Programming with Arrays

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Pattern Matching

Pattern Matching: *Syntax & Semantics*

Let us represent a case as **(*case* e *of* C)** where **C** is

C = P -> e | (P -> e) , C

P = x | CN_0 | $CN_k(P_1, \ldots, P_k)$

The rewriting rules for a case may be stated as follows:

(*case* e *of* P -> e1, C)

⇒ **e1** if match**(P,e)**

⇒ if ~match**(P,e)**

(*case* e *of* P -> e1)

⇒ **e1** if match**(P,e)**

⇒ if ~match**(P,e)**

The match Function

P = x | CN_0 | $CN_k(P_1, \ldots, P_k)$

```
match[[x, t]]     = True

match[[CN0, t]]   = CN0 == tag(t)

match[[CNk(P1, …,Pk), t]] =
                  if tag(t) == CNk
                  then
                    (match[[P1, proj1(t)]] &&
                      .
                      .
                      .
                     match[[Pk, projk(t)]])
                  else
                    False
```


pH Pattern Matching

```
TE[[(case e of C)]]   =
     (let t = e in TC[[t, C]])

TC[[t, (P -> e)]]     =
     if match[[P, t]]
          then (let bind[[P, t]] in e)
          else error "match failure"

TC[[t, ((P -> e),C)]] =
     if match[[P, t]]
          then (let bind[[P, t]] in e)
          else TC[[t, C]]
```


Pattern Matching: bind Function

$$bind[[x, t]] = x = t$$

$$bind[[CN_0, t]] = \varepsilon$$

$$bind[[CN_k(P_1, \ldots, P_k), t]] = bind[[P_1, proj_1(t)]]; \ldots; bind[[P_k, proj_k(t)]]$$

Refutable vs Irrefutable Patterns

Patterns are used in binding for destructuring an expression---but what if a pattern fails to match?

```
let (x1, x2)      = e1
    x : xs        = e2
    y1: y2 : ys   = e3
in
   e
```

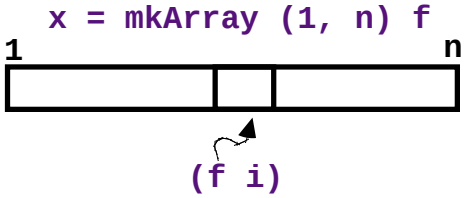
what if **e2** *evaluates to* **[]** *?*
e3 *to a one-element list ?*

Should we disallow refutable patterns in bindings?
Too inconvenient!

Turn each binding into a case expression

Arrays

Cache for function values on a regular subdomain

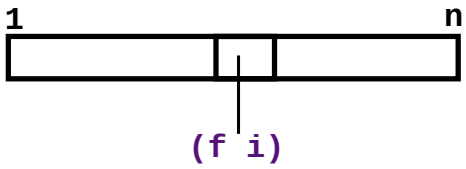


means `x!i = (f i)`
`1 ≤ i ≤ n`

Selection: `x!i` returns the value of the i^{th} slot

Bounds: `(bounds x)` returns the tuple containing the bounds

Efficiency is the Motivation for Arrays



`(f i)` is computed once and stored

`x!i` is simply a fetch of a precomputed value and should take constant time

A Simple Example


```
x = mkArray  (1,10)  (plus 5)
```

Type

```
x  ::  (ArrayI t)
```

assuming

```
f ::  Int -> t
```

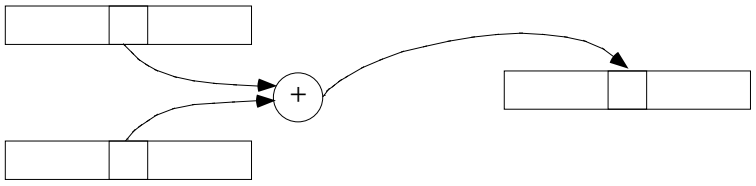

Array: An Abstract Data Type

```
module ArrayI (ArrayI, mkArray, (!), bounds)
   where

   infix 9 (!)

   data ArrayI t
   mkArray  ::(Int,Int)  -> (Int-> t) -> (ArrayI t)
   (!)      ::(ArrayI t) -> Int -> t
    bounds  ::(ArrayI t) -> (Int,Int)
```

