and there is no other type in the union with a method $m$.

There are two special forms for invoking methods: $e1 infixOp e2$ or $prefixOp e$, and $e1.id(e2)$ or $e.id$ or $e.id()$. They are explained in notes [1] and [3] to the expression grammar in the next section. This notation may be familiar from object-oriented languages. Unlike many such languages, Spec makes no provision for varying the method in each object, though it does allow inheritance and overriding.

A method doesn't have to be a routine, though the special forms won't type-check unless the method is a routine. Any method $m$ of $T$ can be referred to by $T.m$.

[4] In $T$ SUCHTHAT $f$, $f$ is a predicate on $T$, that is, a function ($T \rightarrow Bool$). The type $T$ SUCHTHAT $f$ has the same methods as $T$, and its value set is the values of $T$ for which $f$ is true. See section 5 for primary.

[5] If a type is defined by $m[typeList].id$ and $m$ is a parameterized module, the meaning is $m'.id$ where $m'$ is defined by MODULE $m' = m[typeList] END m'$. See section 7 for a full discussion of this kind of type.

[6] Id is the id of a type, obtained from id by dropping trailing ‘ characters and digits, and capitalizing the first letter or all the letters (it’s an error if these capitalizations yield different identifiers that are both known at this point).
5. Expressions

An expression is a partial function from states to results; results are values or exceptions. That is, an expression computes a result for a given state. The state is a function from names to values. This state is supplied by the command containing the expression in a way explained later. The meaning of an expression (that is, the function it denotes) is defined informally in this section. The meanings of invocations and lambda function constructors are somewhat tricky, and the informal explanation here is supplemented by a formal account in Atomic Semantics of Spec. Because expressions don’t have side effects, the order of evaluation of operands is irrelevant (but see [5] and [13]).

Every expression has a type. The result of the expression is a member of this type if it is not an exception. This property is guaranteed by the type-checking rules, which require an expression used as an argument, the right hand side of an assignment, or a routine result to fit the type of the formal, left hand side, or routine range (see section 4 for the definition of ‘fit’). In addition, expressions appearing in certain contexts must have suitable types: in e1(e2), e1 must have a routine type; in e1+e2, e1 must have a type with a + operator, etc. These rules are given in detail in the rest of this section. A union type is suitable if exactly one of the members is suitable. Also, if T is suitable in some context, so are T WITH {...} and T SUCHTHAT F.

An expression can be a literal, a variable known in the scope that contains the expression, or a function invocation. The form of an expression determines both its type and its result in a state:

- **Literal**: A sequence of decimal digits.
- **Variable**: A name that denotes a value.
- **Function Invocation**: An expression with a function (not procedure) type U->T RAISES EX or U->T (note that a sequence is a function), and e must fit U; then f(e) has type T. In more detail, if f has result rf and e has type U' and result re, then U' must fit U (checked statically) and e must have type U (checked dynamically if U' involves a union or SUCHTHAT; if the dynamic check fails the result is a fatal error). Then f(e) has type T.

If either rf or re is undefined, so is f(e). Otherwise, if either is an exception, that exception is the result of f(e); if both are, rf is the result.

If both rf and re are normal, the result of rf at re can be:

- A normal value, which becomes the result of f(e).
- An exception, which becomes the result of f(e). If rf is defined by a function body that loops, the result is a special looping exception that you cannot handle.

Undefinable, in which case f(e) is undefined and the command containing it fails (has no outcome) — failure is explained in section 6.

A function invocation in an expression never affects the state. If the result is an exception, the containing command has an exceptional outcome; for details see section 6.

The other forms of expressions (e.id, constructors, prefix and infix operators, combinations, and quantifications) are all syntactic sugar for function invocations, and their results are obtained by the rule used for invocations. There is a small exception for conditionals [5] and for the conditional logical operators /\, /\, and ==> that are defined in terms of conditionals [13].
The ambiguity of the expression grammar is resolved by taking the\ infixOps to be left associative and using the indicated precedences for the\ prefixOps and\ infixOps (with 8 for IS and\ AND and 5 for : or any operator not listed); higher numbers correspond to tighter binding. The\ precedence is determined by the operator symbol and doesn’t depend on the operand types.

1. The meaning of\ prefixOp\ e is\ T."prefixOp"(e), where\ T is e’s type, and of\ e1\ infixOp\ e2 is\ T1."infixOp"(e1, e2), where\ T1 is e1’s type. The built-in types\ Int (and\ Nat with the same operations),\ Bool, sequences, sets, and functions have the operations given in the grammar. Section 9 on built-in methods specifies the operators for built-in types other than\ Int and\ Bool. Special case:\ e1 IN e2 means\ T2."IN"(e1, e2), where\ T2 is e2’s type.

Note that the\ ! operator does not require that the types of its arguments agree, since both are Any.\ Also,\ = and\ != cannot be overridden by\ WITH. To define your own abstract equality, use a different operator such as\ **="**.

2. The\ exp must have type\ SEQ T or\ SET T. The value is the elements of\ exp combined into a single value by\ infixOps, which must be associative and have an identity, and must also be\ commutative if\ exp is a set. Thus

\[ + : ([i : Int | 0<i \land i<5 | i**2]) = 1 + 4 + 9 + 16 = 30, \]

and if\ s is a sequence of strings,\ + : s is the concatenation of the strings. For another example, see the definition of quantifications in [4]. Note that the entire set is evaluated; see [10].

3. Methods can be invoked by dot notation.

The meaning of\ e.id(e1) is\ T.id(e1), where\ T is e’s type. Section 9 on built-in methods gives the methods for built-in types other than\ Int and\ Bool.

4. A quantification is a conjunction (if the quantifier is ALL) or disjunction (if it is EXISTS) of the\ pred with the id’s in the\ declList bound to every possible value (that is, every value in their types); see section 4 for\ decl. Precisely, (ALL d \mid p) = \land :\ (d \mid p) and\ (EXISTS d \mid p) = \lor :\ (d \mid p). All the expressions in these evolutions are evaluated, unlike e2 in the expressions e1 \land e2 and e1 \lor e2 (see [10] and [13]).

5. A conditional (\ pred => e1) is not exactly an invocation. If\ pred is true, the result is the result of\ e1 even if e2 is undefined or exceptional; if\ pred is false, the result is the result of\ e2 even if e1 is undefined or exceptional. If\ pred is undefined, so is the result; if\ pred raises an exception, that is the result. If (\ *\ e2 is omitted and\ pred is false, the result is undefined.

