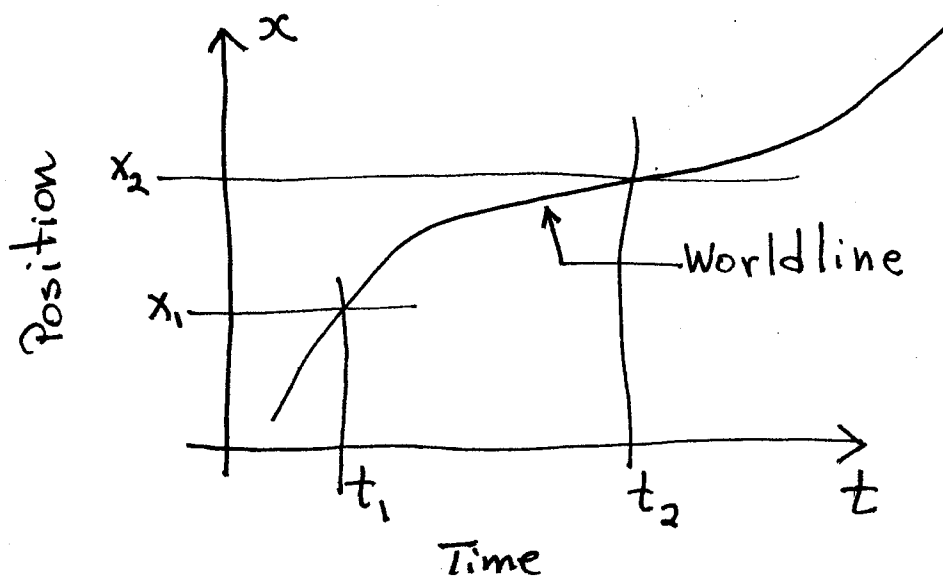


## Kinematics

2-1

- Describes the geometry of motion of a particle
- Uses mathematics to describe the motion in terms of position, velocity and acceleration
- Introduction to dynamics  $\Rightarrow$  study of why things move.
- Next few lectures will involve a study of translational motion.
- Simplify the physics: use concept of an ideal particle:
  - no size
  - no internal structure
  - mass
  - position as a function of time gives a complete description
- world line  $\rightarrow$  position vs time



Speed

Consider a particle moving along a worldline  
 - straight  
 - curved

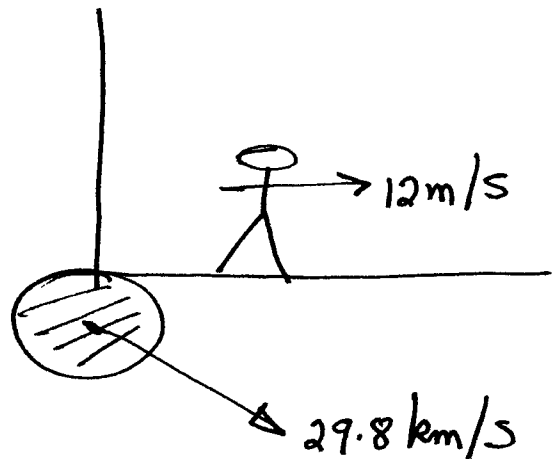
$$\text{Average Speed} = \frac{\text{distance travelled}}{\text{time taken}} \quad \left[ \frac{L}{T} \right]$$

$$s = \frac{d}{t} > 0 \text{ always} \quad \begin{array}{l} (\text{m/s}) \\ (\text{ft/s}) \end{array}$$

Speeds

Light	$3 \times 10^8 \text{ m/s}$
Sound	330
Man	12
Glacier	$10^{-6}$
Continental Drift	$10^{-9}$

