Problem 1: Kinematics (15 pts)

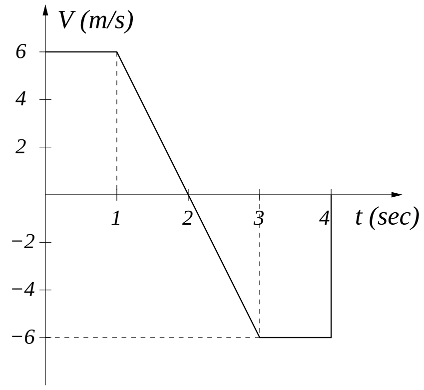
A particle moves along a *straight line* x. At time $t = 0$, its position is at $x = 0$. The *velocity*, V, of the object changes as a function of time, t, as shown in the figure. V is in m/s, x is in meter, and t is in seconds.

(a) What is x at $t = 1$ sec?

(b) What is the *acceleration* (m/s^2) at $t = 2$ sec?

(c) What is x at $t = 4$ sec?

(d) What is the *average speed* (m/s) between $t = 0$ and $t = 3$ sec?



Solution:

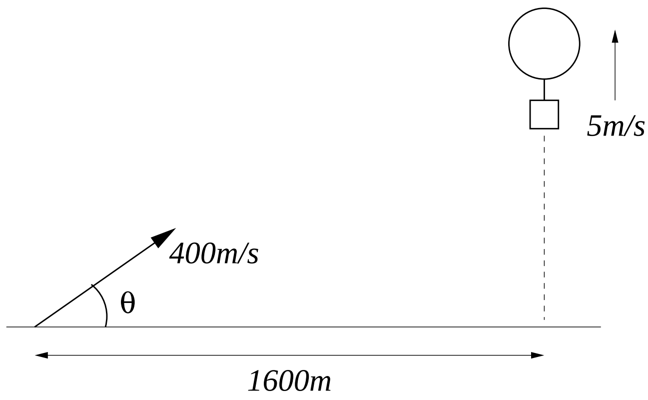
(a) From the area blow the velocity curve, we find $x = 6$m at $t = 1$.

(b) From the slope of the velocity curve at $t = 2$, we find $a = (-6 - 6)/2 = -6m/s^2$.

(c) The area below the $v = 0$-line has a negative contribution to the displacement. We find $x = 0$ at $t = 4$.

(d) The area below the $v = 0$-line has a negative contribution to the total distance traveled. Average speed $=(6{+}3{+}3)/3 = 4m/s$. (Different from average velocity which is $(6{+}3{-}3)/3 = 2m/s$)

Problem 2: Surveillance Balloon (15 pts)
A gun crew observes a remotely controlled balloon launching an instrumented spy package in enemy territory. When first noticed the balloon is at an altitude of 800m and moving vertically upward at a *constant velocity* of 5m/s. It is 1600m down range. Shells fired from the gun have an initial velocity of 400m/s at a *fixed angle* θ ($\sin\theta = 3/5$ and $\cos\theta = 4/5$). The gun crew (using its 8.01 ballistic knowledge) *waits* and *fires* so as to destroy the balloon. Assume $g = 10m/s^2$. Neglect air resistance.
(a) What is the *flight time* of the shell before it strikes the balloon?
(b) What is the *altitude* of the *collision*?
(c) How long did the gun crew wait before they fired?



Solution:

(a) The motion in the x-direction is a constant velocity motion. We find the flight time $= 1600m/v_x = 1600/(400\cos\theta) = 1600/(1600/5) = 5sec.$
Flight time $= 5sec.$

(b) From the flight time, the initial velocity in the y-direction and the acceleration in the y-direction, we can calculate the altitude of the shell: $h = v_y t - \frac{1}{2}gt^2 = \frac{1200}{5}\times 5 - \frac{1}{2}\times 10\times 25 = 1200 - 125 = 1075m.$
Altitude $= 1075m.$

