Homework 2

Date Handed Out: March 21, 2006 *Due Date: Before Class, April 5, 2006*

1. (Observed Optimality of Routing)[25 pts] Consider a simple network of two nodes **S** and **T**. Both nodes are connected by two distinct links e_1 and e_2 with capacity 1 each. The node **S** wishes to send $\rho < 1$ amount of data to **T** over this network. Let $D_{e_i}(x)$ be the delay experienced per unit flow when there is x amount of flow on link $e_i, i \in \{1, 2\}$. An example of such functions: $D_{e_1}(x) = x$ and $D_{e_2}(x) = 2x^2$ for all $x \leq 1$.

Prove or produce a counter-example (by exhibiting example of delay functions) to the following statement: *For any $\rho < 1$, let x be such that $D_{e_1}(x) = D_{e_2}(\rho - x)$. Then routing x amount of data on link e_1 and $\rho - x$ amount of data on e_2 always minimizes the total delay, which is defined as $D = xD_{e_1}(x) + (\rho - x)D_{e_2}(\rho - x)$.*

2. (Network Planning)[25 pts] We will consider the task of building a network, similar to the one described in problem **1**. Specifically, we wish to provision for certain amount of capacity on links e_1 and e_2 so that **S** can send unit (i.e. $\rho = 1$) amount of data to **T**. Let the cost of building links e_1 and e_2 of capacities y_1 and y_2 respectively be y_1^2 and y_2^4 respectively. Let the net-delay on link e_1 be

$$D_1(x_1, y_1) = \frac{y_1}{y_1 - x_1},$$

when link e_1 has capacity y_1 and x_1 amount of data is routed through it. Similarly, for link e_2 the net-delay function be

$$D_2(x_2, y_2) = \frac{2y_2}{y_2 - x_2},$$

when link e_2 has capacity y_2 and x_2 amount of data is routed through it. Design a network so that net-delay for each link is no more than 0.5 and cost of building the network is minimized. That is, find values $y_1^*, y_2^*, x_1^*, x_2^*$ corresponding to link capacities and amount of data routed on the link that satisfies the desired properties.

3. (Optimization)[50pts] Define $f_1(s) = s(1 - s)$, $f_2(s) = s(2 - 3s)$ and $f_3(s) = s(2 - 5s)$. Now, consider the following optimization problem OPT:

$$\max \quad \sum_{i=1}^{3} f_i(x_i),$$

$$\text{Subject to} \quad \sum_{i=1}^{3} x_i = 1.$$

Answer the following questions:

(a) Solve the above problem by the method of Lagrange Multipliers.

(b) Write the Gradient Descent Method for the above problem. You don't need to execute it.

(c) Write down the Dual problem.

(d) Write the Gradient descent method for the dual. You don't need to execute it.