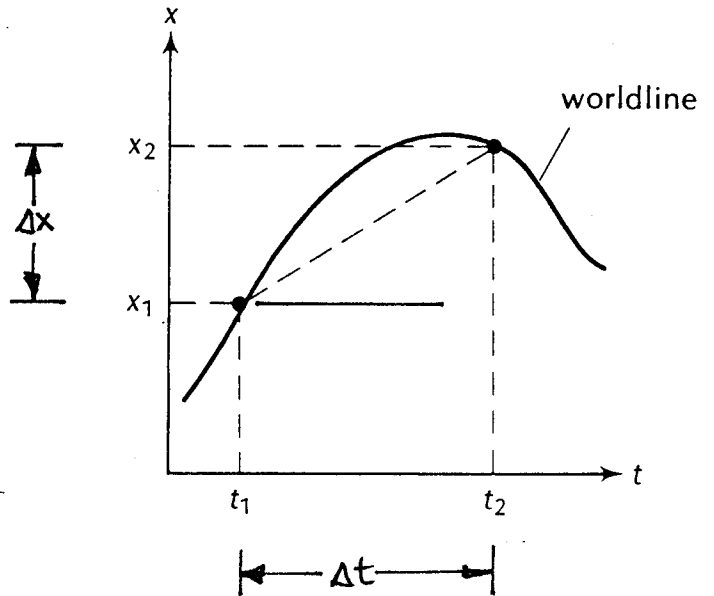
Motion and speed are relative  
 Depend on the frame of reference where they  
 are calculated.



## Average Velocity

2-3

- 1 Dimension look at motion of a particle moving along a straight line.



- if position of particle changes with time, it is moving  $\rightarrow$  velocity.

- We define an average velocity for the particle

$$\bar{v} = \frac{\text{change in position} \leftarrow \text{displacement}}{\text{change in time}}$$

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

$\Delta x$   $\leftarrow$  change in position  
 $\Delta t$   $\leftarrow$  change in time  
 $\pm$  sign of velocity indicates direction of motion

$$\bar{v} = \text{slope of straight line connecting points } (x_1, t_1) \text{ and } (x_2, t_2).$$

$$\bar{v} > 0 \quad \text{motion to right along } x\text{-axis}$$

$$\bar{v} < 0 \quad \text{motion to left along } x\text{-axis}$$

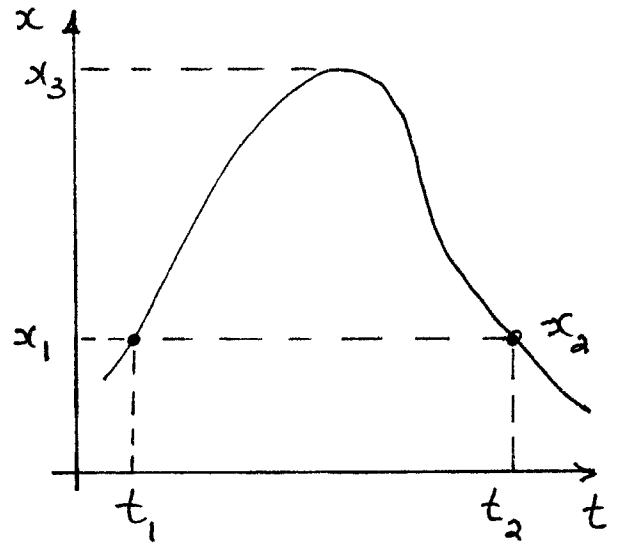
$$\text{average speed} = \frac{\text{distance travelled}}{\text{time}}$$