Selection: **x!i** returns the value of the ith slot
Bounds: **(bounds x)** returns the tuple containing the bounds

Vector Sum

```
vs a b = let
             esum i = a!i + b!i
         in
             mkArray (bounds a) esum
```



Vector Sum - Error Behavior

```
vs  a b  = let
               esum i = a!i + b!i
           in
               mkArray (bounds a) esum
```

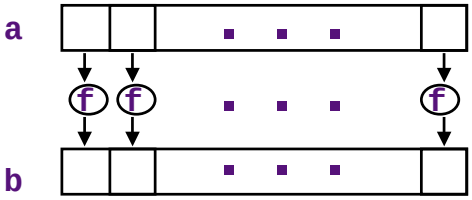
Suppose

1. 

2. 

3. 

Map Array

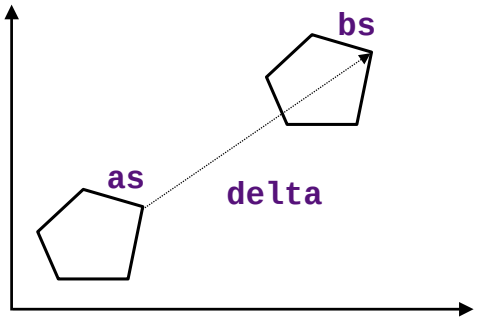



```
mapArray f a = let
                   g i = f (a!i)
               in
                   mkArray (bounds a) g
```

Example: scale a vector, that is, produce b such that $b_i = s * a_i$

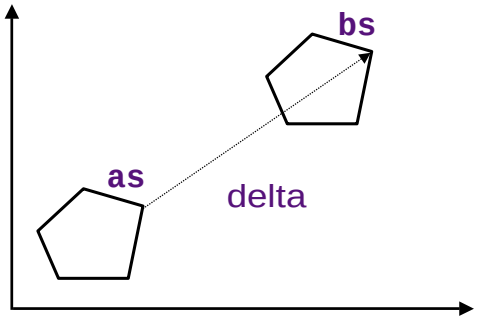
```
vscale a s = mapArray ((*) s)  a   ?
```

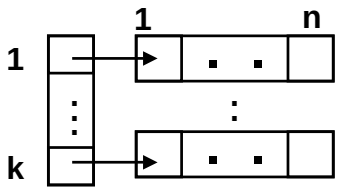

Dragging a Shape



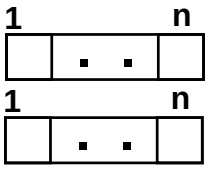
Move a k-sided polygon in an n-dimensional space by distance delta

k-sided polygon: An array of points





A point in n-dimensional space

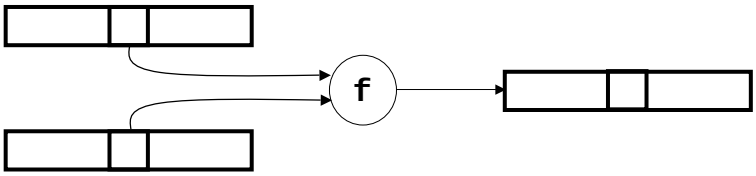


distance delta in n-dimensional space

```
move_shape as delta =
    mapArray (scale delta) as
```

?

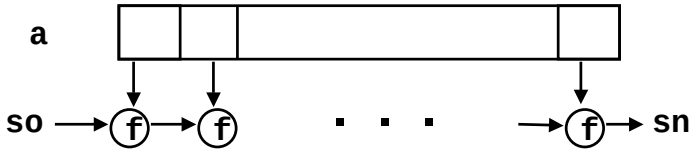
High-level Programming




```
mapArray2 f  a  b =
                  let
                      elem i = f (a!i) (b!i)
              in
                   mkArray (bounds a) elem
```

```
vs   = mapArray2  (+)
vvs  = mapArray2  vs
vvvs = mapArray2  vvs
...
```


Fold Array




```
foldArray a f so =
    let (l,u) = bounds a
         one_fold s i =
           if i > u then s
           else one_fold (f s (a!i)) (i+1)
    in
        one_fold so l
```

```
foldArray  a  (+)  0
foldArray  a  min  infinity
```