6. In a constructor (\ expList) each\ exp must have the same type\ T, the type of the\ constructor is (\ SEQ T +\ SET T), and its value is the sequence containing the values of the\ exps in the given order, which can also be viewed as the set containing these values.

If\ expList is empty the type is the union of all function, sequence and set types, and the value is the empty sequence or set, or a function undefined everywhere. If desired, these constructors can be prefixed by a\ name denoting a suitable set or sequence type.

A constructor\ T(e1, ..., en), where\ T is a record type [f1: T1, ..., fn: Tn], is short for a record constructor (see [7])\ T(f1=e1, ..., fn=en).

7. The primary must have a record type, and the constructor has the same type as its primary and denotes the same value except that the fields named in the\ fieldDefList have the given
values. Each value must fit the type declared for its \(\text{id}\) in the record type. The \text{primary} may also denote a record type, in which case any fields missing from the \text{fieldDefList} are given arbitrary (but deterministic) values. Thus if \(R=a: \text{Int}, b: \text{Int}\), \(R(a := 3, b := 4)\) is a record of type \(R\) with \(a\) and \(b\) set to \(3\) and \(4\). If the record type is qualified by a \text{SUCHTHAT}, the fields get values that satisfy it, and the constructor is undefined if that’s not possible.

[8] The \text{primary} must have a function or sequence type, and the constructor has the same type as its primar\(\text{y}\) and denotes a value equal to the value denoted by the \(\text{primary}\) except that it may map the argument value given by \(\text{exp}\) (which must fit the domain type of the function or sequence) to result (which must fit the range type if it is an \(\text{exp}\)). For a function, if \(\text{result}\) is empty the constructed function is undefined at \(\text{exp}\), and if \(\text{result}\) is RAISE exception, then \text{result} must be in the RAISES set of \(\text{primary}\)'s \text{type}. For a sequence \(\text{result}\) must not be empty or RAISE, and \(\text{exp}\) must be in \text{primary}.

In the \text{primary} must have a function type or a function, and the value of the constructor is a function whose result is \text{result} at every value of the \text{domain} \text{type} (the \text{type} on the left of the \(\Rightarrow\)). Thus if \(f=(\text{Int} \rightarrow \text{Int})\) and \(f=F(\text{\Rightarrow}0)\), then \(f\) is everywhere zero and \(f(4 \rightarrow 1)\) is zero except at 4, where it is 1. If this value doesn’t have the \text{function} \text{type}, the \text{constructor} is undefined; this can happen if the \text{type} has a \text{SUCHTHAT} \text{clause}. For example, the \text{type} can’t be a sequence.

[9] A \text{LAMBDA} constructor is a statically scoped function definition. When it is invoked, the meaning of the body is determined by the local state when the \text{LAMBDA} \text{was} evaluated and the global state when it is invoked; this is ad-hoc but convenient. See section 7 for \text{signature} and section 6 for \text{cmd}. The returns in the \text{signature} may not be empty. Note that a \text{function} can’t have side effects.

The form \((\text{\textbackslash declList} | \text{exp})\) is short for \((\text{LAMBDA} (\text{declList}) \rightarrow \text{T} = \text{RET} \text{exp})\), where \(\text{T}\) is the \text{type} of \text{exp}. See section 4 for \text{decl}.

[10] A set \text{constructor} \((\text{\textbackslash declList} | \text{pred} | \text{exp})\) has type \(\text{SET} \text{T}\), where \(\text{exp}\) has \text{type} \(\text{T}\) in the current state augmented by \text{declList}; see section 4 for \text{decl}. Its \text{value} is a set that contains \(\text{\textbackslash iF \text{\textbackslash x} = \text{exp}}\). Thus
\[
\{i: \text{Int} | 0 \leq i \leq 5 | i \times 2 \} = \{1, 2, 3, 4, 5\}
\]
and both have type \(\text{SET} \text{Int}\). If \(\text{pred}\) is omitted it defaults to \text{true}. If \(\text{seqGenList}\) is omitted it defaults to the last \text{id} declared:
\[
\{i: \text{Int} | i \leq 5 \} = \{1, 2, 3, 4, 5\}
\]
Note that if \(s\) is a set or sequence, \(\text{IN} \text{s}\) is a \text{type} (see section 4), so you can write a constructor like \((i : \text{\textbackslash IN s} | i > 4)\) for the elements of \(s\) greater than 4. This is shorter and clearer than \((i | i \text{\textbackslash IN s} \land i > 4)\) if there are any values of the declared \text{id}'s for which \text{pred} is undefined, or \text{pred} is true and \text{exp} is undefined, then the result is undefined. If nothing is undefined, the same holds for exceptions; if more than one exception is raised, the result exception is an arbitrary choice among them.

[11] A sequence \text{constructor} \((\text{\textbackslash seqGenList} | \text{pred} | \text{exp})\) has type \(\text{SEQ} \text{T}\), where \(\text{exp}\) has \text{type} \(\text{T}\) in the current state augmented by \text{seqGenList}, as follows. The value of
\[
\{x1 := \text{e01 BY e1 WHILE pl, } \ldots, xn := \text{e0n BY en WHILE pn} | \text{pred} | \text{exp}\}
\]
is the sequence which is the value of \text{result} produced by the following program. Here \(\text{exp}\) has type \(\text{T}\) and \text{result} is a fresh identifier (that is, one that doesn’t appear elsewhere in the program). There’s an informal explanation after the program.

\[
\begin{align*}
V A R & x2 := e02, \ldots, x n := e0n, r e s u l t := T(), x 1 := e 01 | \\
& D O p 1 \Rightarrow x 2 := e 2; p 2 \Rightarrow \ldots \Rightarrow x n := e n ; p n \Rightarrow \\
& I F \ pred \Rightarrow \text{result} := \text{result} + \langle \text{exp} \rangle | \ast \Rightarrow \text{SKIP F}1; \\
& x 1 := e 1 \\
& \text{OD}
\end{align*}
\]
However, \text{e01} and \text{ei} are not allowed to refer to \text{xj} if \(j > i\). Thus the \text{sequences} are unrolled in parallel until one of them ends, as follows. All but the first are initialized; then the first is initialized and all the others computed, then all are computed repeatedly. In each iteration, once all the \text{xi} have been set, if \text{pred} is true the value of \text{exp} is appended to the result sequence; thus \text{exp} serves to filter the result. As with set constructors, an omitted \text{pred} defaults to \text{true}, and an omitted \text{exp} defaults to \text{null}. An omitted \text{WHILE} \text{p1} default to \text{WHILE} \text{true}. An omitted \text{:= e0i} defaults to
\[
:= \{x: T L | \text{true}\}. \text{choose}
\]
where \(T L\) is the \text{type} of \text{ei}; that is, it defaults to an arbitrary \text{value} of the right \text{type}.