$$\text{average velocity} = \frac{\text{change in position}}{\text{time}}$$

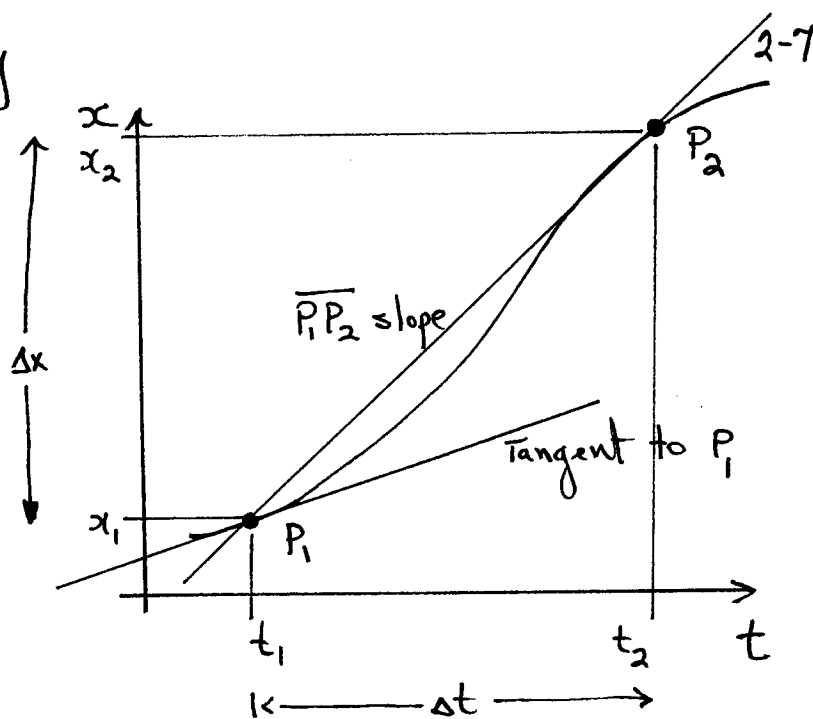
Example

$$\bar{s}_{12} = \frac{x_3 - x_1}{t_2 - t_1} \neq 0$$

$$\bar{v}_{12} = \frac{x_2 - x_1}{t_2 - t_1} = 0$$



## Instantaneous Velocity



Slope of the line  $\overline{P_1P_2}$  represents the average velocity  $\bar{v}$  between  $t_1$  and  $t_2$ .

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

Q: What is the velocity, exactly at  $P_1$ ?

Pick a new point closer to  $P_1$  to get a better measure.

$$\left. \begin{array}{l} \text{Reduce } \Delta t \\ \text{Reduce } \Delta x \end{array} \right\} \bar{v} = \frac{x_i - x_1}{t_i - t_1} = \frac{\Delta x_i}{\Delta t_i}$$

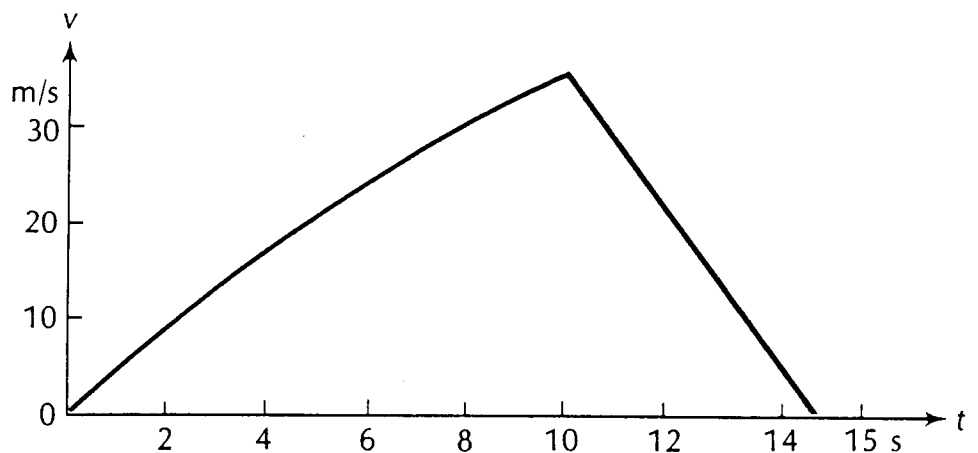
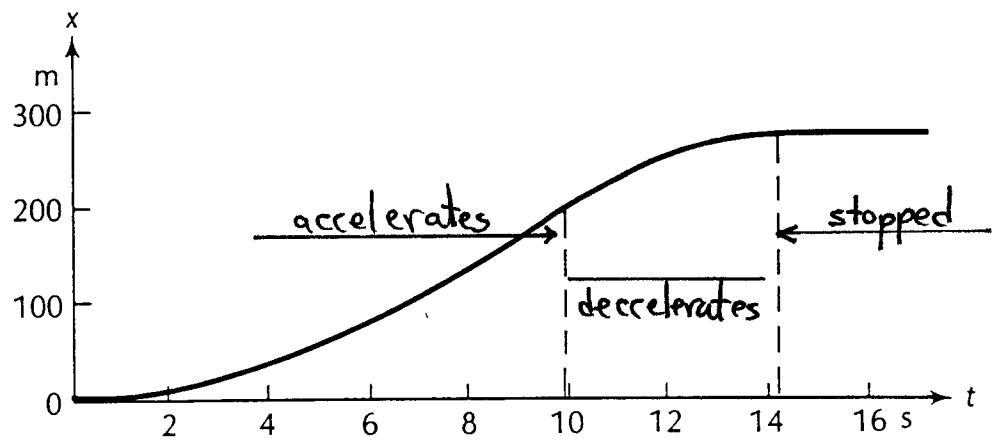
Instantaneous velocity  $\equiv$  velocity defined as the limit as we let  $\Delta t \rightarrow 0$ .

