The Problem with Problems

BY ERIC MAZUR

In a previous column in *Optics & Photonics News* (March 1992), I wrote about the inefficiency of the passive lecturing method in introductory science classes. Here I address another inefficiency: textbook problems. Standard, end-of-chapter textbook problems can generally be solved by rote memorization of sets of formulas and so-called "problem-solving techniques." Often, students solve problems by identifying equivalent problems that they have solved before. Don't we want our students to be able to tackle new and less familiar terrain?

Enrico Fermi was well known for his legendary ability to solve seemingly intractable problems in subjects entirely unfamiliar to him, *e.g.*, "How many piano tuners in Chicago?" Such "Fermi problems" cannot be solved by deduction alone and require assumptions, models, order-of-magnitude estimates, and a great deal of self-confidence. We often use back-of-the-envelope estimates to familiarize ourselves with new problems.

So why do we keep testing our students with conventional problems that contain the same number of unknowns and givens and frequently require nothing but mathematical skills? What distinguishes the successful scientist is not the ability to solve an integral, a differential equation, or a set of coupled equations, but rather, the ability to develop models, to make assumptions, to estimate magnitudes, the very skills developed in Fermi problems.

Let me use a simple example to illustrate what I believe is a serious problem with standard physics textbook exercises. I purposely selected an example outside the realm of physics for the following reason: unless one has thought about this example, one is on equal footing with a student looking for the first time at a problem in a textbook. My example is based on a situation I encountered a while ago: I wanted to go shopping and pulled my car into a public parking lot near the stores. All spots were taken. Wanting to know if the best strategy was to roam around the lot or stay put in one spot, I decided to estimate the time I would have to wait if I stayed put. Using some rough estimates, I obtained a time of three minutes, and, sure enough, after roughly that time someone freed up a space for me.

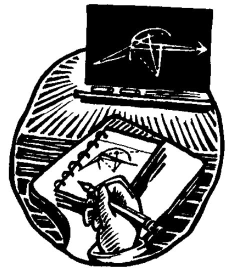
Suppose we turn this into a problem as follows:

On a Saturday afternoon, you pull into a parking lot with unmetered spaces near a shopping area. You circle around, but there are no empty spots. You decide to wait at one end of the lot where you can see (and command) about 20 spaces. How long do you have to wait before someone frees up a space?

This is a classic Fermi problem, requiring students to a) make assumptions, b) make estimates, c) develop a model, and d) work out that model. Putting a question like this on an exam would surely cause a revolt among students. Let's therefore turn it into a typical textbook problem by removing, one-by-one, requirements a through d. Because making assumptions typically is the last thing students are willing to do, let's start by making the assumption for them. This can be accomplished by adding a single sentence to the problem:

On a Saturday afternoon, you pull into a parking lot with unmetered spaces near a shopping area. You circle around, but there are no empty spots. You decide to wait at one end of the lot where you can see (and command) about 20 spaces. On average people shop for about two hours. How long do you have to wait before someone frees up a space?

The assumption that people shop for about two hours is rough, but certainly in the right ballpark. In this form the problem is still intractable to all but the best students because it presents an unfamiliar situation for which they have not yet developed (or seen) any model. So, let's simplify even more by implicitly stating the result one would get by statistically averaging over a large number of events:



On a Saturday afternoon, you pull into a parking lot with unmetered spaces near a shopping area. You circle around, but there are no empty spots. You decide to wait at one end of the lot where you can see (and command) about 20 spaces. On average people shop for about two hours. If people leave at regularly spaced intervals, how long do you have to wait before someone frees up a space?

In this form the problem still would not fly because it presents an unfamiliar situation and the model is not stated explicitly. In the standard textbook form this problem may, at first glance, not look very different from the one we started with:

On a Saturday afternoon, you pull into a parking lot with unmetered spaces near a shopping area where people are known to shop, on average, for about two hours. You circle around, but there are no empty spots. You decide to wait at one end of the lot where you can see (and command) about 20 spaces. How long do you have to wait before someone frees up a space?

The crucial point is, however, that somewhere in the book the students have encountered (and subsequently highlighted and memorized) the equation:

$$t_{wait} = \frac{1}{2}\frac{T_{shop}}{N_{spaces}}$$

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All that is left is for the students to classify the problem as pertaining to the above equation, plug in the numbers, and use their calculator! In four steps we have, "thrown out the baby with the bathwater." We have turned a question that requires a combination of skills relevant for solving the type of problems scientists face into one that requires hardly any skills at all. The original analytical challenges are now contained in the equation and in the statement of the problem. All opportunities to develop logical reasoning, to build confidence are lost.

The general idea illustrated by this example is this: Most textbook problems test mathematical, instead of analytical, thinking skills. Is this what we want to accomplish? In my opinion the numerical or algebraic answer to a problem and the mathematical manipulations that lead up to it are perhaps the least interesting aspect of problem solving—they should certainly not be ignored, but they shouldn't be the exclusive focus either. Even though we manage to produce first-rate scientists with the conventional way of teaching introductory science courses, those who currently succeed in the sciences may well do so in spite of the current educational system, not because of it. Standard textbook problems perpetuate the students' impression that science is a complicated web of facts, equations, and algorithms. We shouldn't be satisfied when a student just knows how to plug numbers into an equation in familiar situations, how to solve a differential equation, or how to recite a law of physics. We must insist on more meaningful problems that will better prepare our students for the demands of their future careers.

Eric Mazur is Gordon McKay Professor of Applied Physics and professor of physics at Harvard University. The above article is adapted from his forthcoming book entitled* Peer Instruction: A User's Manual, *which will be published by Prentice-Hall this summer. Visit Mazur's web site at http://www.harvard.edu.