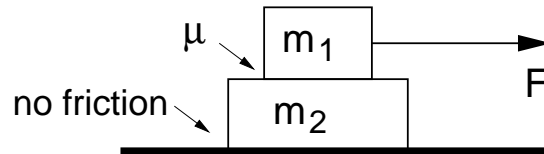


Problem 1: Dynamics (15 pts)

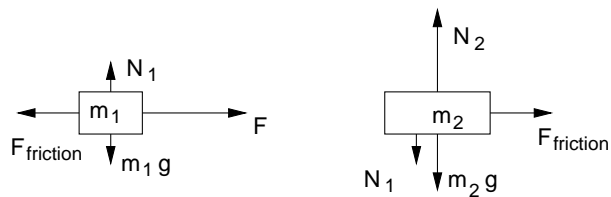
Two blocks of mass m_1 and m_2 are put on a frictionless level surface as shown in the figure below. The static coefficient of friction between the two blocks is μ . A force F acts on the top block m_1 .



- (a) When the force F is small, the two blocks move together. Draw the free-body diagrams of the block m_1 and the block m_2 .
- (b) Find the acceleration of the two blocks for small F .
- (c) Find the magnitude of the force F above which the block m_1 starts to slide relative to the block m_2 .

Solution:

(a)



(b) $a = F/(m_1 + m_2)$

(c) m_1 start to slide when

$$F_{friction} = \mu m_1 g = m_2 a = F m_2 / (m_1 + m_2).$$

We find

$$F = \frac{\mu m_1 g (m_1 + m_2)}{m_2}$$

Problem 2: Circular motion (15 pts)

A car of mass $m = 1000kg$ is traveling around a flat circular race track of radius $100m$. The static coefficient of friction between the tire and the road (against transverse motion) is $\mu = 0.5$. (Assume $g = 10m/s^2$)

- (a) How fast can the car travel before it starts to skid? Express the speed in the units of m/s .
- (b) What is the angular velocity ω of the car at the speed calculated in (a).
- (c) The driver of the car wants to drive faster. He loads $500kg$ of weight into the car to increase the friction force. Now how fast can the car travel without skidding?

Solution:

- (a) The max speed of the car should satisfy

$$m \frac{v^2}{r} = \mu mg$$

We find

$$v = \sqrt{\mu gr} = \sqrt{0.5 * 100 * 10} = 10\sqrt{5}m/s = 22m/s$$

- (b) The angular velocity is

$$\omega = v/r = 0.22/s$$

- (c) v does not depend on the mass. So the max speed is not changed.

Problem 3: Balance and energy (15 pts)

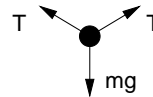
A block of mass m is tied to two strings as shown in the figure below. Each string has a length L . The angle $\theta = 30^\circ$. ($\sin \theta = 1/2$ and $\cos \theta = \sqrt{3}/2$.) Assume the strings are massless.



- Draw the free-body diagram of the block.
- Find the tension of each string.
- We cut one string and the block starts to swing down. Find the speed of the block when it reaches the lowest point.
- Find the tension in the string when the block reaches the lowest point.

Solution:

(a)



(b) From the balance of the force in the vertical direction, we find

$$mg = 2T \sin \theta = T$$

Thus $T = mg$.

(c) The change in the potential energy is $mg(L - L \sin \theta) = mgL/2$. Thus

$$\frac{1}{2}mv^2 = \frac{mgL}{2}$$

We find

$$v = \sqrt{gL}$$

(d) The tension minus weight should provide the acceleration for the circular motion:

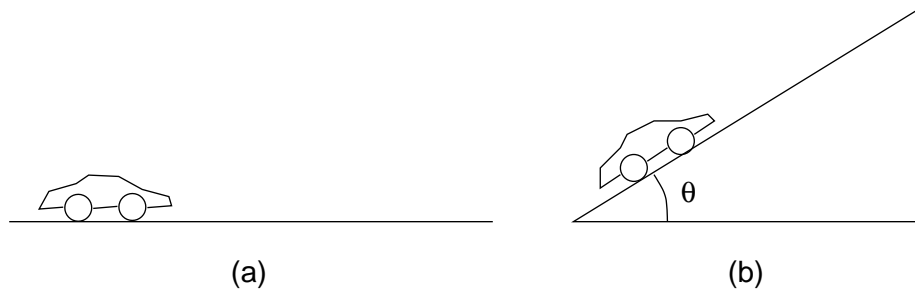
$$T - mg = m\frac{v^2}{L}$$

So

$$T = mg + mg = 2mg$$

Problem 4: Power

A small car's engine can deliver $90kW$ of power (about 120hp). The car's mass is $1000kg$. (Assume $g = 10m/s^2$)



(a) Assume the total resistive force is proportional to the velocity: $F_{friction} = \alpha v$. The drag coefficient α is $\alpha = 100N/m$. How fast can the car move on a level road? Express the speed in the units of m/s .

(b) How fast can the car travel up a slope if we ignore all friction? The angle of the slope is θ ($\sin(\theta) = 3/5$ and $\cos(\theta) = 4/5$). Express the speed in the units of m/s .

Solution:

(a) From $P = vF_{friction}$, we find $P = \alpha v^2$ or

$$v = \sqrt{P/\alpha} = \sqrt{90000/100} = 30m/s = 108km/hr$$

(b) From $Work = P * \Delta t = mg\Delta h = mg\Delta x \sin \theta = mgv\Delta t \sin \theta$, we find $P = mgv \sin \theta$ or

$$v = \frac{P}{mg \sin \theta} = \frac{90000}{1000 * 10 * 3/5} = 15m/s = 54km/hr$$