It is equal to the slope of the tangent to the curve at the point.

$$v(t) = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

↑ calculus

- velocity is the general time derivative of the position function.



Example: Racing car.

- Position vs. time.
- Resulting velocity vs. time.

Note: Neither  $\bar{v}(t)$  nor  $v(t)$  depend on the choice of a coordinate system (if there is no relative motion) since they involve only differences in position.

↔ in variant to choice of origins / systems

### Example

Ideal particle moving in a straight line with position given by:

$$x = 2.1 t^2 + 2.80 \quad (\text{m}) \quad t \rightarrow \text{sec.}$$

Q: What is average velocity between  $t_1 = 3.0\text{s}$  and  $t_2 = 5.0\text{s}$ ?

$$t_1 = 3.0\text{s} \quad x_1 = 2.1 (3.0)^2 + 2.80 = 21.7\text{m}$$

$$t_2 = 5.0\text{s} \quad x_2 = 2.1 (5.0)^2 + 2.80 = 55.3\text{m}$$

$$\text{Average } \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{55.3 - 21.7}{5.0 - 3.0} = 16.8 \text{ m/s}$$

### Instantaneous $v$ ?

$$\textcircled{1} \quad x = 2.1 t^2 + 2.80$$

$$x + \Delta x = 2.1 (t + \Delta t)^2 + 2.80$$

$$\textcircled{2} \quad x + \Delta x = 2.1 t^2 + 4.2 t (\Delta t) + 2.1 (\Delta t)^2 + 2.80$$

$$\textcircled{3} - \textcircled{1} \quad \Delta x = 4.2 t (\Delta t) + 2.1 (\Delta t)^2$$