The generator \(x i := \text{IN ei}\) generates the elements of the \text{sequence} \text{ei} in order. It is short for
\[
(j := 0 \text{ BY } j + 1 \text{ WHILE } j < \text{ei.size}; x i := x i(e i(j))
\]
where \(j\) is a fresh identifier. Note that if the \text{IN} isn’t the first \text{generator} then the first \text{element} of \text{ei} is skipped, which is probably not what you want. Note that \text{IN} in a sequence \text{constructor} overrides the normal use of \text{IN} \text{s} as a \text{type} (see [10]).

Undefined exceptions and \text{results} are handled the same way as in set \text{constructors}.

Examples

\[
\begin{align*}
& \{i := 0 \text{ BY } i + 1 \text{ WHILE } i < n\} = 0..n = \{0, 1, \ldots, n\} \\
& \{r := \text{head BY r.next WHILE r != nil} | r.val\} = \text{val fields of a list starting at head} \\
& \{x : \text{IN s, sum := 0 \text{ BY } sum + x}\} = \text{partial sums of } s \\
& \{x : \text{IN s, sum := 0 BY sum + x}.last\} = \text{last partial sum} \\
& \{x : \text{IN s | i \text{= f}(x)}\} = \text{reverse of } s \\
& \{i : \text{IN 1..n | i \text{=} 2 \# 0 | i \times i}\} = \text{squares of odd numbers } \leq n \\
& \{f(x), f'(e), \ldots, f''(e)\}
\end{align*}
\]

[12] These \text{operations} are defined in section 9.

[13] The conditional logical \text{operators} are defined in terms of conditionals:
\[
\begin{align*}
& e 1 \text{\ OR} e 2 = (e 1 \Rightarrow \text{true} \ast\ast \ast e 2) \\
& e 1 \text{\ AND} e 2 = (\neg e 1 \Rightarrow \text{false} \ast\ast \ast e 2) \\
& e 1 \Rightarrow e 2 = (\neg e 1 \Rightarrow \text{true} \ast\ast \ast e 2)
\end{align*}
\]
Thus the second \text{operand} is not evaluated if the value of the first one determines the \text{result}.

[14] \text{AS} changes only the \text{type} of the \text{expression}, not its \text{value}. Thus if \(\text{exp AS type}\) the \text{value} of \(\text{exp AS type}\) is the \text{value} of \text{exp}, but its \text{type} is \text{type} rather than the \text{type} of \text{exp}.
6. Commands

A command changes the state (or does nothing). Recall that the state is a mapping from names to values; we denote it by \texttt{state}. Commands are non-deterministic. An atomic command is one that is inside \texttt{<<...>>} brackets.

The meaning of an atomic command is a set of possible transitions (that is, a relation) between a state and an outcome (a state plus an optional exception); there can be any number of outcomes from a given state. One possibility is a looping exceptional outcome. Another is no outcomes. In this case we say that the atomic command \texttt{fails}; this happens because all possible choices within it encounter a false guard or an undefined invocation.

If a subcommand fails, an atomic command containing it may still succeed. This can happen because it’s one operand of [ ] or [*] and the other operand succeeds. If can also happen because a non-deterministic construct in the language that might make a different choice. Leaving exceptions aside, the commands with this property are [ ] and \texttt{VAR} (because it chooses arbitrary values for the new variables). If we gave an operational semantics for atomic commands, this situation would correspond to backtracking. In the relational semantics that we actually give (\textit{Atomic Semantics of Spec}), it corresponds to the fact that the predicate defining the relation is the “or” of predicates for the subcommands. Look there for more discussion of this point.

A non-atomic command defines a collection of possible transitions, roughly one for each \texttt{<<...>>} command that is part of it. If it has simple commands not in atomic brackets, each one also defines a possible transition, except for \texttt{assignment} and \texttt{invocations}. An \texttt{assignment} defines two transitions, one to evaluate the right hand side, and the other to change the value of the left hand side. An \texttt{invocation} defines a transition for evaluating the arguments and doing the call and one for evaluating the result and doing the return, plus all the transitions of the body. These rules are somewhat arbitrary and their details are not very important, since you can always write separate commands to express more transitions, or atomic brackets to express fewer transitions. The motivation for the rules is to have as many transitions as possible, consistent with the idea that an expression is evaluated atomically.

A complete collection of possible transitions defines the possible sequences of states or histories; there can be any number of histories from a given state. A non-atomic command still makes choices, but it does not backtrack and therefore can have histories in which it gets stuck, even though in other histories a different choice allows it to run to completion. For the details, see handout 17 on formal concurrency.

\begin{verbatim}
6.826—Principles of Computer Systems 1999

6. Commands

A command changes the state (or does nothing). Recall that the state is a mapping from names to values; we denote it by state. Commands are non-deterministic. An atomic command is one that is inside <<...>> brackets.

The meaning of an atomic command is a set of possible transitions (that is, a relation) between a state and an outcome (a state plus an optional exception); there can be any number of outcomes from a given state. One possibility is a looping exceptional outcome. Another is no outcomes. In this case we say that the atomic command fails; this happens because all possible choices within it encounter a false guard or an undefined invocation.

If a subcommand fails, an atomic command containing it may still succeed. This can happen because it’s one operand of [ ] or [*] and the other operand succeeds. If can also happen because a non-deterministic construct in the language that might make a different choice. Leaving exceptions aside, the commands with this property are [ ] and \texttt{VAR} (because it chooses arbitrary values for the new variables). If we gave an operational semantics for atomic commands, this situation would correspond to backtracking. In the relational semantics that we actually give (\textit{Atomic Semantics of Spec}), it corresponds to the fact that the predicate defining the relation is the “or” of predicates for the subcommands. Look there for more discussion of this point.

A non-atomic command defines a collection of possible transitions, roughly one for each \texttt{<<...>>} command that is part of it. If it has simple commands not in atomic brackets, each one also defines a possible transition, except for \texttt{assignment} and \texttt{invocations}. An \texttt{assignment} defines two transitions, one to evaluate the right hand side, and the other to change the value of the left hand side. An \texttt{invocation} defines a transition for evaluating the arguments and doing the call and one for evaluating the result and doing the return, plus all the transitions of the body. These rules are somewhat arbitrary and their details are not very important, since you can always write separate commands to express more transitions, or atomic brackets to express fewer transitions. The motivation for the rules is to have as many transitions as possible, consistent with the idea that an expression is evaluated atomically.