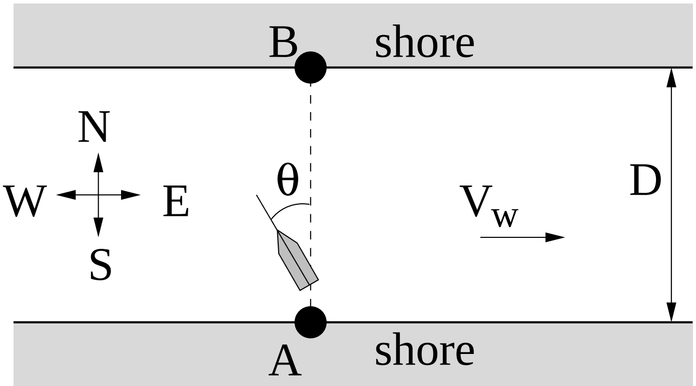
(c) After the waiting time plus the flight time, the balloon should reach the same altitude as the shell. Let t_w be the waiting time. We have $h = (t_w + 5)\times 5 + 800 = 1075.$
$t_w + 5 = 275/5 = 55sec.$ So $t_w = 50sec.$
The waiting time $= 50sec.$

Problem 3: Crossing a river (25 pts)

Two ports, A and B, on a North-South line are separated by a river of with D. The river flows east with speed V_W. A boat cross the river from port A to port B. The speed of the boat relative to the water is V_B. Assume $V_B = 2V_W$. State all your answers in terms of V_B and D.

(a) What is the *direction* of the boat, θ, relative to the North so that it crosses directly on a line from A to B? How long does the trip take?

(b) Suppose the boat wants to cross the river from A to the other side in the *shortest* possible *time*. What *direction* should it head? (Hint: Think carefully about what this means.) How *long* does the trip take? How *far* is the boat from the port B after crossing?



Solution:

(a) To reach the port B, the x-component of the total velocity must be zero: $V_B \sin\theta - V_w = 0$. So $\sin\theta = 1/2$.

The y-component of the total velocity is $V_B \cos\theta$. So $t = \frac{D}{V_B \cos\theta} = \frac{2D}{V_B\sqrt{3}}$.

The direction θ is 30° relative to the North, or 30° West of the North.

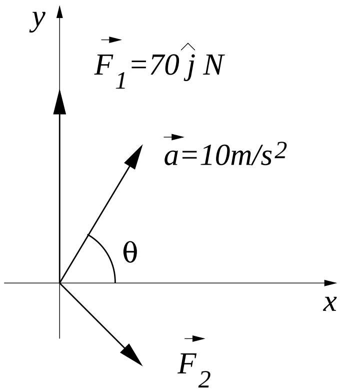
The trip takes $t = \frac{2D}{V_B\sqrt{3}}$.

(b) To cross the river the fastest, we need to maximize the the y-component of the total velocity is $V_B \cos\theta$. So $\theta = 0$. The boat should head straight to the North.

The trip takes $t = \frac{D}{V_B}$.

The y-component of the total velocity is V_W. So the boat is a distance $tV_W = DV_W/V_B$ down stream from the port B after crossing.

Problem 4: Force and Acceleration (25 pts)
A particle of mass $m = 5$kg, is momentarily at rest at $x = 0$ at $t = 0$. It is acted upon by two forces $\vec{F}_1$ and $\vec{F}_2$. $\vec{F}_1 = 70\hat{j}$N. The *direction* and *magnitude* of $\vec{F}_2$ are *unknown*. The particle experiences a *constant acceleration*, $\vec{a}$, in the direction as shown. Note: $\sin\theta = 4/5$, $\cos\theta = 3/5$, and $\tan\theta = 4/3$. Neglect gravity.
(a) Find the missing force $\vec{F}_2$. Either give *magnitude* and *direction* of $\vec{F}_2$ *or* its components. Plot $\vec{F}_2$ on the figure. What angle does $\vec{F}_2$ make to the x-axis?
(b) What is the *velocity vector* of the particle at $t = 10$sec?
(c) What *third* force, $\vec{F}_3$, is required to make the acceleration of the particle zero? Either give *magnitude* and *direction* of $\vec{F}_3$ *or* its components.
(d) What is the *vector sum* of the three forces: $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 =$?



Solution:

(a) $F_{2x} = ma_x - F_{1x} = 5 \times 10 \times cos\theta - 0 = 30N$.
$F_{2y} = ma_y - F_{1y} = 5 \times 10 \times \sin\theta - 70 = -30N$.
$\vec{F}_2 = (30, -30)N$.

(b) $\vec{v} = 10 \times \vec{a} = (60, 80)m/s$, or
$|\vec{v}| = 100m/s$ with direction θ relative to x-axis.

(c) The force $\vec{F}_3$ cancel the total acceleration. So $\vec{F}_3 = -m\vec{a}$.
$\vec{F}_2 = (-30, -40)N$.

(d) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (0, 70) + (30, -30) + (-30, -40) = (0, 0) = 0$.