$$\bar{v} = \frac{\Delta x}{\Delta t} = 4.2 t + 2.1 (\Delta t)$$

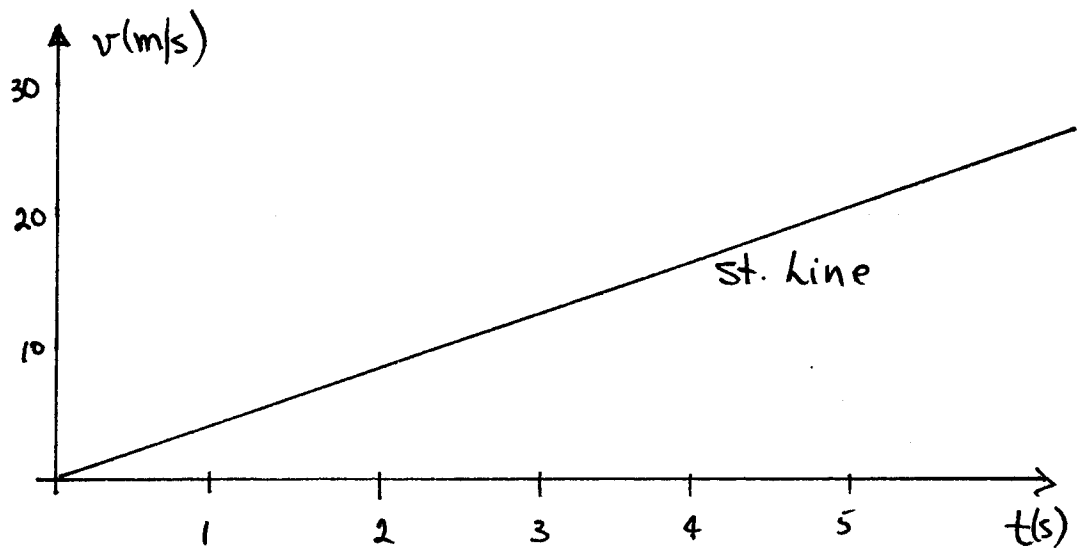
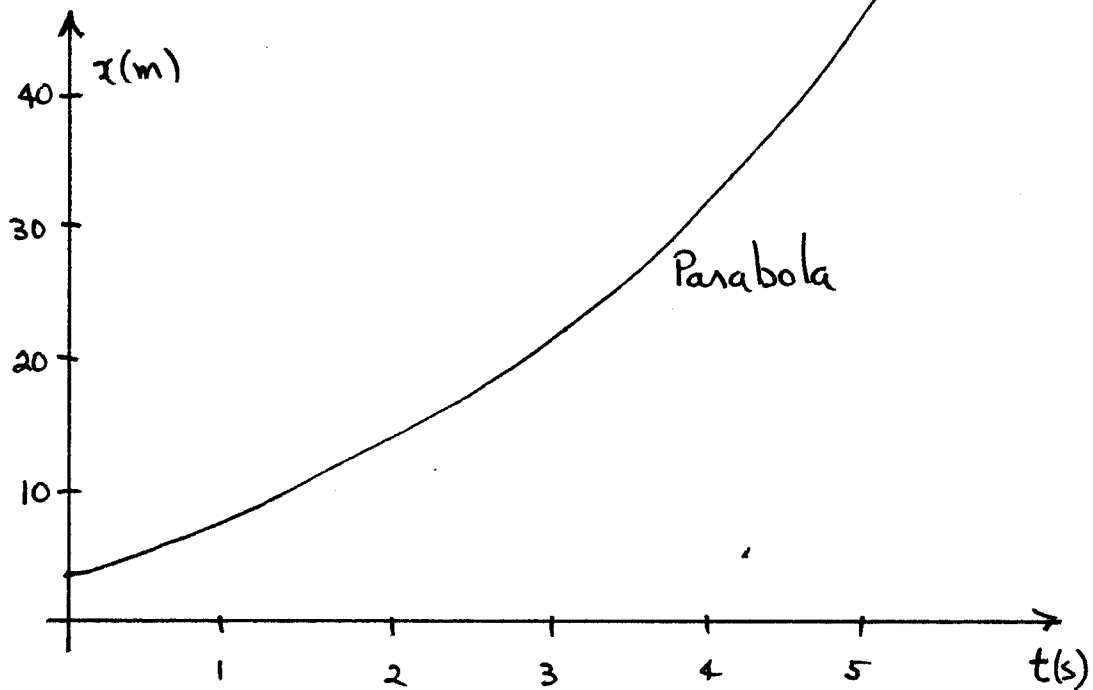
$$\text{Instantaneous } v(t) = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = 4.2 t$$

$$\begin{aligned} v(t=5) &= 4.2 \times 5 = 21.0 \text{ m/s} \\ v(t=3) &= 4.2 \times 3 = 12.6 \text{ m/s} \end{aligned} \left. \vphantom{\begin{aligned} v(t=5) \\ v(t=3) \end{aligned}} \right] 16.8 \text{ m/s} !!$$

Using calculus

$$x = 2.1t^2 + 2.80$$

$$v(t) = \frac{dx}{dt} = 4.2t + 0.$$



## Constant Velocity Motion

2-11

- particle moves with a position-time dependence which is a st. line.
- $\therefore$  slope of  $x(t) \equiv$  constant