A complete collection of possible transitions defines the possible sequences of states or histories; there can be any number of histories from a given state. A non-atomic command still makes choices, but it does not backtrack and therefore can have histories in which it gets stuck, even though in other histories a different choice allows it to run to completion. For the details, see handout 17 on formal concurrency.

\end{verbatim}
atomic command, or one of the simple commands \texttt{SKIP}, \texttt{HAVOC}, \texttt{RET}, or \texttt{RAISE}. The type-checking rule for \texttt{invocations} is the same as for function invocations in expressions.

[4] You can only assign to a name declared with \texttt{VAR} or in a signature. In an assignment the \texttt{exp} must fit the type of the \texttt{lhs}, or there is a fatal error. In a function body \texttt{assignments} must be to names declared in the signature or the body, to ensure that the function can’t have side effects.

An assignment to a left hand side that is not a name is short for assigning a constructor to a name. In particular,
\begin{verbatim}
  lhs(arguments) := exp short for lhs := lhs(arguments->exp), and
  lhs . id := exp short for lhs := lhs[id := exp].
\end{verbatim}

These abbreviations are expanded repeatedly until \texttt{lhs} is a name.

In an assignment the right hand side may be an \texttt{invocation} (of a procedure) as well as an ordinary expression (which can only invoke a function). The meaning of \texttt{lhs} := \texttt{exp} or \texttt{lhs} := \texttt{invocation} is to first evaluate the \texttt{exp} or do the \texttt{invocation} and assign the result to a temporary variable \texttt{v}, and then do \texttt{lhs} := \texttt{v}. Thus the assignment command is not atomic unless it is inside \texttt{<<...>>}.

If the left hand side of an assignment is a \texttt{(lhsList)}, the \texttt{exp} must be a tuple of the same length, and each component must fit the type of the corresponding \texttt{lhs}. Note that you cannot write a tuple constructor that contains procedure invocations.

[5] A guarded command fails if the result of \texttt{pred} is undefined or \texttt{false}. It is equivalent to \texttt{cmd} if the result of \texttt{pred} is \texttt{true}. A \texttt{pred} is just a Boolean \texttt{exp}; see section 4.

\texttt{S1} \texttt{[]} \texttt{S2} chooses one of the \texttt{S1} to execute. It chooses one that doesn’t fail. Usually \texttt{S1} and \texttt{S2} will be guarded. For example,
\begin{verbatim}
  x=1 \Rightarrow y:=0 []; x>1 \Rightarrow y:=1
\end{verbatim}
and \texttt{S1} has no outcome if \texttt{x<1}. But \texttt{S1} [] \texttt{S2} is the same as \texttt{S1} unless \texttt{S1} fails, in which case it’s the same as \texttt{S2}.

\texttt{IF} ... \texttt{FI} are just command brackets, but it often makes the program clearer to put them around a sequence of guarded commands, thus:
\begin{verbatim}
  IF \ x < 0 \Rightarrow y := 3 \ldots \texttt{FI}
\end{verbatim}

[6] In a \texttt{VAR} the unadorned form of \texttt{declInit} initializes a new variable to an arbitrary value of the declared type. The \texttt{:=} form initializes a new variable to \texttt{exp}. Precisely,
\begin{verbatim}
  VAR id: T := exp | S.
\end{verbatim}
is equivalent to
\begin{verbatim}
  VAR id: T | id := exp; S.
\end{verbatim}

The \texttt{exp} could also be a procedure invocation, as in an assignment.

Several \texttt{declInit}s after \texttt{VAR} is short for nested \texttt{VAR}s. Precisely,
\begin{verbatim}
  VAR declInit , declInitList | cmd
\end{verbatim}
is short for
\begin{verbatim}
  VAR declInit | VAR declInitList | cmd
\end{verbatim}
This is unlike a module, where all the names are introduced in parallel.

[7] In an atomic command the atomic brackets can be used for grouping instead of \texttt{BEGIN ... END}; since the command can’t be any more atomic, they have no other meaning in this context.

[8] Execute \texttt{cmd} repeatedly until it fails. If \texttt{cmd} never fails, the result is a looping exception that doesn’t have a name and therefore can’t be handled. Note that this is \texttt{not} the same as failure.

[9] Exception handling is as in Clu, but a bit simplified. Exceptions are named by literal strings (which are written without the enclosing quotes). A module can also declare an identifier that denotes a set of exceptions. A command can have an attached exception \texttt{handler}, which gets to look at any exceptions produced in the command (by \texttt{RAISE} or by an invocation) and not handled closer to the point of origin. If an exception is not handled in the body of a routine, it is raised by the routine’s invocation.

An exception \texttt{ex} must be in the \texttt{RAISE} set of a routine \texttt{r} if either \texttt{RAISE ex} or an invocation of a routine \texttt{with ex in its RAISES} set occurs in the body of \texttt{r} outside the scope of a handler for \texttt{ex}.

7. Modules
A program is some global declarations plus a set of modules. Each module contains variable, routine, exception, and type declarations.

Module definitions can be parameterized with \texttt{mformals} after the \texttt{module id} and a parameterized module can be instantiated. Instantiation is like macro expansion: the formal parameters are replaced by the arguments throughout the body to yield the expanded body. The parameters must be types, and the body must type-check without any assumptions about the argument that replaces a formal other than the presence of a \texttt{WITH} clause that contains all the methods mentioned in the formal parameter list (that is, formals are treated as distinct from all other types).

Each module is a separate scope, and there is also a \texttt{Global} scope for the identifiers declared at the top level of the program. An identifier \texttt{id} declared at the top level of a non-parameterized module \texttt{m} is short for \texttt{m.id} when it occurs in \texttt{m}. If it appears in the \texttt{exports}, it can be denoted by \texttt{m.id} anywhere. When an identifier \texttt{id} that is declared globally occurs anywhere, it is short for \texttt{Global.id}. \texttt{Global} cannot be used as a module \texttt{id}.