Inner Product: $\Sigma\, a_i\, b_i$

```
vp a  b  = let
               elem i = a!i * b!i
           in
               mkArray (bounds a) elem

ip a  b  = foldArray (vp a b) (+) 0
```


Index Type Class

pH allows arrays to be indexed by any type that can be regarded as having a contiguous enumerable range

```
class Ix a where
   range          :: (a,a) -> [a]
   index          :: (a,a) -> a -> Int
   inRange  :: (a,a) -> a -> Bool
```

range: Returns the list of *index* elements between a lower and an upper bound

index : Given a *range* and an *index*, it returns an integer specifying the position of the index in the range based on 0

inRange : Tests if an *index* is in the *range*

Examples of Index Type

```
data Day = Sun | Mon | Tue | Wed | Thu | Fri | Sat
```

An index function may be defined as follows:

```
index (Sun,Sat) Wed = 3
index (Sun,Sat) Sat = 6
   ...
```

A two dimentional space may be indexed as followed:

```
index ((li,lj), (ui,uj)) (i,j) =
              (i-li)*((uj-lj)+1) + j - lj
```

This indexing function enumerates the space in the *row major* order

Arrays With Other Index Types

```
module Array (Array, mkArray, (!), bounds)
   where

   infix 9 (!)

   data (Ix a) => Array a t
   mkArray  :: (Ix a) => (a,a) -> (a->t) ->
                                    (Array a t)
   (!)      :: (Ix a) => (Array a t) -> a -> t
    bounds  :: (Ix a) => (Array a t) -> (a,a)
```

Thus,

```
   type ArrayI  t = Array Int t
   type MatrixI t = Array (Int,Int) t
```


Higher Dimensional Arrays

```
x = mkArray ((l1,l2),(u1,u2)) f
```

means `x!(i,j) = f (i,j)` $l1 \leq i \leq u1$, $l2 \leq j \leq u2$

Type

```
x  ::(Array (Int,Int) t)
```

Assuming

```
f :: (Int,Int) -> t
```

mkArray will work for higher dimensional matrices as well.

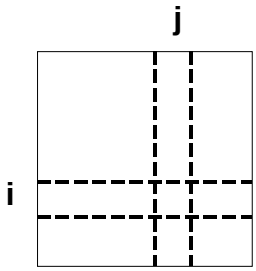
Array of Arrays

```
(Array a (Array a t)) ≠ (Array (a,a) t)
```

This allows flexibility in the implementation of higher dimensional arrays.

Matrices

```
add  (i,j) =  i + j

mkArray  ((1,1),(n,n))  add      ?
```



Transpose

```
transpose a =
     let
        ((l1,l2),(u1,u2)) = bounds a

        f (i,j) =  (j,i)              ?

     in
        mkArray  ((l2,l1),(u2,ui))  f
```


The *Wavefront* Example

$$x_{i,j} = x_{i-1,j} + x_{i,j-1}$$

1	**1**	**1**	**1**	**1**	**1**	**1**	**1**
1							
1							
1							
1							
1							
1							
1							

```
x = mkArray ((1,1),(n,n)) (f x)

f x (i, j) = if  i == 1 then 1
             else if  j == l  then  1
                  else x!(i-1,j) + x!(i,j-1)
```


Compute the least fix point.

⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥
⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥
⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥
⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥
⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥
⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥
⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥
⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥

1	**1**	**1**	**1**	**1**	**1**	**1**	**1**
1	⊥	⊥	⊥	⊥	⊥	⊥	⊥
1	⊥	⊥	⊥	⊥	⊥	⊥	⊥
1	⊥	⊥	⊥	⊥	⊥	⊥	⊥
1	⊥	⊥	⊥	⊥	⊥	⊥	⊥
1	⊥	⊥	⊥	⊥	⊥	⊥	⊥
1	⊥	⊥	⊥	⊥	⊥	⊥	⊥
1	⊥	⊥	⊥	⊥	⊥	⊥	⊥

```
x = mkArray ((1,1),(n,n)) (f x)

f x (i, j) = if  i == 1 then 1
             else if  j == l  then  1
                  else x!(i-1,j) + x!(i,j-1)
```