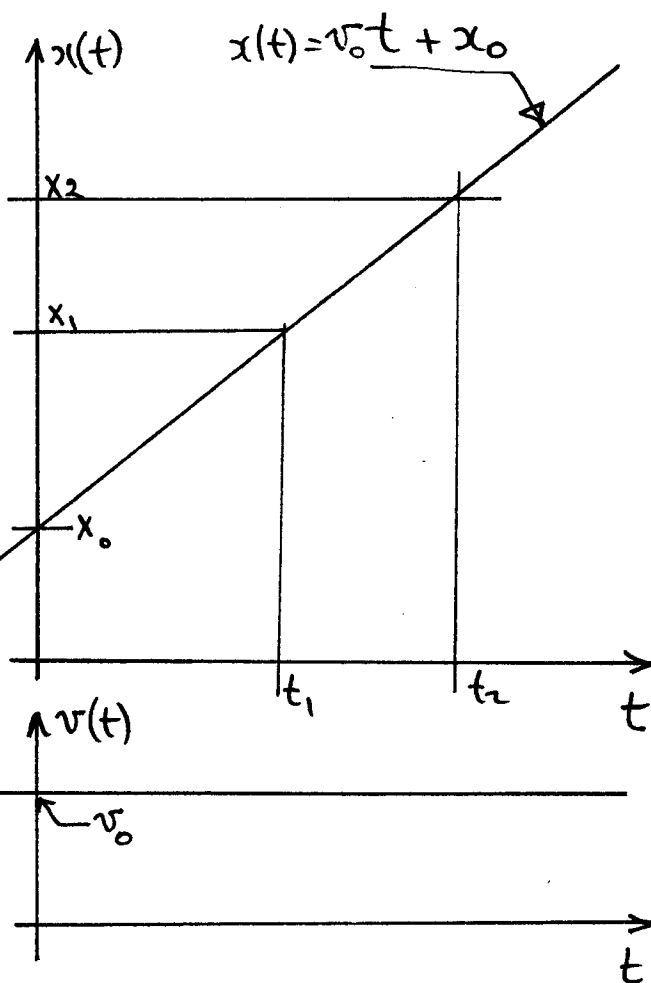
$$\bar{v} = \frac{\Delta x}{\Delta t} = \text{constant} = v_0$$

Also

$$v(t) = \frac{dx}{dt} = \text{constant} \equiv v_0$$

$$\bar{v} = v$$

average  $\equiv$  instantaneous!!



- Motion at constant velocity is called uniform linear motion.

Let  $\bar{v} = v(t) = v_0$ ; a constant.

Suppose at time  $t=0$  the position of the particle is at  $x=x_0$ . Then at any time  $t$  its position is at  $x(t)$ .

$$\therefore \bar{v} = v_0 = \frac{x(t) - x_0}{t - 0}$$

$$\therefore x(t) = x_0 + v_0 t$$

Eg. of a st. line.

$\Rightarrow$  General Desc. for 1-Dim. motion at Const.  $v$ .

# Speed of Rifle Bullet

2-12

[15-20] min

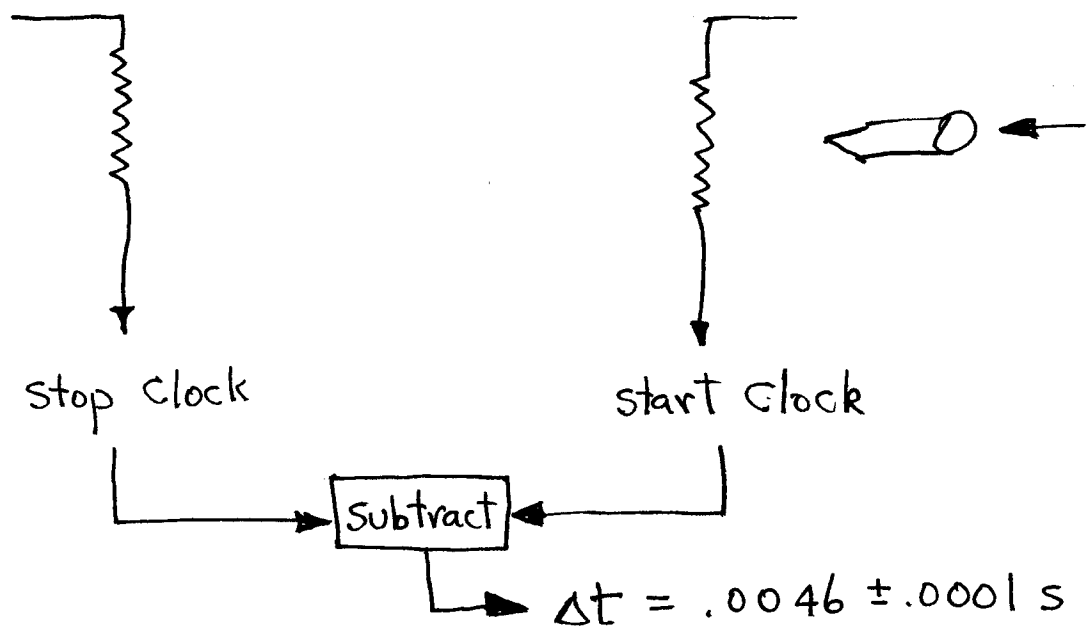
- High speed
- Need to use special techniques