An exported \texttt{id} must be declared in the module. If an exported \texttt{id} has a \texttt{WITH} clause, it must be declared in the module as a type with at least those methods, and only those methods are accessible outside the module; if there is no \texttt{WITH} clause, all its methods and constructors are accessible. This is Spec’s version of data abstraction.

\begin{verbatim}
  program ::= toplevel* module* END
  module ::= modclass id mformals exports = body END id
  modclass ::= MODULE [CLASS] % [4]
  exports ::= EXPORT exportList
  export ::= id WITH (methodList) % see section 4 for method
  mformals ::= empty
  mfp ::= id [ mfpList ] % module formal parameter
  id WITH { declList } % see section 4 for decl
\end{verbatim}
body ::= toplevel* % id must be the module id
toplevel ::= VAR declInit* % instance of parameterized module
    CONST declInit* % declares the decl ids [1]
routineDecl ::= funcDecl, procDecl, threadDecl % declares the routine id
    EXCEPTION exSetDecl* % declares the exception set ids
    TYPE typeDecl* % declares the type ids and any
        % ids in ENUM
routineDecl ::= funcDecl, procDecl, threadDecl % declares the
APROC id signature = <<cmd>> % function
    THREAD id signature = cmd % one thread for each possible
signature ::= ( declList ) returns raises % see section 4 for returns
    ( ) returns raises % and raises
exSetDecl ::= id = exceptionSet % see section 4 for exceptionSet
typeDecl ::= id = type % see section 4 for type
    id = ENUM [ idList ] % a value is one of the id's [3]

[1] The ":= exp" in a declInit (defined in section 6) specifies an initial value for the variable. The exp is evaluated in a state in which the variable used during the evaluation has been initialized, and the result must be a normal value, not an exception. The exp sees all the names known in the scope, not just the ones that textually precede it, but the relation "used during evaluation of initial values" on the variables must be a partial order so that initialization makes sense. As an assignment, the exp may be a procedure invocation as well as an ordinary expression. It's a fatal error if the exp is undefined or the invocation fails.

[2] Instead of being invoked by the client of the module or by another procedure, a thread is automatically invoked in parallel once for every possible value of its arguments. The thread is named by the id in the declaration together with the argument values. So

```
VAR sum := 0, count := 0
THREAD P(i: Int) = i IN 0 .. 9 =>
    VAR t | t := F(i); <<sum := sum + t>>; <<count := count + 1>>
adds up the values of F(0) ... F(9) in parallel. It creates a thread P(i) for every integer i; the threads P(0), ..., P(9) for which the guard is true invoke F(0), ..., F(9) in parallel and total the results in sum. When count = 10 the total is complete. A thread is the only way to get an entire program to do anything (except evaluate initializing expressions, which could have side effects), since transitions only happen as part of some thread.

[3] The id's in the list are declared in the module; their type is the ENUM type. There are no operations on enumeration values except the ones that apply to all types: equality, assignment, and routine argument and result communication.

[4] A class is shorthand for a module that declares a convenient object type. The next few paragraphs specify the shorthand, and the last one explains the intended usage.

If the class id is Obj, the module id is ObjMod. Each variable declared in a top level VAR in the class becomes a field of the ObjRec record type in the module. The module exports only a type Obj that is also declared globally. Obj indexes a collection of state records of type ObjRec stored in the module's objRec variable, which is a function Obj -> ObjRec. Obj's methods are all the names declared at top level in the class except the variables, plus the new method described below; the exported Obj's methods are all the ones that the class exports plus new.

To make a class routine suitable as a method, it needs access to an ObjRec that holds the state of the object. It gets this access through a self parameter of type Obj, which it uses to refer to the object state objRec(Obj). To carry out this scheme, each routine in the module, unless it appears in a WITH clause in the class, is 'objectified' by giving it an extra self parameter of type Obj. In addition, in a routine body every occurrence of a variable v declared at top level in the class is replaced by objRec(Obj).v in the module, and every invocation of an objectified class routine gets self as an extra first parameter.

The module also gets a synthesized and objectified StdNew procedure that adds a state record to objRec, initializes it from the class's variable initializations (rewritten like the routine bodies), and returns its ObjRec index; this procedure becomes the new method of Obj unless the class already has a new routine.

A class cannot declare a THREAD.

The effect of this transformation is that a variable obj of type Obj behaves like an object. The state of the object is objRec(obj). The invocation obj.m or obj.m(x) is short for ObjMod.m(objRec(obj)) or ObjMod.m(obj, x) by the usual rule for methods, and it invokes the method m in the module object each occurrence of a class variable refers to the corresponding field in objRec's state.

obj.new() returns a new and initialized ObjRec object. The following example shows how a class is transformed into a module.

```
CLASS Obj EXPORT T1, f, p, ... = MODULE ObjMod EXPORT Obj WITH {T1, f, p, new } =
    TYPE T1 = ... WITH {add:=AddT}
    CONST c := ...
    VAR v1:T1:=ei, v2:T2:=pi(v1), ...
    TYPE ObjRec = [v1: T1, v2: T2, ...]
    VAR objs: Obj -> ObjRec := {}% in T1
    THREAD P(i: Int) = i IN 0 .. 9 =>
        VAR t | t := F(i); <<sum := sum + t>>; <<count := count + 1>>
    adds up the values of F(0) ... F(9) in parallel. When count = 10 the total is complete.

In abstraction functions and invariants we also write obj.n for field n in objRec's state, that is, for ObjRec.objRec(obj).n.
8. Scope

The declaration of an identifier is known throughout the smallest scope in which the declaration appears (redeclaration is not allowed). This section summarizes how scopes work in Spec; terms defined before section 7 have pointers to their definitions. A scope is one of:

- the whole program, in which just the predefined (section 3), module, and globally declared identifiers are declared;
- a module;
- the part of a routineDecl or LAMBDA expression (section 5) after the :=;
- the part of a VAR declInit | cmd command after the := (section 6);
- the part of a constructor or quantification after the first := (section 5);
- a record type or methodDefList (section 4);

An identifier is declared by

- a module id, mfp, or toplevel (for types, exception sets, ENUM elements, and named routines),
- a decl in a record type (section 4), | constructor or quantification (section 5), declInit (section 6), routine signature, or WITH clause of a mfp, or
- a methodDef in the WITH clause of a type (section 4).

An identifier may not be declared in a scope where it is already known. An occurrence of an identifier id always refers to the declaration of id which is known at that point, except when id is being declared (precedes a := of a toplevel, the := of a record constructor, or the := or BY in a seqGen), or follows a dot. There are four cases for dot:

- moduleId . id — the id must be exported from the basic module moduleId, and this expression denotes the meaning of id in that module.
- record . id — the id must be declared as a field of the record type, and this expression denotes that field of record. In an assignment’s lhs see [7] in section 6 for the meaning.
- typeId . id — the typeId denotes a type, id must be a method of this type, and this expression denotes that method.
- primary . id — the primary must be a method of primary’s type, and this expression, together with any following arguments, denotes an invocation of that method; see [2] in section 5 on expressions.

