6.976/ESD.937 Quantitative Foundations of Engineering Systems

Homework 1

Date Handed Out: Feb 22, 2006 *Due Date: Before Class, March 6, 2006*

1. (Architecture Design)[25 pts] Choose your favorite (complex) engineering system, different from the one covered in the class. Some examples of such system are network of health care system, network of railway system for Europe, network of Postal system, communication system for poor rural country, etc. After you select such system (not restricted to the above examples), carry out the following steps in detail:

(a) List three primary desired performance goals of the system.

(b) Derive three corresponding implications of the goals on the system architecture.

(c) Present a modular architecture design containing at least three modules. If you require fewer modules, provide appropriate justification.

2. (Explanation)[25 pts] Please answer the following questions regarding architecture design.

(a) Explain the rationale behind the packet-switched Internet architecture.

(b) Do you think packet switched architecture always improves resource utilization (you may use example to answer this)?

(c) How would you describe the architectural contribution of C. E. Shannon for Digital communication?

3. (Capacity Computation)[25pts] Consider the transmission over Internet. Everytime you send a packet, it is dropped with probability δ and with probability $1-\delta$ it reaches the destination. Suppose you are going to send L packets over the internet.

(a) What is the average number of packets that will reach the destination ?

(b) Compare answer of (a) with the channel capacity of the following channel, also known as *L bit erasure channel*. Input set $\mathcal{X} = \{0,1\}^L$ and output set $\mathcal{Y} = \{0,1,\phi\}^L$. For each channel transmission $x \in \{0,1\}$, $\Pr[x|x] = 1-\delta$ and $\Pr[\phi|x] = \delta$. The probability of received output depends only on the transmitted input symbol at that time and is independent of all other transmissions.

Hint: To answer (b), you need to evaluate the channel capacity formula that was given in the class or can be found in any standard information theory text.

4. (Kuhn-Tucker conditions [25pts]) Consider the following optimization problem:

$$\begin{aligned} & \arg\max f_0(\mathbf{x}) \\ \text{subject to} \quad & f_i(\mathbf{x}) \leq 0, 1 \leq i \leq m \end{aligned}$$

where $\mathbf{x} \in \mathbf{R^n}$ and $f_j(\mathbf{x}), 0 \leq j \leq m$ are twice continuously differentiable functions from $\mathbf{R^n} \rightarrow \mathbf{R}$. Let $\mathbf{x}^*$ be an optimal point. Let $I(\mathbf{x}^*) = \{i | f_i(\mathbf{x}) = 0, 1 \leq i \leq m\}$, that is, the subset of constraints which are satisfied with equality at $\mathbf{x}^*$. In the class we studied that if the *constraint qualification* condition holds, the Kuhn - Tucker conditions're satisfied, that is,

$$\exists \lambda_i \geq 0 \quad \text{such that} \quad \nabla f_0(\mathbf{x}^*) = \sum_{i \in I(\mathbf{x}^*)} \lambda_i \nabla f_i(\mathbf{x}^*)$$

Give an example where the *constraint qualification* condition does not hold, and the Kuhn-Tucker conditions are not satisfied. This shows that the constraint qualification condition is required, though it might not be necessary. Give another example where the constraint qualification does not hold, but at an optimal point, the Kuhn-Tucker conditions are satisfied.