$$s = \frac{d}{t} = \frac{\text{distance}}{\text{time}} = \text{speed}$$

Distance : 1.50 m  $\pm$  .005 (meter-stick)

Time: 2 methods

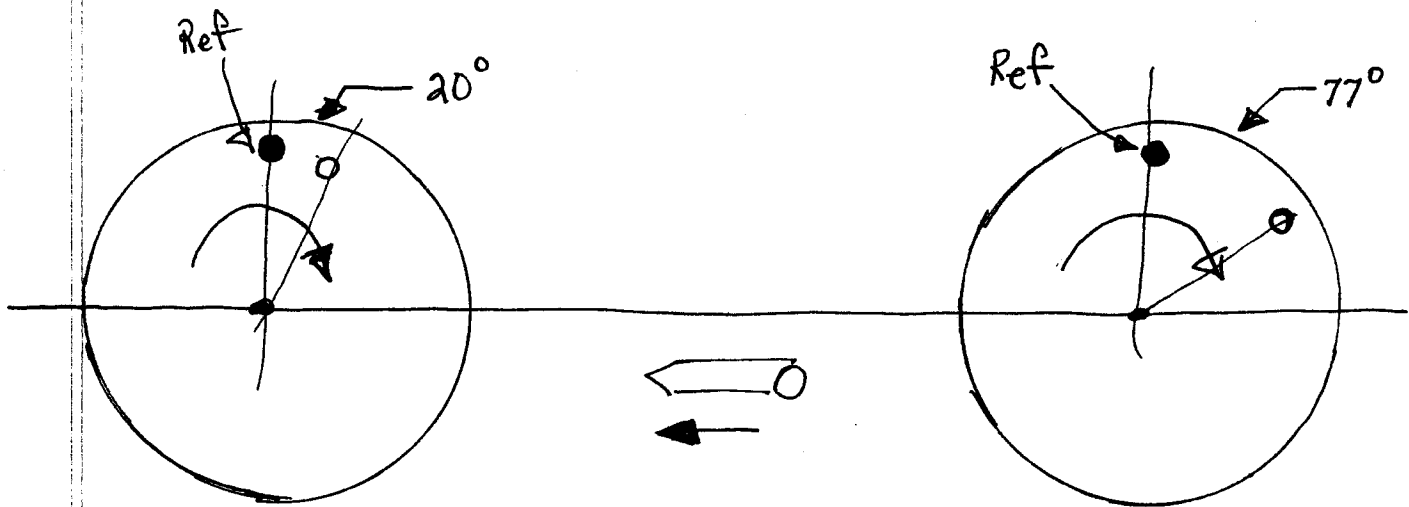
Method I: Direct Timing



$$s = \frac{1.50}{.0046} = 326 \text{ m/s}$$

Sound = 330 m/s





Time-of-Flight

$$\begin{aligned}\Delta t &= \frac{\Delta \theta}{360^\circ} \times .0293 \\ &= \frac{77^\circ - 20^\circ}{360^\circ} \times .0293 \\ &= .0046 \text{ s}\end{aligned}$$

$$s = \frac{d}{\Delta t} = \frac{1.50 \text{ m}}{.0046 \text{ s}} = 323 \text{ m/s}$$

Uncertainty / Errors

- Time measurement of shaft rotation

$$\Delta t_r = .0001$$

< 1/2%

- Location of Holes (angle measurement)

$$\Delta \theta \sim (5 \rightarrow 10)\%$$

- Length measurement

$$\Delta L \sim .01$$

< 1%