If id refers to an identifier declared by a toplevel in the current module m, it is short for m.id. If it refers to an identifier declared by a toplevel in the program, it is short for Global.id. Once these abbreviations have been expanded, every name in the state is either global (contains a dot and is declared in a toplevel), or local (does not contain a dot and is declared in some other way).

Exceptions look like identifiers, but they are actually string literals, written without the enclosing quotes for convenience. Therefore they do not have scope.
There are constructors {} for the empty set, {e1, e2, ...} for a set with specific elements, and {declList | pred | exp} for a set whose elements satisfy a predicate. These constructors are described in [6] and [10] of section 5. Note that \( (t \ | \ p).pred = \{ t \ | \ p \} \), and similarly \( (t \ | \ p).set = \{ t \ | \ p \} \).

**Functions**

The function types \( T->U \) and \( T->U \) RAISES XS have methods for composition, overlay, inverse, and restriction; testing whether a function is defined at an argument and whether it produces a normal (non-exceptional) result at an argument, and for the domain and range; converting a function to a relation (the inverse is the relation’s func method).

In other words, they behave as though they were \( \text{Function}[T, U].F \), where (making allowances for the fact that XS and V are pulled out of thin air):

```
MODULE Function[T, U]
EXPORT F =
TYPE F = T->U RAISES XS WITH {"*":=Compose, "+":=Overlay,
   inv:=Inverse, restrict:=Restrict,
   "!":=Defined, "!!":=Normal,
   dom:=Domain, rng:=Range, rel:=Rel}
R = (T, U) -> Bool
FUNC Compose(f, g: U -> V) -> (T -> V) = RET \( \{ t | g(f(t)) \} \)
FUNC Overlay(f1, f2) -> F = RET \( \{ t | (f2!(t) => f2(t) [*] f1(t)) \} \)
% \( (f1 + f2) \) is \( f2(x) \) if that is defined, otherwise \( f1(x) \)
FUNC Inverse(f) -> (U -> T) = RET f.rel.inv.func
FUNC Restrict(f, s: SET T) -> F = RET \( \{ t | (t IN s => f(t)) \} \)
FUNC Defined(f, t)->Bool = IF f(t)=f(t) => RET true [*] RET false FI EXCEPT XS => RET true
FUNC Normal(f, t)->Bool = IF f(t)=f(t) => RET false FI EXCEPTION XS => RET false
FUNC Domain(f) -> SET T = RET \( \{ t, u | f(t, u) \} \)
FUNC Range (f) -> SET U = RET \( \{ t, u | f(t, u) \} \)
FUNC Rel(f) -> R = RET \( \{ t, u | f(t, u) \} \)
END Function
```

A total function \( T->Bool \) is a predicate and has an additional method to compute the set of \( \{ t \} \)'s that satisfy the predicate (the inverse is the set’s pred method). In other words, a predicate behaves as though it were \( \text{Predicate}[T].P \), where

```
MODULE Predicate[T] EXPORT P =
TYPE P = T -> Bool WITH {set:=Set}
FUNC Set(p) -> SET T = RET \( \{ t | p(t) \} \)
END Predicate
```

A predicate with \( T = (T0, U0) \) is a relation and has additional methods to turn it into a function, a total function, or a function to sets of \( U0 \)'s, and to get its domain and range, invert it or compose it (overriding the methods for a function). In other words, it behaves as though it were \( \text{Relation}[T0, U0].R \), where (making allowances for the fact that \( V \) is pulled out of thin air in Compose):

```
MODULE Relation[T, U] EXPORT R =
TYPE R = (T, U) -> Bool WITH {func:=Func, totalF:=TotalFunc, setF:=SetFunc,
   dom:=Domain, rng :=Range,
   inv:=Inverse, "+":=Compose}
FUNC Func(r) -> (T -> U) =
% The result function is defined at \( t \) iff \( r \) relates \( t \) to a single \( u \).
   RET \( \{ t | (r.setF(t).size = 1 => r.setF(t).choose) \} \)
FUNC TotalFunc(r) -> (T -> (U + Null)) =
% The result function is defined everywhere, returning some related \( U \), or nil if there is none.
   RET \( \{ t | (r.setF(t).size = 0 | r.setF(t).choose [*] nil) \} \)
FUNC SetFunc(r) -> (T -> SET U) = RET \( \{ t, u | r(t, u) \} \)
% The result function is defined everywhere, returning the set of related \( U \)’s.
FUNC Domain(r) -> SET T = RET \( \{ t, u | r(t, u) \} \)
FUNC Range (r) -> SET U = RET \( \{ t, u | r(t, u) \} \)
FUNC Inverse(r) -> (U, T) -> Bool = RET \( \{ u, t | r(t, u) \} \)
FUNC Compose(r: R, s: (U, V)->Bool) -> (T, V)->Bool =
   RET \( \{ t, v | (EXISTS u | r(t, u) \&\& s(u, v) \} \}
END Relation
```

A relation with \( T = U \) is a graph and has additional methods to test whether a sequence of \( T \)'s is a path in the graph and to compute the transitive closure. In other words, it behaves as though it were \( \text{Graph}[T].G \), where

```
MODULE Graph[T] EXPORT G =
TYPE G = (T, T) -> Bool WITH {IsPath:=IsPath, closure:=TransitiveClosure}
P = SEQ T
FUNC IsPath(g, p) = RET \( \{ t | p.dom = \{ 0 \} \| g(p(i-1), p(i)) \} \)
% Any p of size <= 1 is a path by this definition.
FUNC TransitiveClosure(g) -> G = RET \( \{ t1, t2 | \{ EXISTS p | p.size > 1 \& p.head = t1 \& p.last = t2 \& g.isPath(p) \} \} \)
END Graph
```
Sequences

A function is called a sequence if its domain is a finite set of consecutive Ints starting at 0, that is, if it has type

\[ S : \text{Int} \to T \text{ SUCHTHAT } \forall s \mathbf{|} (\exists s : \text{Int} \rightarrow T . \text{s.dom} = (0 \ldots \text{s.size}-1).\text{set}) \]

We denote this type (with the methods defined below) by SEQ T. A sequence inherits the methods of the function (though it overrides +), and it also has methods for

- detaching or attaching the first or last element,
- extracting a segment of a sequence, concatenating two sequences, or finding the size, making a sequence with all elements the same
- making a sequence into a tuple or set (set also makes it into a set),
- testing for empty, prefix, or sub-sequence (not necessarily contiguous),
- lexical comparison, permuting, and sorting,
- treating a sequence as a multiset with operations to:

  - count the number of times an element appears, test membership and multisets equality, take differences, and remove an element (*"union and addl adds an element).

