Massachusetts Institute of Technology

Physics Department

8.01 **Fall 2004**

Exam 3 - Solutions

1. Collision (15 points)

a. (5 points)

$$\left.\begin{array}{ll} \text{along x}: & p_x = 2Mv\cos\theta + M\left(\frac{3}{2}v\right)\cos\phi = Mv\left(\frac{8}{5}+\frac{9}{10}\right) = \frac{5}{2}Mv \\ \text{along y} & p_y = -2Mv\sin\theta + M\left(\frac{3}{2}v\right)\sin\phi = Mv\left(\frac{6}{5}-\frac{12}{10}\right) = 0 \end{array}\right\}$$

b. (5 points) The total linear momentum along the *y* axis is zero. If the collision were anything other than completely inelastic, mass *2M* would be moving in the +*y* direction and mass *M* would move in the would move in the –*y* direction. So the collision is completely inelastic. To find the momentum for mass *M* after the collision, subtract the momneumt for mass *2M* after the collision from the total momentum before the collision: $\frac{5}{2}Mv - \frac{5}{4}2M = 0$ and the velocity of mass *M* after the collision is zero.

c. (15 points) The velocity of the center of mass along *y* is zero. For *x* is is *u* such that $2M(v\cos\theta - u) + M(v\cos\phi - u) = 0 = \frac{5}{2}Mv - 3Mu \Rightarrow u = \frac{5}{6}v$.

2. Colliding bullet (15 points)

a. (5 points) The change in momentum of the bullet is $\Delta p = m\left(v - \frac{v}{2}\right) = \frac{mv}{2}$ and this impulse is delivered to the pendulum, so $v_p = \frac{\Delta p}{M} = \frac{mv}{2M}$.

OR

The momentum before the collision is $L = Mvl$. Angular momentum is conserved over the very short duration of the collision, we have

$$L = Mvl = mv_p l + \frac{Mvl}{2} \Rightarrow v_p = \frac{mv}{2M}.$$

b. (5 points) At the top of the circle, the pendulum has potential energy $U = Mg(2l)$ which must be equal to the initial kinetic energy

$$\frac{1}{2}mv_p^2 = 2mgl \text{ and } v_p = \frac{mv}{2M} > 2\sqrt{gl} \Rightarrow v > \frac{4M}{m}\sqrt{gl}.$$

c. (3 points) At the top, v_p=0, so there is no centripetal acceleration. The rod is then only under compression due to gravity and $|T|=Mg$.

d. (2 points) The acceleration is zero.

3. Rotational dynamics (15 points)

a. Call the position of the pulley *A y* which which increases when *A* goes up.
$R\alpha_B = -a_y - R\alpha_A$

b. The angular acceleration of *B* $I_B\alpha_B = \tau = TR \Rightarrow \alpha_B = \frac{TR}{I_B}$, for *A* the angular accelertion is $I_A\alpha_A = TR \Rightarrow \alpha_A = \frac{TR}{I_A}$ and the linear acceleration is *A* is $a_y = \frac{T}{M} - g$. $I_A = \frac{1}{2}MR^2$ and $I_B = MR^2$. Then

$$R\left(\frac{TR}{MR^2}\right) = -\left(\frac{T}{M} - g\right) - R\left(\frac{TR}{\frac{1}{2}MR^2}\right) \Rightarrow \frac{T}{M} + \frac{T}{M} + \frac{2T}{M} = g \Rightarrow T = \frac{Mg}{4}.$$

c. $a_y = \frac{T}{M} - g = g\left(\frac{1}{4} - 1\right) = -\frac{3g}{4}$

d. $\alpha_A = \frac{Mg}{4}\frac{R}{\frac{1}{2}MR^2} = \frac{1}{2}\frac{g}{R}$ and $\alpha_B = \frac{Mg}{4}\frac{R}{MR^2} = \frac{g}{4R}$.

4. Multiple choice (15 points)

a. The total impulse is

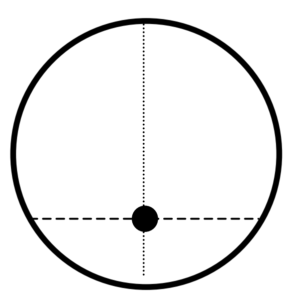
$$\Delta p = \int_0^4 F_x(t)dt = -8Ns - 4Ns + 16Ns = 4kg - m/s \Rightarrow \Delta v = 2m/s \Rightarrow v(4s) = 4m/s.$$

so A is the correct answer. Explicitely, $\begin{matrix} F(t) = -8N & 0 < t < 1s \\ F(t) = 8\frac{N}{s}t - 16N & 1s < t < 4s \end{matrix}$,

then
$$\int_0^4 F(t)dt = \int_0^1 F(t)dt + \int_1^4 F(t)dt = \left(-8 + 8\frac{t^2}{2}\Big|_1^4 - 16t\Big|_1^4\right)\frac{kg-m}{s}$$
$$= \left(-8 + 4(16-1) - 16(4-1)\right)\frac{kg-m}{s} = 4\frac{kg-m}{s}.$$

b. The easiest way to think about this is in the center of mass frame in which the masses approach each other with each and opposite mopmenta. In order to conserve the total momentum, after the collision, they must have uqal and opposite momenta. The impulse is just the change in momentum for each mass and hence must be equal and opposite for all collisions, answer b.

c. Easiest to make a sketch: The center of mass is halfway down the line connecting



the two cars *and* $v_{cm} < v$, so b.

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