All these operations are undefined if they use out-of-range subscripts, except that an empty sub-sequence is defined regardless of the subscripts.

We define the sequence methods with a module. Precisely, SEQ T is Sequence[T].S, where:

\[
\text{MODULE } \text{Sequence}\![T]\text{ EXPORTS } S =
\]

\[
\text{TYPE } I = \text{Int} \\
S = (I \to T) \text{ SUCHTHAT } \forall s \mathbf{|} (\forall i \mathbf{|} s!i = (0 <= i \land i < s\text{.size}))
\]

\[\text{WITH } \{ \text{size}:\text{Int}, \text{sub}:\text{Sub}, **+:\text{Concatenate}, \text{head}:\text{Head}, \text{tail}:\text{Tail}, \text{addh}:\text{AddHead}, \text{remh}:\text{Tail}, \text{last}:\text{Last}, \text{reml}:\text{RemoveLast}, \text{addil}:\text{AddLast}, \text{seg}:\text{Seg}, \text{fill}:\text{Fill}, \text{tuple}:\text{Tuple}, \text{isEmpty}:\text{Empty}, \text{"<=":Prefix, \"<":SubSeq, \text{lexLE}:\text{lexLE}, \text{perms}:\text{Perms}, \text{fsorter}:\text{FSorter}, \text{sort}:\text{Sort}, \text{sort}:\text{Sort}, \text{set}:\text{S.rng} \}
\]

\[\% \text{ These methods treat a sequence as a multiset (or bag).}
\text{count}:\text{Count}, \text{"IN":EqElem}, \text{"-":Diff}, \text{set}:S\text{.rng} \}
\]

\[\text{FUNC Size(s)} \rightarrow \text{Int} = \text{S.dom.size} \]

\[\text{FUNC Sub(s, i1, i2)} \rightarrow S = \{ \text{RET s.sub(i1, i2)} \}
\]

\[\text{FUNC Concatenate(s1, s2)} \rightarrow S = \text{VAR s} | \text{RET s.sub(0, s.size-1)} = s2 \Rightarrow \text{RET s} \]

\[\text{FUNC Head(s)} \rightarrow T = \text{RET s.0} \]

\[\text{FUNC Tail(s)} \rightarrow S = \{ \text{RET s.sub(1, s.size-1)} \}
\]

\[\text{FUNC AddHead(s, t)} \rightarrow S = \{ \text{RET s + t} \}
\]

\[\text{FUNC Last(s)} \rightarrow T = \{ \text{RET s.last} \}
\]

\[\text{FUNC RemoveLast(s)} \rightarrow S = \{ \text{RET s.reml} \}
\]

\[\text{FUNC AddLast(s, t)} \rightarrow S = \{ \text{RET s + {t}} \}
\]

\[\text{FUNC Seg(s, i, n)} \rightarrow S = \{ \text{RET s.sub(i, i+n-1)} \}
\]

\[\text{FUNC Fill(t, n)} \rightarrow S = \{ \text{RET t} \}
\]

\[\text{FUNC IsEmpty(s)} \rightarrow \text{Bool} = \{ \text{RET s.size == 0} \}
\]

\[\text{FUNC Prefix(s1, s2)} \rightarrow \text{Bool} = \{ \text{RET s1 <= s2} \}
\]

\[\text{FUNC SubSeq(s1, s2)} \rightarrow \text{Bool} = \{ \text{RET s1 <= s2} \}
\]

\[\text{FUNC LexLE(s1, s2, f)} \rightarrow \text{Bool} = \{ \text{RET s1.lexLE(s2, f)} \}
\]

\[\text{FUNC Perms(s)} \rightarrow \text{SET} S = \{ \text{RET s.perms} \}
\]

\[\text{FUNC FSort(s, f)} \rightarrow S = \{ \text{RET s.fsorter(f)} \}
\]

\[\text{FUNC Sort(s)} \rightarrow S = \{ \text{RET s.fsorter(T."<")} \}
\]

\[\text{FUNC Fill(t, n)} \rightarrow S = \{ \text{RET t} \}
\]

\[\text{FUNC EqElem(s1, s2)} \rightarrow \text{Bool} = \{ \text{RET s1 == s2} \}
\]

\[\text{FUNC Count(s, t)} \rightarrow \text{Int} = \{ \text{RET s.count(t)} \}
\]

\[\text{FUNC Diff(s1, s2)} \rightarrow S = \{ \text{RET s1 - s2} \}
\]

\[\text{FUNC Head(s)} \rightarrow T = \{ \text{RET s.head} \}
\]

\[\text{FUNC Tail(s)} \rightarrow S = \{ \text{RET s.tail} \}
\]

\[\text{FUNC AddHead(s, t)} \rightarrow S = \{ \text{RET s + {t}} \}
\]

\[\text{FUNC Last(s)} \rightarrow T = \{ \text{RET s.last} \}
\]

\[\text{FUNC RemoveLast(s)} \rightarrow S = \{ \text{RET s.reml} \}
\]

\[\text{FUNC AddLast(s, t)} \rightarrow S = \{ \text{RET s + {t}} \}
\]

\[\text{FUNC Seg(s, i, n)} \rightarrow S = \{ \text{RET s.sub(i, i+n-1)} \}
\]

\[\text{FUNC Fill(t, n)} \rightarrow S = \{ \text{RET t} \}
\]

\[\text{FUNC IsEmpty(s)} \rightarrow \text{Bool} = \{ \text{RET s.size == 0} \}
\]

\[\text{FUNC Prefix(s1, s2)} \rightarrow \text{Bool} = \{ \text{RET s1 <= s2} \}
\]

\[\text{FUNC SubSeq(s1, s2)} \rightarrow \text{Bool} = \{ \text{RET s1 <= s2} \}
\]

\[\text{FUNC LexLE(s1, s2, f)} \rightarrow \text{Bool} = \{ \text{RET s1.lexLE(s2, f)} \}
\]

\[\text{FUNC Perms(s)} \rightarrow \text{SET} S = \{ \text{RET s.perms} \}
\]

\[\text{FUNC FSort(s, f)} \rightarrow S = \{ \text{RET s.fsorter(f)} \}
\]

\[\text{FUNC Sort(s)} \rightarrow S = \{ \text{RET s.fsorter(T."<")} \}
\]

\[\text{FUNC Fill(t, n)} \rightarrow S = \{ \text{RET t} \}
\]

\[\text{FUNC EqElem(s1, s2)} \rightarrow \text{Bool} = \{ \text{RET s1 == s2} \}
\]

\[\text{FUNC Count(s, t)} \rightarrow \text{Int} = \{ \text{RET s.count(t)} \}
\]

\[\text{FUNC Diff(s1, s2)} \rightarrow S = \{ \text{RET s1 - s2} \}
\]
There is a constructor \{e1, e2, ...\} for a sequence with specific elements and a constructor \{\}\ for the empty sequence. There is also a constructor \(s \mapsto e\) which is equal to \(s\) except at \(e\) (and undefined if \(e\) is out of range). For the constructors see \[6\] and \[8\] of section 5. To generate a sequence there are constructors \{x :IN s | pred | exp\} and \{x := e1 BY e2 WHILE pred1 | pred2 | exp\}. For these see \[11\] of section 5.

To map each element \(t\) of \(s\) to \(f(t)\) use function composition \(s * f\). Thus if \(s: SEQ Int\), \(s * (i: Int \mapsto i*i)\) yields a sequence of squares. You can also write this \{i :IN s | i*i\}.

Index

- 9, 19, 21
- !, 9, 22
- \%, 9, 10
- \{\}, 3, 8, 16
- (expList), 8
- (typeList), 5
- *, 3, 9, 21, 22
- **, 9
- \/, 9
- \/, 9
- :, 8, 14
- \:, 3, 5
- :=, 3, 8, 14, 18
- ;, 14
- [], 3, 14
- [declList], 5
- [\*], 3, 8, 14
- [n], 3
- \\/, 8
- \(\ast \mapsto\) result, 8
- ( ), 3, 8
- (declList | pred | exp), 8
- (exceptionList), 5
- (exp -> result), 8
- (expList), 8
- (methodDefList), 5, 6
- [\!], 21
- (e1, e2, ...), 21
- [\|], 3, 14
- ~, 9
- +, 5, 9, 19, 21, 22
- >, 9
- &&, 14, 16
- << >>, 3
- <=, 9, 22
- <=, 19, 21
- =, 9, 10
- >=, 9
- =>, 9
- ~> 3, 8, 14
- ~>, 3, 5, 8
- >, 9
- <=>>, 9
- >>, 14
- abstract equality, 10
- add, 9
- add an element, 9
- adding an element, 19, 22
- addl, 22
- ALL, 8
- ambiguity, 10, 14
- Any, 5, 10
- append an element, 9
- APROC, 5, 16
- arguments, 8
- AS, 8
- assignment, 14
- associative, 6, 10, 14
- atomic command, 1, 13, 14
- atomic procedure, 2
- Atomic Semantics of Spec, 1, 7, 13
- backtracking, 13
- bag, 22
- BEGIN, 14
- body, 16
- Bool, 5
- built-in methods, 19
- capital letter, 3
- Char, 5
- characteristic predicate, 19
- choice, 14
- choose, 19
- choosing an element, 19
- CLASS, 16
- class, 17
- closure, 21
- Clu, 15
- cmd, 14
- command, 1, 13
- comment, 3
- composition, 20
- concatenation, 9
- conditional, 10
- conditional and, 9
- conditional or, 9
- CONST, 17
- constructor, 8
- count, 22
- data abstraction, 6
- decl, 5
- declaration, 18
- defined, 9, 20
- difference, 22
- divide, 9
- DO, 14
- dot, 18
- e.id, 10
- e.id(), 10
- e1 infixOp e2, 10
- e1.id(e2), 10
- else, 14
- empty, 3, 10
- empty sequence, 22
- empty set, 19
- END, 14, 16
- ENUM, 16
- equal, 9
- equal types, 4
- EXCEPT, 14
- exception, 5, 6, 7, 15, 16
- exceptionSet, 5, 16
- EXISTS, 8
- exp, 8
- expanded definitions, 4
- expression, 1, 7
- expression has a type, 7
- fail, 7, 13
- FI, 14
- fill, 22
- fit, 4, 7, 11, 14, 15
- formal parameters, 16
- FUNC, 16, 23
- function, 2, 6, 14, 19, 20
- function undefined everywhere, 20
- general procedure, 2
- Global.id, 16, 18
- grammar, 3
- graph, 21
- greater or equal, 9
- greater than, 9
- grouping, 15
- guard, 13, 14
- handler, 14
- has a routine type, 4
- has type T, 4
- HAVOC, 14
- head, 22
- id, 3, 6
- id := exp, 8
- id [ typeList ], 5
- identifier, 3
- if, 14
- implies, 9
- IN, 9, 19, 22
- includes, 4
- infixOp, 9
- initial value, 17
- initialize, 15
- instantiate, 16
- intersection, 9, 19
- Introduction to Spec, 1

invocation, 7, 8, 10, 14
isEmpty, 22
isPath, 21
local, 18
less than, 9
lexical comparison, 22
isEmpty, 22
isPath, 21
method, 4, 5, 6, 19
mfp, 16
min, 19
module, 2, 16
multiply, 9
mutiset, 22
multiset, 22
multiset difference, 9
name, 1, 5, 18
new variable, 15
non-atomic command, 2, 13
Non-Atomic Semantics of Spec, 1
non-deterministic, 1
nonterminal symbol, 3
normal result, 20
not equal, 9
NULL, 5
OD, 14
operator, 3, 6
OrderedList, 19
organizing your program, 2
outcome, 13
parameterized module, 16
path in the graph, 21
precedence, 6, 9, 14
precondition, 14
pred, 8, 19
predefined identifiers, 3
predicate, 20
prefix, 9, 22
prefixOp, 9
prefixOp e, 10
primary, 8
PROC, 5, 16
program, 2, 16
punctuation, 3
quant if, 8
quanti fication, 10
quoted character, 3
RAISE, 8, 14
RAISE exception, 11
RAISES, 5, 11
RAISES set, 15
record, 5, 10
redeclaration, 18
relation, 21
remove an element, 9, 19, 22
result, 7
result type, 14
RET, 14
routine, 2, 14, 16
scope, 18
seq, 22
SEQ, 5, 6, 22
SEQ Char, 6
sequence, 22
sequential composition, 14
set difference, 9, 19
set of sequences of states, 2
set of values, 4
set with specific elements, 19
setF, 21
side effects, 15
signature, 15, 16
SKIP, 14
specifications, 1
tail, 22
transitive closure, 21
tuple, 5, 14, 15
tuple constructor, 8
type, 2, 4, 5, 16
type equality, 4
type inclusion, 4
type-checking, 4, 7, 14
undefined, 7, 10, 13
UNION, 5, 6, 7, 9, 19, 22
upper case, 3
variable, 1, 14, 15
white space, 3
WITH, 5, 6, 10, 16